SISSA ISAS

SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI
INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

Supersymmetry Breaking in Grand Unified Theories

Thesis submitted for the degree of
Doctor Philosophiae

PhD Thesis of:
Robert Ziegler

Supervisor:
Prof. Andrea Romanino

September 2010

Abstract

Supersymmetric grand unified theories (SUSY GUTs) are very interesting extensions of the minimal supersymmetric standard model (MSSM). They unify strong and electroweak interactions explaining the MSSM quantum numbers, account for presicion gauge coupling unification and stabilize the weak scale. Most importantly, they make remarkable predictions that allow to test them experimentally. Here we discuss two aspects of SUSY GUTs. First we present a short work on the impact of intermediate scales on gauge coupling unification. We provide a concise and systematic description of the subject by introducing "magic" fields contents. These are sets of chiral superfields that do not form complete SU(5) multiplets, but exactly preserve the one-loop unification of the MSSM independently of their mass scale. Unlike full SU(5) multiplets, these fields can raise (or lower) the GUT scale. Magic fields can play an important role in GUT model building, as we illustrate in two examples in the context of Orbifold GUTs and Gauge Mediation.

The second part contains the main subject of this thesis. We propose a new mechanism to explain the origin of soft SUSY breaking terms in the MSSM, which we call tree-level gauge mediation (TGM). SUSY breaking is communicated by the tree-level, renormalizable exchange of superheavy gauge messengers, which naturally arise in the context of grand unified theories. We demonstrate that this mechanism is viable despite the well-known arguments against tree-level SUSY breaking. In TGM sfermion masses are generated at tree-level and are flavor-universal, while gaugino masses arise at one-loop, but the loop factor is partially (or fully) compensated by numerical factors. The ratio of different sfermion masses is determined by group theoretical factors only and thus provides a distinct prediction that allows to test this mechanism at the LHC. We discuss the basic ideas and their implementation both in a general setup and a simple SO(10) model.

Contents

Introduction and Outline

The unified description of apparently different phenomena in nature has been one of the main goals of modern physics since its very beginning. In the late 17th century Isaac Newton demonstrated that the motion of celestial bodies and falling objects on earth can be described by the same equations derived from a universal gravitational theory [1]. Almost two hundred years later James Clerk Maxwell showed that magnetism, electricity and light were all manifestations of the same phenomenon that can be described by a single theory of electromagnetism [2]. The electromagnetic forces were then unified with the weak interactions about a century later in the electroweak theory by Sheldon Lee Glashow [3], which is today part of the standard model of particle physics [3, 4].

In modern physics unification is closely related to symmetry. Apparently different concepts are connected by symmetry transformations and unified into a single entity on which the symmetry group acts. In the case of electrodynamics for example magnetic and electric fields are related by Lorentz transformations and form the components of a single tensor of the Lorentz group. During the process of understanding weak interactions it became clear that the underlying symmetries do not have to be manifest [5]. If the symmetries are spontaneously broken at some energy scale, the resulting phenomenology below that scale does not even approximately exhibit the features of the unified theory above. Indeed weak and electromagnetic interactions look very different at energies below the weak scale, where the $SU(2)\times U(1)$ gauge symmetry is broken spontaneously. Symmetries play an important role in the attempt to unify matter and forces also beyond the standard model (SM). Such theories can make remarkable predictions that allow to test them experimentally, even if the symmetry breaking scale is very high.

One possible path is taken by grand unified theories (GUTs). These theories unify strong and electroweak interactions by embedding the SM gauge group into a simple group like SU(5) [6] or SO(10) [7]. Quarks and leptons are unified into irreducible representations of this group which allows the derivation of their SM quantum numbers. In particular the quantization of hypercharge in the SM can be explained, because the unified group is simple. The embedding of quarks and leptons into a single multiplet indicates that baryon and lepton number are in general not conserved. Nothing can therefore prevent the decay of the proton, which is indeed the main phenomenological consequence of grand unification [8]. While additional proton decay channels are strongly model-dependent, the decay via the exchange of the additional gauge bosons always takes place, but the decay rate is suppressed by the fourth power of their mass scale. The experimental bounds on proton lifetime require this mass scale and therefore the breaking scale of the unified group to be extremely large, around 10^{16} GeV, which is already enough to rule out simple models [9]. The breaking of the enlarged gauge symmetry to the SM gauge group at such large energy scales also implies that the SM gauge couplings are strongly affected by the renormalization group (RG) evolution. Under the assumption that this evolution is determined solely by the low-energy field content one can test the unification of gauge couplings experimentally. While unification in the standard model works rather poorly, it works remarkably well in the presence of supersymmetry [10, 11].

