Entrance exam for the PhD program in Theoretical Particle Physics

1. Consider the Standard Model of Electoweak interactions (with gauge group $SU(2) \times U(1)_Y$) in the limit $g' \to 0$, where g' is the $U(1)_Y$ gauge coupling. Assume that the Higgs mass satisfies the condition $m_h > 2m_Z, 2m_W$. What will be the ratio of the decay width $\frac{\Gamma(h \to ZZ)}{\Gamma(h \to WW)}$ in this case?

   (A) $\frac{1}{2}$

   (B) 1

   (C) These decay channels are forbidden.

   (D) $\frac{1}{3}$

2. Which of the following statements for the muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ width is correct?

   (A) $\Gamma \propto G_F m_\mu^3 m_e^2$

   (B) $\Gamma \propto G_F m_\mu^3$

   (C) $\Gamma \propto G_F^2 m_\mu^5$

   (D) $\Gamma \propto G_F^2 m_\mu^3 m_e^2$

   where $G_F \sim 1.17 \times 10^{-5}$ GeV^{-2} is Fermi constant and m_μ and m_e are the masses of muon and electron respectively

3. In a d-dimensional quantum field theory, an operator $\mathcal{O}$ of dimension Δ (in mass units) is said to be irrelevant if $\Delta > d$, marginal if $\Delta = d$ and relevant if $\Delta < d$. Consider a theory of a free scalar field ϕ in $d = 3, 4, 5$ spacetime dimensions, respectively, and the operator $\mathcal{O} = \phi^4$. What is the dimension of $\mathcal{O}$?

   (A) Irrelevant in $d = 3$, Marginal in $d = 4$ and Relevant in $d = 5$.

   (B) Relevant in $d = 3$, Marginal in $d = 4$ and Irrelevant in $d = 5$.

   (C) Marginal in all spacetime dimensions $d = 3, 4, 5$.

   (D) Marginal in $d = 3$, Relevant in $d = 4$ and Irrelevant in $d = 5$.

4. Consider a system of four spin-5/2 particles. (That is each particle transforms in the dimension 6 irreducible representation of $SU(2)$). The particles are pairwise different.

   The total Hilbert space has dimension 6^4. Compute the dimension of the sub Hilbert space made of the states with total $SU(2)$ spin zero.

   (A) 0

   (B) 15

   (C) 36

   (D) 6

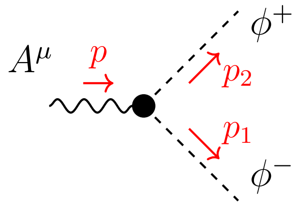
5. Consider an extension of QED with an additional massive complex scalar field

$$\phi^{\pm} \equiv \frac{1}{\sqrt{2}} (\phi_1 \mp i\phi_2) \ , \tag{1}$$

which couples with the photon field A^μ by means of the following Lagrangian density

$$\mathcal{L}_{\phi A} \equiv ieA^\mu \phi^- (\partial_\mu \phi^+) + h.c.\,. \tag{2}$$

Considering the Feynman diagram (red arrows indicate incoming and outgoing momenta)



What is the corresponding Feynman rule?

(A) $ie(p_1 - p_2)^\mu$

(B) The photon is massless and cannot decay into two massive particles; the Feynman rule, therefore, vanishes.

(C) $ie(p_1 + p_2)^\mu$

(D) $ie\epsilon^\mu(p)(p_1 \cdot p_2)$, where $\epsilon^\mu(p)$ is the photon polarization four-vector.

6. Consider a unitary relativistic quantum field theory with N scalar fields ϕ_i and N scalar fields ψ_i in four dimensions, with bare Lagrangian

$$\mathcal{L}_{bare} = \sum_i (\partial_\mu \phi_i)^2 + \sum_i (\partial_\mu \psi_i)^2 - \lambda (\sum_i \phi_i^2)(\sum_j \psi_j^2)$$

where $\lambda > 0$. Does the renormalized Lagrangian at 1-loop include additional quartic interaction terms?

(A) No.

(B) Yes, of the form $-g_2(\sum_i \phi_i \psi_i)^2$

(C) Yes, of the form $-g_1((\sum_i \phi_i^2)^2 + (\sum_j \psi_j^2)^2)$

(D) Yes, of the form $-g_1((\sum_i \phi_i^2)^2 + (\sum_j \psi_j^2)^2) - g_2(\sum_i \phi_i \psi_i)^2$

where $g_1, g_2 \neq 0$.

7. Consider a decay of the particle $A \to B + C$. What will be the energy of the particle B in the rest frame of A? m_A, m_B, m_C are the masses of the particles A, B, C respectively and satisfy the conditions $m_A > m_B + m_C, m_B \neq m_C$.

