SISSA
Entrance
Examination

Elementary
Particle Theory
Sector

Trieste, 22 July 2002

FOUR PROBLEMS are given. You are requested to solve two of them.

Problem 1.

A particle of mass m is constrained on the x axis and subject to the potential

$$V = \frac{1}{2} m\omega^2 x^2 \tag{1}$$

1) Sketch the derivation of the energy eigenvalues E_n. Sketch the derivation of the eigenfunctions $\psi_n(x)$ from the vacuum one, $\psi_0(x) = \frac{1}{\pi^{\frac{1}{4}}} e^{-\frac{1}{2}\frac{m\omega}{\hbar}x^2}$.

2) Suppose the particle is electrically charged, with charge $e > 0$, and an electric field is switched on along the x axis. Find the new eigenvalues E'_n and describe the modifications of the new eigenfunctions $\psi'_n(x)$ with respect to $\psi_n(x)$,

- A) in the case the potential is (constant electric field E_0)

$$V_A = \frac{1}{2} m\omega^2 x^2 - eE_0 x \tag{2}$$

  where E_0 is a constant electric field;

- B) in the case the potential is (linearly rising electric field)

$$V_B = \frac{1}{2} m\omega^2 x^2 - \frac{1}{2} eK_0 x^2 \tag{3}$$

  where K_0 is a constant. Find the bound on K_0 so that the system remains a harmonic oscillator.

3) If the system is initially in the fundamental eigenstate of V, and the electric field is switched on at $t = 0$, describe the time evolution of the system for $t > 0$.

Hints:

1) Write the Hamiltonian $\mathcal{H} = \frac{1}{2} m(\dot{x}^2 + \omega^2 x^2) \equiv \hbar\omega H$ and express H in terms of $q = \sqrt{\frac{m\omega}{\hbar}}x$ and $p = \sqrt{\frac{m}{\hbar\omega}}\dot{x}$. Then assume the quantum bracket $[q, p] = i$ and define $a = \frac{1}{\sqrt{2}}(q + ip)$, $a^\dagger = \frac{1}{\sqrt{2}}(q - ip)$, etc.

3) Integrals involving eigenfunctions are to be left indicated.

Problem 2.

Consider the action

$$S = \int d^4x[\frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - \frac{1}{2}\phi^T M^2 \phi - \frac{1}{4!}\lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l] \tag{1}$$

where ϕ is a N component real scalar field with components ϕ^i, $i = 1, ...N$, M is an arbitrary $N \times N$ real symmetric matrix and λ_{ijkl} are real parameters.

1) Draw a 1-loop diagram with 4 external lines and write down the momentum space expression for it.

2) List all the global transformations on the vector ϕ which leave this action invariant for a general choice of M and λ. What happens if $M = 0$?

3) What is the largest internal invariance group of this action and for what choice of the matrix M and the couplings λ_{ijkl} this group is realized?

4) Choose the matrix M and the couplings λ such that the internal symmetry is maximum.

- i) Write down the conserved Noether current and construct the corresponding conserved charges in terms of the fields ϕ^i and their conjugate momenta π_i.
- ii) Calculate the commutation between these charges.

PROBLEM 3.

CONSIDER the elastic scattering of muon neutrino (ν_μ) and antineutrino ($\bar{\nu}_\mu$) on electron e^-,

$$\nu_\mu(k) + e^-(p) \to \nu_\mu(k') + e^-(p'), \tag{1}$$

$$\bar{\nu}_\mu(k) + e^-(p) \to \bar{\nu}_\mu(k') + e^-(p'), \tag{2}$$

k and p (k' and p') being respectively the four-momenta of the initial (final) state neutrino and electron. Assume that the muon neutrino is massless and that the processes (1) and (2) are generated by the weak neutral current interaction described by the Standard Theory effective Lagrangian:

$$L_{eff} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}_{\mu L}(x)\gamma_\alpha(1+\gamma_5)\nu_{\mu L}(x)\, \bar{e}(x)\gamma_\alpha[g_L(1+\gamma_5)+g_R(1-\gamma_5)]e(x)\ . \tag{3}$$

Here G_F is the Fermi coupling constant, $e(x)$ and $\nu_{\mu L}(x)$ are the electron field and the left-handed field of the muon neutrino, and g_L and g_R are the left-handed and right-handed electron neutral current couplings ($g_L = -0.5 + \sin^2\theta_W$, $g_R = \sin^2\theta_W$).