Supersymmetry (SUSY) is a spacetime symmetry that unifies fermions and bosons in single entities called superfields [12]. This leads to a dramatic improve of the UV-behavior of supersymmetric field theories, because scalar mass terms are now protected from the influence of heavy scales by the same chiral symmetries as fermions. In particular this is true for the Higgs mass terms which determine the weak scale, and thus supersymmetry provides a solution to the technical aspect of the Hierarchy Problem. It is however clear that SUSY cannot be an exact symmetry of nature because it predicts equal masses of fermion and bosons in the same superfield and therefore a plethora of unobserved scalar particles. In order to break SUSY but conserving its benefits regarding unification and stabilization of the weak scale, SUSY must be broken in the low-energy theory only softly [13], that is by dimensionful operators with an associated scale that is not much larger than the weak scale. Such soft SUSY breaking operators include mass terms for the unobserved superpartners which allow to shift them beyond the reach of past experiments. These particles should however not be significantly heavier than the TeV scale and are therefore expected to be discovered soon at the Large Hadron Collider (LHC), which finally started to take data for beam energies in the TeV regime half a year ago.

The minimal low-energy realization of supersymmetry is provided by the minimal supersymmetric standard model (MSSM) [14]. Every field of the standard model is promoted to a superfield that contains the SM field and a superpartner with opposite statistics. In particular this requires the existence of a new fermion with the quantum number of the Higgs. This field then contributes to the triangular anomalies and spoils the neat cancellation which takes place among the SM fermions. Therefore another Higgs superfield with conjugate quantum numbers is added in the MSSM to obtain a vectorlike Higgs sector that does not contribute to triangular anomalies. The most general renormalizable supersymmetric Lagrangian that can be written down with this field content is however not realistic. In contrast to the SM baryon and lepton number are not accidental symmetries but are violated by renormalizable operators that induce fast proton decay. In order to forbid these dangerous operators one can define a new discrete symmetry called R-parity [15] which does not commute with SUSY. Under this symmetry SM fields are even while the superpartners are odd. This also implies that the lightest supersymmetric particle (LSP) is stable and can be a good Dark Matter candidate. The most general supersymmetric operators which respect R-parity are then the Yukawa couplings and a mass term for the Higgs fields. In addition the MSSM contains all possible terms which respect R-parity and break SUSY only softly. The associated scale is called the soft susy breaking scale and should not exceed the TeV scale in order to provide a solution to the Hierarchy Problem. The MSSM defined in this way has then many virtues beyond the stabilization of the weak scale. The most prominent success is to account for precision gauge coupling unification which in turn suggests that the MSSM is just the low-energy effective theory of a supersymmetric grand unified theory. This is the setup on which the work presented in this thesis is based on.

The MSSM however gives rise also to new problems, mainly related to its soft SUSY breaking part. The soft terms introduce a lot of new parameters whose structure is constrained by experiments, in particular flavor physics. For example, the soft masses for the sfermions, the scalar superpartners of the SM fermions, are matrices in flavor space which in general are not diagonal in the same basis where the fermion masses are. This gives rise to flavor-violating processes like $K-\overline{K}$ mixing or $\mu \to e\gamma$ that are strongly suppressed or absent in the standard model. The SM however well explains the experimental data and therefore any new source of flavor violation must be very small. This requires a non-generic flavor structure of the soft terms which should be explained by the underlying mechanism of SUSY breaking.

This is referred to as the Flavor Problem of the MSSM, see e.g. [16]. Another problem is related to the vectorlike structure of the MSSM Higgs sector. It allows for a supersymmetric mass term with parameter μ that enters the scalar potential together with the soft SUSY breaking parameters (soft masses for both Higgses and a bilinear term denoted by B_μ) and therefore participates in the determination of the Z mass. This means that it should be of the same order as the soft terms, despite it is a supersymmetric mass term that in principle is allowed to be very large. To explain why the μ-term is connected to the soft SUSY breaking scale is referred to as the μ-Problem [17].

