Quantum Field Theory (Serone) **Exam Summer Session**

Academic Year 2017-2018 **10.07.2018**

Solve the following two/three exercises:

Exercise 1.

Discuss the Goldstone theorem and the main implications of adding a small explicit symmetry breaking term in a theory undergoing spontaneous symmetry breaking.

Exercise 2.

Compute the contribution to the one-loop β-function in QED due to a Dirac fermion with mass m and charge q and discuss the UV and IR behaviours of the theory.

[Hint: Use gauge invariance to extract the β-function from the one-loop photon polarization $\Pi_{\mu\nu}(p) = (\eta_{\mu\nu}p^2 - p_\mu p_\nu)\Pi(p^2)$]

Exercise 3.

Consider the renormalized 1PI 4-point function in the ϕ^4 theory

$$\Gamma_R^{(4)}(E) = \Gamma_R^{(4)}(s = t = u = -E^2) \tag{1}$$

for off-shell values of the Mandelstam variables. For $E, \mu \gg m$, it can be shown that to all orders in perturbation theory

$$\Gamma_R^{(4)}(E) \approx \sum_{l=0}^{\infty} \lambda^{l+1} \sum_{n=0}^{l} C_{n,l} \log^n \left(\frac{\mu}{E}\right), \tag{2}$$

namely at each loop l, $\Gamma_R^{(4)}(E)$ is given by a polynomial of degree l in $\log(\mu/E)$. Use the Callan-Symanzik equation to prove that all the coefficients of the form $C_{l,l}$ (leading logs) are determined in terms of the one-loop β-function coefficient β_0 ($\beta = \beta_0\lambda^2 + \ldots$). Show that the resummation of these leading logs is captured by the tree-level expression

$$\Gamma_R^{(4)}(E) = -\lambda(E), \tag{3}$$

where $\lambda(E)$ is the running coupling constant.