(A) $\frac{m_A^2 + m_C^2 - m_B^2}{2m_B}$

(B) $m_A - m_C$

(C) $m_A/2$

(D) $\frac{m_A^2 + m_B^2 - m_C^2}{2m_A}$

8. Consider the classical field theory in four space-time dimensions described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)\,, \qquad V(\phi) = \frac{\lambda}{4!}\phi^4\,.$$

and canonical energy-momentum tensor $T^{\mu\nu}$,

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - \eta^{\mu\nu}\mathcal{L}.$$

Consider the transformation

$$x'_\mu = a x_\mu\,, \qquad \phi'(x') = a^{-1}\phi(x)\,,$$

which is called *dilatation*. What is the Noether current associated with this symmetry transformation?

(A) $J^\mu_{\text{dil}} = T^{\mu\rho}x_\rho + \phi^2(\partial^\mu \phi)$.

(B) $J^\mu_{\text{dil}} = T^{\mu\rho}x_\rho + \phi(\partial^\mu \phi)$.

(C) $J^\mu_{\text{dil}} = \phi(\partial^\mu \partial^\nu \phi)$.

(D) $J^\mu_{\text{dil}} = (\partial_\nu \phi)T^{\mu\nu} + \partial^\mu \phi$.

9. Which one of the following combinations has dimension of length in d-dimensional Minkowski space-time? $G_N^{(d)}$, $\hbar$ and c are, respectively, the Newton's constant in d dimensions, the reduced Planck constant and the speed of light in the vacuum.

(A) $\left[\frac{G_N^{(d)}\hbar^2}{c}\right]^{1/(d-2)}$

(B) $\left[\frac{G_N^{(d)}\hbar}{c}\right]^{1/(d-2)}$

(C) $\left[\frac{G_N^{(d)}\hbar}{c^3}\right]^{1/(d-2)}$

(D) $\left[\frac{G_N^{(d)}\hbar}{c^3}\right]^{2/(d)}$

10. Consider the energy spectrum of a quantum mechanical non-relativistic particle of finite mass in one dimension subject to the potential

$$V(x) = \begin{cases} 0 & \text{if} \quad |x| > L \\ -V_0 & \text{if} \quad |x| < L \end{cases}$$

where $V_0 > 0, L > 0$ are positive and finite. Which of the following statements is correct?

(A) The spectrum is purely continuous.

(B) The spectrum is purely discrete.

(C) The spectrum has a continuous component and a discrete component containing infinitely many points.

(D) The spectrum has a continuous component and a discrete finite component containing at least a point.

11. Consider an isotropic harmonic oscillator in d dimensions whose Hamiltonian reads

$$H = \frac{1}{2m}\vec{p}^2 + \frac{1}{2}\omega^2 r^2$$

where $\vec{p}$ is the d-momentum, ω the frequency and $r = \sum_{i=1}^{d} x_i^2$. The degeneracy of its first excited level is d. What are the degeneracies of the second and third excited levels?

(A) $(d^2 - d)/2$ and $(d^3 - d^2 + d)/6$.

(B) $(d^2 + d)/2$ and $(d^3 + 3d^2 + 2d)/6$.

(C) d^2 and d^3.

(D) $(d^2 + d)/2$ and $(2d^3 + d^2 + 3d)/6$.

12. Consider a non relativistic quantum mechanical particle of mass m subject to a time independent potential $V(x)$ in one dimension. Its quantum states at given energy are described by the solutions of the Schrödinger equation. Suppose that the wave function

$$\psi(x) = Ce^{-\frac{[cosh(x/l)]^2}{2}},$$

where C is a normalisation constant and l a length scale, is an eigenfunction of energy

$$E = \frac{5\hbar^2}{8ml^2}$$

Then

(A) $V(x) = \frac{\hbar^2}{8ml^2}\left(\cosh(2x/l) - 2\right)^2$

(B) $V(x) = \frac{\hbar^2}{2ml^4}\left(\cosh(x/l) - 2\right)^2$

(C) $V(x) = \frac{\hbar^2}{4ml^2}\left(\sinh(x/l) - 1\right)$

(D) $V(x) = \frac{\hbar^2}{2ml^2}\left(\cosh(4x/l)\right)^2$

13. Consider a complex scalar field with the potential

$$V = -m^2(\phi^2 + (\phi^*)^2) + \lambda(\phi\phi^*)^2$$

where $*$ stand for complex conjugation and $m^2 > 0, \lambda > 0$. What is the mass spectrum of the theory?

(A) Two massive fields $m_1 > 0, m_2 > 0$.

(B) Two fields with mass zero $m_1 = m_2 = 0$.

(C) One massless field and one massive field $m_1 = 0, m_2 > 0$.

(D) The potential is unbounded from below and the system is unstable.