

Solve the following two/three exercises:

Exercise 1.

Discuss the Källèn-Lehmann representation for scalar field theories.

Exercise 2.

Consider the scalar QED theory in four space-time dimensions:

$$\mathcal{L} = |D_\mu \phi_0|^2 - m_0^2 \phi_0^\dagger \phi_0 + \frac{\lambda_0}{6} (\phi_0^\dagger \phi_0)^2 - \frac{1}{4} F_{0,\mu\nu} F_0^{\mu\nu}. \quad (1)$$

- (i) Draw the one-loop Feynman diagrams that are relevant for the wave function renormalization of the field ϕ and explain why other Feynman diagrams do not matter.
- (ii) Compute the anomalous dimension γ of the field ϕ in the Feynman gauge $\xi = 1$. Do you expect γ to be gauge-dependent? Motivate your answer.
- (iii) (*Mandatory for TPP students only*) Compute the anomalous dimension γ in a generic ξ -gauge.

Exercise 3. (*Mandatory for TPP students only.*)

Discuss the notion of chiral anomaly and derive its form using a path integral derivation.