The soft terms of the MSSM have to be generated by some mechanism of SUSY breaking, which should explain their non-generic flavor structure and provide a solution to the μ-Problem. Generically SUSY is broken spontaneously by vacuum expectation values (vevs) of the auxiliary components of some superfields. Since it is difficult to couple such SUSY breaking fields directly to the MSSM, the common paradigm is that SUSY is broken in some "hidden" sector of the theory and then communicated to the "observable" sector by means of a "messenger" sector. What typically governs the structure of the soft terms is not how SUSY is broken in the hidden sector, but how it is communicated to the observable sector. In four spacetime dimensions there are two popular scenarios : either hidden and observable sector are coupled only gravitationally or there are additional chiral superfields that couple directly to SUSY breaking but only to the gauge fields of the MSSM. In the first case, referred to as gravity mediation [18], all soft terms arise from Planck suppressed operators. In the second case, which is referred to as gauge mediation [19], soft terms arise at loop-level. While gravity mediation can elegantly explain the μ-Problem with the Giudice-Masiero mechanism [20], it has no good solution to the Flavor Problem. Gauge mediation instead provides flavor-universal soft terms, but has difficulties with the μ-term. More precisely, it is not complicated to generate the μ-term at loop-level, but typically the same operator induces also the B_μ term at a scale which is a loop factor too large [21]. The problem of generating the soft terms in the MSSM is therefore far from being solved. One might hope that LHC will find some of the superpartners and give a hint of their mass patterns. This could provide the experimental input needed to reveal the origin of soft SUSY breaking in the MSSM.

In summary supersymmetric grand unified theories are very interesting extensions of the standard model. They unify strong and electroweak interactions and can explain the quantization of hypercharge and the smallness of the weak scale.

They also make remarkable predictions that allow to test them at currently running experiments. The low-energy effective theory of SUSY GUTs should be given by the MSSM, possibly up to additional light gauge singlets. One prediction is therefore the existence of superpartners at the TeV scale that could be tested at the LHC, depending on the structure of the soft SUSY breaking terms. These terms also influence the prediction of proton decay by grand unification. It is therefore difficult to make precise predictions on the decay rate without making assumptions on the soft terms. Nevertheless simple models are already ruled out [22] and others are pushed to their theoretical limits [23]. If nature is indeed described by SUSY GUTs, proton decay should therefore be observed soon at Super-Kamiokande or the next generation of water Cherenkov detectors.

In this thesis we consider two aspects of supersymmetric grand unified theories. First we present a short work on the impact of intermediate scales on gauge coupling unification in SUSY GUTs [24]. We introduce "magic" fields, which are sets of SM chiral superfields that do not form complete SU(5) multiplets, but preserve exactly the one-loop unification of the MSSM independently of their mass scale. Unlike full SU(5) multiplets, such magic field sets can have an impact on the GUT scale. We analyze the consequences of magic fields for unification and discuss their origin in the context of SO(10) grand unified theories. Then we extent these considerations to the important case of a two-step breaking of SO(10). We discuss two applications of magic fields and finally give a systematic list of examples.

In the second part we propose a new mechanism to explain the origin of soft terms in the MSSM, which we called tree-level gauge mediation (TGM) [25, 26]. In this scheme SUSY breaking is communicated by the tree-level, renormalizable exchange of superheavy gauge messengers, which naturally arise in the context of grand unified theories. We begin with an overview of the basic ideas of TGM and illustrate their implementation in a simple SO(10) model. In particular we demonstrate that this mechanism is viable despite the well-known arguments against tree-level SUSY breaking. In the following chapters we discuss TGM under more general aspects. First we analyze the viability of this mechanism in the context of a generic supersymmetric gauge theory and provide general expressions for the resulting soft terms. Based on this analysis, we proceed to study the model-building guidelines for a possible realization of TGM in the framework of SUSY GUTs. Finally, we discuss several possible options for an implemention of the μ-term in these models.

Part I

Intermediate Scales in SUSY GUTs

1 Introduction

The unification of gauge couplings within the MSSM is regarded as one of the major successes of low-scale supersymmetry and gives strong support to the idea of grand unification. Gauge coupling unification together with low-energy data on the electroweak gauge sector allows the prediction of the strong coupling α_3 and the unification scale $M_{\mathrm{GUT}} \approx 2 \cdot 10^{16}\,\mathrm{GeV}$. While the former is in good agreement with the measured value, within the uncertainties associated with low-energy and high-energy thresholds, the latter is large enough to avoid rapid proton decay, despite it is starting to be challenged by currently running experiments on proton lifetime.

