Quantum Field Theory (Serone/Gambassi) **Exam Winter Session**

Academic Year 2014-2015 **08.01.2015**

Solve the following two/three exercises:

Exercise 1.

Discuss the optical theorem.

Exercise 2.

Consider the following non-relativistic scalar theory in $d+1$ space-time dimensions:

$$S = \int \mathrm{d}x_0\, \mathrm{d}^d\vec{x}\, \mathscr{L} \quad \text{with} \quad \mathscr{L} = \frac{1}{2}(\partial_0\phi)^2 - \frac{1}{2\Lambda^2}(\sum_{i=1}^{d}\partial_i^2\phi)^2 - \frac{\lambda}{n!}\phi^n\,, \tag{1}$$

where $i = 1,\ldots,d$ are spatial dimensions and n is an integer.

(a) Determine the region in the (n,d)-plane within which $\mathscr{L}$ is renormalizable. In particular, determine the critical spatial dimension d^* below which $\mathscr{L}$ is renormalizable when $n=4$, and the maximum value n^* of n for which the theory is renormalizable in $d=3$. *[Hint: Assign different scaling dimensions to x_0 and $\vec{x}$ so that the quadratic terms in eq. (1) have the same transformation properties under scaling.]*

(b) Add to $\mathscr{L}$ in eq. (1) a standard kinetic term:

$$\mathscr{L}' = \mathscr{L} - \frac{c}{2}\sum_{i=1}^{d}(\partial_i\phi)^2\,, \tag{2}$$

where c is a coupling. Determine whether this term is a relevant, irrelevant or marginal perturbation to $\mathscr{L}$ and explain why.

(c) Consider $d=3$ and $n=n^*$ in eq. (1). Determine the classical continuous and discrete global symmetries of the action (1). Argue that S does not contain all the operators compatible with these symmetries and write down the missing one.

Determine the additional six renormalizable operators that can be added to S, when, among the continuous symmetries, only spatial rotations and the internal ones are preserved. Use dimensional analysis in order to classify the seven operators identified above into marginal and relevant ones.

Exercise 3. *(Mandatory for TPP students only.)*

A system with global $G = SU(2)$ symmetry is spontaneously broken to $H = U(1)$.

(a) Write down the leading invariant term in the effective Lagrangian of the NG bosons.

(b) Denoting by "3" the unbroken direction in field space, write down the leading term in (a) for the case in which H is gauged. Determine the unitary gauge (if it exists) and the mass (if any) for the gauge field.

(c) Repeat point (b) for the case in which the gauged $U(1)$ is aligned along the directions "$1+3$".