Consider the kinematic regime in which

$$s = (p+k)^2 = m_e^2 + 2(pk) \cong 2(pk) >> m_e^2, \tag{4}$$

m_e being the electron mass. In the rest frame of the initial electron (which coincides with the laboratory frame), the above kinematic regime corresponds to $E >> m_e$, where $E = k_0$ is the initial neutrino energy. In the indicated kinematic regime and neglecting terms of the order of m_e^2/s, the differential cross sections of the processes (1) and (2) to leading order in G_F have the simple form:

$$\frac{d\sigma(\nu_\mu e^-)}{dy} = \sigma_0\ [g_L^2 + g_R^2(1-y)^2],\quad \frac{d\sigma(\bar{\nu}_\mu e^-)}{dy} = \sigma_0\ [g_R^2 + g_L^2(1-y)^2], \tag{5}$$

where $\sigma_0 \equiv G_F^2 s/\pi$ and $y = (pq)/(pk)$, $q = k - k' = p' - p$. In the laboratory frame one has: $y = T/E$, where $T = p'_0 - m_e$ is the *kinetic* energy of the final state electron.

1) Find the interval of values the kinematic variable y can take in the lab. system. What is this interval if $m_e/E << 1$ and terms of the order of m_e/E are neglected?

2) Show that in the laboratory system ($p = (m_e, 0, 0, 0)$) the angle between the momenta of the initial state neutrino and the final state e^- is given for a relativistic final state e^- (i.e., when $T = p'_0 - m_e >> m_e$) approximately by

$$\frac{\theta^2}{2} \cong \frac{m_e}{T}\,(1-y) \leq \frac{m_e}{T} << 1. \qquad (6)$$

Give a physical interpretation of this result.

3) As it follows from eq. (5), for $y = 1$ both the contribution of the right-handed electron (the term with the factor g_R in L_{eff}) to the cross-section of the process (1) and the contribution of the left-handed electron (the term $\sim g_L$ in L_{eff}) to the cross section of the process (2), vanish. Give a physical explanation of this fact.

Hint:

3) neglecting the electron mass, consider the processes (1) and (2) as well as the condition y = 1, in the center of mass system of the initial state particles.

PROBLEM 4.

According to naturalness principles, the Early Universe is usually described as a particle-antiparticle environment with zero net quantum numbers. The process which leads from such baryon-symmetric state to the nowadays Universe where the amount of antimatter seems to be negligible is called "baryogenesis".

Consider an initial setup with net baryon number equal to zero and a particle physics theory which conserves CPT.

1) Show that the two conditions below are necessary ingredients for baryogenesis:

- 1a) Baryon number violation
- 1b) CP or C violation

(together with a third condition, departure from thermal equilibrium, these are known as Sakharov criteria)

2) Postulate the existence in the initial state of some supermassive spin-0 boson X and of its antiparticle $\bar{X}$, and assume that X and $\bar{X}$ have the same abundances. Assume that X can decay, through baryon number violating processes, into light fermions f_i. Let the baryon number of the fermion f_i be B_i, the baryon number of X be zero, and assume that all fermion masses are much smaller that the mass of X. For simplicity, consider the case in which X has only two decay modes, those allowed by the interaction:

$$L_{int} = g_a \, X \bar{f}_2 f_1 + g_b \, X \bar{f}_4 f_3 + h.c. \,. \tag{1}$$

- 2a) Verify, in this example, that both conditions 1a) and 1b) are needed by writing an expression for the induced net baryon number ϵ_X, in terms of the values B_i and of the branching ratio of one of the two decay modes. Why are at least two decay modes needed?
- 2b) Show that ϵ_X is zero at the lowest order in perturbation theory (it is not required to perform explicit calculation of kinematic factors)