These predictions for α_3 and M_{GUT} in the MSSM are however based on the assumption that there are no new degrees of freedom all the way up to the unification scale, referred to as the bleak scenario of the "grand desert". Regarding the fact that many models for BSM physics are based on additional matter at intermediate scales, it is important to derive constraints on the new particle content, requiring that unification of gauge couplings and thus the successful prediction for α_3 are (approximately) maintained. Moreover, since unification might be fixed simply by carefully adjusting the threshold scales, it is desirable to obtain constraints that involve gauge quantum numbers only, so that unification is preserved *independently* of the mass scale of the new fields.

An important aspect is the possible impact of new fields on the unification scale, since keeping the prediction for α_3 does not imply that M_{GUT} is unchanged. In particular it might happen that the new fields raise the GUT scale, which is interesting both for phenomenology regarding the present bound on proton decay, as well as for theoretical reasons, because many string theory models predict a GUT scale that is about one order of magnitude larger than in the MSSM [27].

It is well known that fields forming complete SU(5) multiplets do not affect the prediction of α_3 nor M_{GUT} at one-loop, independently of their mass scale. Many authors studied the impact on gauge coupling unification of a more general particle content, in particular with the motivation to increase the unification scale [27, 28, 29]. In this part we will try to give a concise and systematic description of the subject, by introducing what we called magic field contents [24]. These are sets of vectorlike matter superfields that do not form full SU(5) multiplets, but share their benefits regarding gauge coupling unification: i) they exactly preserve the one-loop MSSM prediction for α_3 and ii) they do it independently of the value of their (common) mass. Therefore they maintain the predictivity of the MSSM, in the sense that their mass does not represent an additional parameter that can be tuned in order to fix α_3. On the other hand magic sets do not form full SU(5) multiplets and therefore typically do have an impact on M_{GUT} and can raise the GUT scale.

This part is organized as follows: We begin with the definition of magic field sets and discuss their impact on the GUT scale by classifying them according to five different scenarios of unification. Then we investigate the origin of these fields and show some examples how they can be obtained in the context of a unified theory. After that we consider the important special case in which the unified group SO(10) is broken in two steps, so that the gauge group below the unification scale is not the SM one. Then we discuss a few applications of magic fields. First we consider the possibility to suppress Kaluza-Klein threshold effects in the context of unified theories with extra dimensions (Orbifold GUTs). Then we briefly analyze gauge mediated supersymmetry breaking models that have magic fields as messengers. In the last chapter, we present a systematic collection of magic field sets.

2

Magic Fields

2.1 Definition

We consider the MSSM with additional vectorlike matter superfields at a scale $Q_0 > M_Z$. Let us denote by b_i, $i = 1, 2, 3$ the one-loop beta function coefficients for the three SM gauge couplings. At scales $M_Z < \mu < Q_0$, the MSSM spectrum gives $(b_1, b_2, b_3) = (33/5, 1, -3) \equiv (b_1^0, b_2^0, b_3^0)$. At $\mu > Q_0$, the beta coefficients include the contribution b_i^N of the new fields, $b_i = b_i^0 + b_i^N$ and the one-loop running gives

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i^0}{2\pi} \log\left(\frac{\mu}{M_Z}\right) - \frac{b_i^N}{2\pi} \log\left(\frac{\mu}{Q_0}\right). \tag{2.1}$$

The MSSM one-loop prediction for α_3,

$$\frac{1}{\alpha_3} = \frac{1}{\alpha_2} + \frac{b_3^0 - b_2^0}{b_2^0 - b_1^0}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right) \tag{2.2}$$

is exactly preserved independently of the scale Q_0 if [28]

$$\frac{b_3^N - b_2^N}{b_2^N - b_1^N} = \frac{b_3^0 - b_2^0}{b_2^0 - b_1^0} = \frac{5}{7}. \tag{2.3}$$

In this case, the unification scale M_{GUT} becomes

$$M_{\text{GUT}} = M_{\text{GUT}}^0 \left(\frac{Q_0}{M_{\text{GUT}}^0}\right)^r, \tag{2.4}$$

and the unified gauge coupling α_U is

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_U^0} - \frac{(1-r)b_i^N - rb_i^0}{2\pi} \log\left(\frac{M_{\rm GUT}^0}{Q_0}\right), \tag{2.5}$$

with the parameter

$$r = \frac{b_3^N - b_2^N}{b_3 - b_2}, \tag{2.6}$$

and $M_{\rm GUT}^0 \approx 2 \cdot 10^{16}\,\text{GeV}$ and $\alpha_U^0 \approx 1/24$ denote the corresponding values in the MSSM.

