Quantum Field Theory (Serone/Marzocca) **Exam Winter Session**

Academic Year 2021-2022 **14.1.2022**

Solve the following two/three exercises:

Exercise 1.

Discuss the implications of symmetries in QFT and in particular derive the transversality of the photon propagator to all orders in perturbation theory.

Exercise 2.

Consider a Yukawa theory of a massless pseudoscalar ϕ and a massless Dirac fermion ψ in four spacetime dimensions, with renormalized Lagrangian:

$$\mathscr{L} = (1+\delta_\phi)\frac{1}{2}(\partial_\mu\phi)^2 + (1+\delta_\psi)\bar{\psi}i\partial\!\!\!/\psi + (g+\delta_g)\phi\bar{\psi}i\gamma_5\psi - (\lambda+\delta_\lambda)\frac{\phi^4}{4!}\,. \tag{1}$$

1. Find the form of the one-loop β functions β_g and β_λ in terms of the five parameters $a_\phi, a_\psi, a_g, a_\lambda, b_g$, defined in the MS scheme as

$$\delta_\phi = a_\phi\frac{g^2}{\varepsilon}\,, \qquad \delta_\psi = a_\psi\frac{g^2}{\varepsilon}\,, \qquad \delta_g = a_g\frac{g^3}{\varepsilon}\,, \qquad \delta_\lambda = a_\lambda\frac{\lambda^2}{\varepsilon} + b_g\frac{g^4}{\varepsilon}\,, \tag{2}$$

   and draw all the relevant one-loop Feynman diagrams.

2. Compute the actual values of the parameters a_g and b_g.

3. Determine the explicit form of the one-loop β functions for both g and λ knowing that

$$a_\phi = -\frac{1}{4\pi^2}\,, \qquad a_\psi = -\frac{1}{16\pi^2}\,, \qquad a_\lambda = \frac{3}{16\pi^2}\,. \tag{3}$$

4. Let $\rho \equiv \lambda/g^2$, and compute $\beta_\rho = d\rho/d\log\mu$. Express the answer as function of g and ρ and find the two fixed points ρ_-^* and ρ_+^*. Discuss their RG stability.

[*Hint: Recall that for* $d = 4-\varepsilon$, $\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^4} = \frac{i}{(4\pi)^2}\frac{2}{\varepsilon} + \text{finite}$.]

Exercise 3. *(Mandatory for TPP students only.)*

1. Discuss the notion of naturalness and of hierarchy problem in QFT.

2. Consider a $SU(N)$ gauge theory ($N > 2$) coupled to three massless Dirac fermions in the fundamental representation of the gauge group. Is the vacuum trivially gapped? Motivate your answer.