Quantum Field Theory (Serone) **Exam Winter Session**

Academic Year 2015-2016 **08.01.2016**

Solve the following two/three exercises:

Exercise 1.

Discuss the notion of naturalness and the hierarchy problem.

Exercise 2.

Consider the pseudo-scalar Yukawa theory in four space-time dimensions:

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi_0)^2 - \frac{\lambda_0}{4!}\phi_0^4 + \bar{\psi}_0 i\partial\!\!\!/ \psi_0 - g_0 \bar{\psi}_0 i\gamma^5 \psi_0 \phi_0 \,. \tag{1}$$

Introduce in dimensional regularization the counterterms Z_ϕ, Z_ψ, δ_λ and δ_g defined as follows:

$$\phi_0 = \sqrt{Z_\phi}\phi, \quad \psi_0 = \sqrt{Z_\psi}\psi, \quad \mu^{-\varepsilon} Z_\phi^2 \lambda_0 = \lambda + \delta_\lambda \,, \quad \mu^{-\frac{\varepsilon}{2}} Z_\psi \sqrt{Z_\phi} g_0 = g + \delta g \,. \tag{2}$$

At one-loop level, in Minimal Subtraction scheme, one has ($\varepsilon = 4 - d$)

$$\delta_\lambda = \frac{(3\lambda^2 - 48 g^4)}{(4\pi)^2 \varepsilon}, \quad \delta_g = \frac{2g^3}{(4\pi)^2\varepsilon}, \quad Z_\phi = 1 + \frac{a g^2}{(4\pi)^2\varepsilon}, \quad Z_\psi = 1 + \frac{b g^2}{(4\pi)^2\varepsilon} \,. \tag{3}$$

(i) Compute the coefficients a and b appearing in Z_ϕ and Z_ψ in eq.(3).

(ii) Using eq.(3) and the results found in (i), compute the one-loop β-functions $\beta_\lambda(\lambda, g)$ and $\beta_g(\lambda, g)$.

(iii) (*Mandatory for TPP students only*) Assume that at some energy scale

$$\lambda \propto g^n \,, \tag{4}$$

with $g \ll 1$. What is the highest possible value of n for which the relation (4) is stable at the quantum level in perturbation theory?

Exercise 3. *(Mandatory for TPP students only.)*

Consider a QCD-like theory with gauge group $G = SU(N_c)$ and four massless quarks in the fundamental representation of $SU(N_c)$. Two quarks are left-handed and two are right-handed. The global symmetry group is $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$.

(a) Compute the values of the D_{abc} coefficients involving at least 1 gauge $SU(N_c)$ current and $0, 1, 2$ global currents. Show in particular that only one $U(1)$ factor is anomalous.

(b) Weakly gauge the axial generator $U(1)_A$. Is the resulting theory consistent? Motivate your answer.

(c) Weakly gauge the axial generator $U(1)'_A \subset SU(2)_A$ along the σ_3 direction. Is the resulting theory consistent? Motivate your answer.