Complete GUT multiplets give the same contribution to the three beta functions and thus trivially satisfy Eq. (2.3); they preserve gauge coupling unification and leave the GUT scale unchanged. We call "magic" all other vectorlike sets of fields that satisfy Eq. (2.3) and therefore preserve the one-loop MSSM prediction for α_3. They fall into two categories: those with $r = 0$, which just mimic the effect of complete GUT multiplets and those with $r \neq 0$, which change the GUT scale according to Eq. (2.4).

The parameter r also determines the relative order of the three scales Q_0, $M_{\rm GUT}^0$ and $M_{\rm GUT}$. There are five different possibilities:

- $r = 0 \Rightarrow Q_0 < M_{\rm GUT}^0 = M_{\rm GUT}$: **Standard unification**.

  This corresponds to $b_3^N = b_2^N = b_1^N$. The GUT scale is unchanged. The new fields can form complete GUT multiplets, but do not necessarily have to.

- $-\infty < r < 0 \Rightarrow Q_0 < M_{\rm GUT}^0 < M_{\rm GUT}$: **Retarded unification**.

  The new fields slow down the convergence of the gauge couplings. The simplest example of magic fields leading to retarded unification is $\left(Q + \overline{Q}\right) + G$[1], which gives $(b_3^N, b_2^N, b_1^N) = (5, 3, 1/5)$ and $r = -1$. The running of the gauge couplings is shown in Fig 2.1.

- $r = \pm\infty \Rightarrow Q_0 = M_{\rm GUT}^0 < M_{\rm GUT}$: **Fake unification**.

  This case corresponds to $b_3 = b_2 = b_1$. The unified group is broken at a scale $M_{\rm GUT} \geq M_{\rm GUT}^0$, but the couplings run together between $Q_0 = M_{\rm GUT}^0$ and $M_{\rm GUT}$, thus faking unification at the lower scale $M_{\rm GUT}^0$. Note that in this case $M_{\rm GUT}$ is undetermined, while Q_0 is fixed.

[1]Here and below we denote the new fields according to their quantum numbers as in Table 2.1 which can be found in Section 2.5.

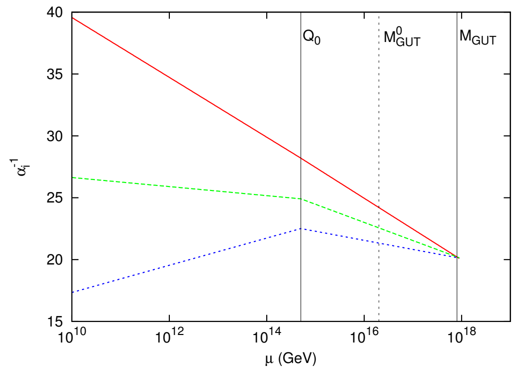

Figure 2.1 Example of retarded unification. The fields $\left(Q+\overline{Q}\right)+G$ have been added at the scale Q_0.

A simple example is provided by adding the fields $(6,2)_{-1/6}$ + c.c.[2], which gives $(b_3^N, b_2^N, b_1^N) = (10, 6, 2/5)$ (see Fig 2.2). This possibility was previously considered in [30].

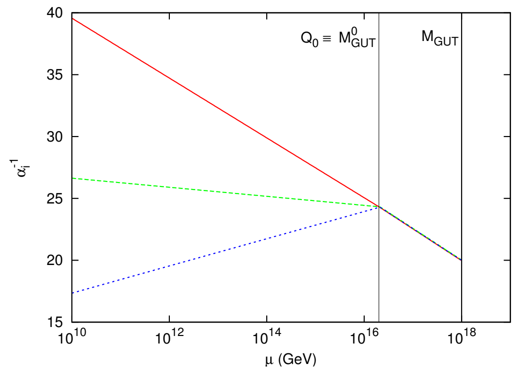

Figure 2.2 Example of fake unification. The fields $(6,2)_{-1/6}$ + c.c. have been added at the scale $Q_0 = M^0_{\rm GUT}$.

- $1 < r < +\infty \Rightarrow M^0_{\rm GUT} < Q_0 < M_{\rm GUT}$: **Hoax unification**.

