Exercise 1.

In a toy version of QED, with two charged fermions with masses $m_1 = e^8$ GeV and $m_2 = e^2$ GeV, the value of α at the energy scale $E = e^{10}$ GeV is measured to be $\alpha(E) = 1/128$. Compute the approximate value of α at energies below the mass scale m_2. Neglect threshold effects and keep only the one-loop beta function. Recall that for a single fermion $\beta(\alpha) = 2\alpha^2/(3\pi)$.

Exercise 2.

Consider the following interaction of a massive photon with a charged scalar, which we call $L_{int,2}$ as a piece of the Lagrangian:

$$L_{int,2} = e^2 \bar{\phi}(x)\phi(x)A^\mu(x)A_\mu(x) \tag{1}$$

Take 4 space-time dimensions.

The propagator of the massive photon is, in the unitary gauge,

$$G^{(AA)}_{\mu\nu}(k) = -i\frac{\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}}{k^2 - M^2 - i\varepsilon} \tag{2}$$

where M is the photon mass.

(We are not interested on which mechanism might have given mass to the photon. The previous scalar field ϕ is not a Higgs field).

Compute in dimensional regularization the divergent part $\Delta\Sigma^\phi_{L_{int,2}}$ of the one-loop contribution to the ϕ propagator $G^{(\phi\phi)}$ due to the interaction $L_{int,2}$ in the unitary gauge using eq.(2). Recall that $G^{(\phi\phi)}(q) = \frac{i}{q^2 - m^2_\phi + \Delta\Sigma^\phi(q)}$, with $\Delta\Sigma^\phi(q) = \Delta\Sigma^\phi_{L_{int,2}} + \cdots$.

Exercise 3.

Discuss the Renormalization Group for Irrelevant Couplings.