  In this scenario the magic set turns a convergent running into a divergent one and vice versa. Therefore such a field content cannot be added at a scale smaller than $M^0_{\rm GUT}$, or the gauge couplings would diverge above Q_0 and never meet. However unification is preserved if the magic fields are heavier than $M^0_{\rm GUT}$. Then the couplings, after an hoax crossing at $M^0_{\rm GUT}$, diverge between $M^0_{\rm GUT}$ and Q_0, start to converge above Q_0 and finally unify at $M_{\rm GUT}$, the

[2] This representation is contained for example in the **210** of SO(10).

scale where the unified group is broken. For example, the fields $W + 2 \times \big((8,2)_{1/2} + \text{c.c.}\big)$[3] give $(b_3^N, b_2^N, b_1^N) = (24, 18, 48/5)$ and $r = 3$ (see Fig 2.3).

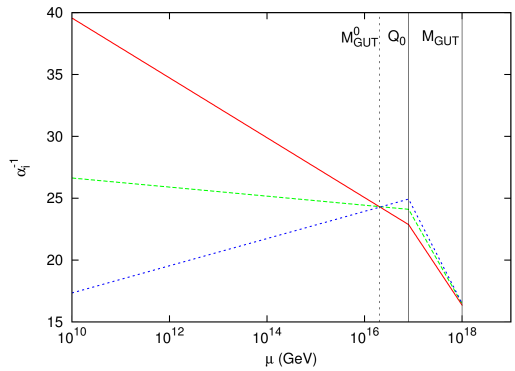

Figure 2.3 Example of hoax unification.The fields $(1,3)_0 + 2 \times \big((8,2)_{1/2} + \text{c.c.}\big)$ have been added at the scale $Q_0 > M^0_{\text{GUT}}$.

- $0 < r < 1 \Rightarrow Q_0 < M_{\text{GUT}} < M^0_{\text{GUT}}$: **Anticipated unification**.

  The magic content accelerates the convergence of the gauge couplings and the unification takes place before the usual GUT scale. This possibility can be useful in combination with other types of magic sets at different scales.

Some comments are in order:

- In the above considerations, the scale Q_0 is arbitrary, as long as unification takes place before the Planck scale, $M_{\text{GUT}} \lesssim M_{\text{Pl}} \sim 2 \cdot 10^{18}$ GeV and the unified gauge coupling is in the perturbative regime, $\alpha_U \lesssim 4\pi$.

- If we restrict our analysis to representations that can be obtained from the decomposition of SU(5) multiplets under G_{SM}, then both $b_3^N - b_2^N$ and $\frac{5}{2}(b_2^N - b_1^N)$ are integers. In this case the magic condition requires $b_3^N - b_2^N$ to be even and $b_2^N - b_1^N$ to be a multiple of 14/5 [28]. Therefore in the case of retarded unification the only possibility is $b_3^N - b_2^N = 2$, which corresponds to $r = -1$. The expression for the GUT scale (2.4) becomes particularly simple:

$$\frac{M_{\text{GUT}}}{M^0_{\text{GUT}}} = \frac{M^0_{\text{GUT}}}{Q_0}. \tag{2.7}$$

  In this scenario therefore Q_0 cannot be lower than $10^{13} - 10^{14}$ GeV, in order to keep $M_{\text{GUT}} \lesssim M_{\text{Pl}}$.

[3] $\big((8,2)_{1/2} + \text{c.c.}\big)$ is contained both in the **120** and **126** of SO(10).

- An important property following from Eq. (2.3) is that combinations of magic sets at different scales do not spoil unification. In particular, merging two or more sets at the same scale gives again a magic set. Two simple rules are: adding two retarded solutions gives a fake solution, and adding a fake to a retarded solution or to another fake gives a hoax solution[4].

2.2 The Origin of Magic Fields

By definition magic field sets do not form complete SU(5) multiplets. Therefore the question arises if they can easily be obtained from complete multiplets of a unified group. As the example of the doublet-triplet splitting problem illustrates, this might be not a trivial problem. In this section we show that magic field sets at a scale $Q_0 < M_{\text{GUT}}$ can indeed arise from the spontaneous breaking of a supersymmetric SO(10) GUT at the scale M_{GUT}. We will illustrate this in three examples for the case of retarded, fake, and hoax unification.

- Retarded unification

  The simplest magic field content leading to retarded unification is

$$\left(Q + \overline{Q}\right) + G,$$

  which can be obtained by splitting the components of a $\mathbf{16}+\overline{\mathbf{16}}+\mathbf{45}$ of SO(10). As an example, such a splitting is provided by the following superpotential:

$$\begin{aligned} W = &\ 16\ 45_H \overline{16} + 16_H\ 16\ 10 + \overline{16}_H\ \overline{16}\ 10 + 45_H\ 45\ 54 \\ &+ 16_H\ 45\ \overline{16}' + \overline{16}_H\ 45\ 16' + M\ 10\ 10 + M\ 54\ 54 + M\ \overline{16}' 16'. \end{aligned} \tag{2.8}$$

  Here and below, all dimensionless couplings are supposed to be $\mathcal{O}(1)$ and $M \sim M_{\text{GUT}}$. The 45_H is assumed to get a vev of order M_{GUT} along the T_{3R} direction, while 16_H gets a vev in its SU(5) singlet. Then the above superpotential gives a mass of order M_{GUT} to all matter fields except Q, $\overline{Q}$,

[4] Note that the classification based on r can be rewritten in terms of the parameter $q = b_3^N - b_2^N$ used by [28]. Anticipated unification then corresponds to $q < 0$, standard unification to $q = 0$, retarded to $q = 2$, fake to $q = 4$, and hoax to $q > 4$. The q of a combination of magic fields sets is the sum of the individual q's, from which the rules follow trivially.

G, which are assumed to get a mass at a lower scale Q_0. A two-loop analysis shows that the prediction for $\alpha_3(M_Z)$ does not significantly differ from the MSSM one.

- Fake unification

  An example of fields leading to fake unification is

$$2 \times (L + \overline{L}) + 2 \times G + 2 \times W + 2 \times (E + \overline{E}) + \left((8, 2)_{1/2} + \text{c.c}\right),$$

  which can be embedded into a $\mathbf{45} + \mathbf{45} + \mathbf{120}$ of SO(10). This magic field set can be obtained from the following superpotential

$$W = 45\ 45_H\ 45' + 120\ 45_H\ 120' + M\ 120'\ 120', \tag{2.9}$$

  if 45_H gets a vev of order $M \sim M_{\text{GUT}}$ along the $B - L$ direction. Another example is $2 \times (Q + \overline{Q} + G)$, which can be obtained by a generalization of the superpotential in Eq. (2.8).

- Hoax unification

  As an example for hoax unification, we consider the set

$$4 \times (L + \overline{L}) + 3 \times \left((8, 2)_{1/2} + \text{c.c}\right),$$

  which can be embedded into a $\mathbf{120} + 2 \times (\mathbf{126} + \overline{\mathbf{126}})$ of SO(10). This field set can for example be obtained from the superpotential

$$W = 126\ 45_H\ \overline{126} + 126'\ 45_H\ \overline{126}' + 120\ 45_H\ 120' + M\ 120'\ 120', \tag{2.10}$$

  again with a vev of 45_H along the $B - L$ direction.

2.3 Magic Fields in two-step Breaking of SO(10)

The necessity of achieving gauge coupling unification in the presence of fields not forming full unified multiplets is particularly important in the context of a two-step breaking of SO(10), meaning that SO(10) is broken at the scale M_{GUT} to the intermediate group G_i, which is then broken to the SM at a lower scale M_i. In this case, the presence of an intermediate gauge group at a lower scale $M_i < M_{\rm GUT}$ often spoils gauge unification if no further fields are added. This is because the additional gauge bosons of $G_i/G_{\rm SM}$ are not necessarily in full SU(5) multiplets, as in the case of the Pati-Salam (PS) group $G_i = G_{\rm PS} \equiv {\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm SU}(4)_c$ and the Left-Right group $G_i = G_{LR} \equiv {\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm SU}(3)_c \times {\rm U}(1)_{B-L}$. In this section we repeat the previous analysis of magic fields for these two important cases. Some examples which are related to this discussion can be found in [31].

We consider a set of fields at the scale Q_0, with $M_i < Q_0 < M_{\rm GUT}$, which consists of multiplets of the gauge group G_i. The condition (2.3) for preserving unification has to be modified, since we now have to take into account the additional vector superfields and to express the condition in terms of the beta coefficients of the gauge couplings of the group G_i.

2.3.1 The case $G_i = G_{\rm PS}$

We denote a PS multiplet by $(\mathbf{R}_4, \mathbf{R}_L, \mathbf{R}_R)$, where $\mathbf{R}_4, \mathbf{R}_L, \mathbf{R}_R$ are the representations of ${\rm SU}(4)_c$, ${\rm SU}(2)_L$, ${\rm SU}(2)_R$ respectively. The three PS gauge couplings g_4, g_L, g_R are matched to the SM ones at the PS breaking scale $M_{\rm PS}$ as follows:

$$\frac{1}{\alpha_4} = \frac{1}{\alpha_3}, \qquad \frac{1}{\alpha_L} = \frac{1}{\alpha_2}, \qquad \frac{1}{\alpha_R} = \frac{5}{3}\frac{1}{\alpha_1} - \frac{2}{3}\frac{1}{\alpha_3}. \tag{2.11}$$

In terms of the corresponding beta function coefficients b_4,b_L, b_R, the condition (2.3) becomes

$$\frac{b_4 - b_L}{b_L - b_R} = \frac{1}{3}. \tag{2.12}$$

The contribution of MSSM fields and PS gauge bosons is $(b^0_4, b^0_L, b^0_R) = (-6, 1, 1)$. Thus, the Pati-Salam couplings do not unify if no extra matter is added, because condition (2.12) is not satisfied. A simple possibility to restore unification is to add a single $(\mathbf{6}, \mathbf{1}, \mathbf{3})$ field, which exactly cancels the contribution of the PS gauge bosons to the beta function coefficients. This field acquires a mass together with the PS gauge bosons at the PS breaking scale. Note that some extra matter is also needed in order to achieve this breaking.

If the field content below the PS scale is the MSSM one, the classification given in Section 2.1 can be carried over to the scenario considered here, by simply replacing r in Eq. (2.6) with

$$r = \frac{b^N_4 - 3 - b^N_L}{b_4 - b_L}. \tag{2.13}$$

The formula (2.4) for the GUT scale is then still valid. A more general expression for the new unification scale valid for an arbitrary (magic) field content below M_i is

$$\ln \frac{M_{\rm GUT}}{M^0_{\rm GUT}} = \left(\frac{b_3 - b_2}{b_4 - b_L} - 1 \right) \ln \frac{M^0_{\rm GUT}}{M_{\rm PS}}, \tag{2.14}$$

where b_2, b_3 are the MSSM beta coefficients just below the PS scale.

2.3.2 The case $G_i = G_{LR}$

The magic condition can be written in terms of the beta coefficients (b_L, b_R, b_3, b_{B-L}) as

$$\frac{b_3 - b_{2L}}{b_{2L} - \frac{3}{5} b_{2R} - \frac{16}{15} b_{B-L}} = \frac{5}{7}. \tag{2.15}$$

The contribution of the MSSM and the additional G_{LR} gauge bosons to the beta coefficients is $(b_L, b_R, b_3, b_{B-L}) = (1, 1, -3, 16)$ and the expression for r is the same as in the MSSM Eq. (2.6) with $b_2 = b_L$.

2.4 Applications

Clearly, the previous discussions would be just academic exercises unless there is a good motivation to consider additional fields at intermediate scales, which do not merely serve to change the running of gauge couplings. While there are plenty of specific models which require such fields for various reasons, in this section we focus on two applications of magic field contens. First we discuss Orbifold GUTs where the Kaluza-Klein states form magic sets, then we consider the case of gauge mediation where the messengers of SUSY breaking make up a magic field content. A multi-scale model of fermion masses and mixings that makes extensive use of magic fields was presented in [32].

2.4.1 Orbifold GUTs

An interesting application arises in unified theories with extra dimensions compactified on an orbifold. Such orbifold GUTs have several advantages over unified theories in four dimensions, for example they allow for an easy breaking of the unified group by orbifold boundary conditions, a straightforward solution of the doublet-triplet splitting problem, and the suppression of dangerous dimension-five operators causing fast proton decay [33]. In these theories, fields living in the bulk of the extra dimension correspond to Kaluza-Klein (KK) towers of fields in the effective four-dimensional theory, whose masses are multiples of the compactification scale. Because of the very mechanism of GUT breaking by orbifolding, the KK fields with a given mass do not form full multiplets of the unified group. As a consequence, the KK towers associated to the bulk fields introduce new thresholds affecting the prediction of α_3. While such thresholds are often used to improve the agreement with data (if they are not too large), it is interesting to note that it is possible to get rid of such effects if the fields corresponding to a given KK mass form magic sets.

As an example, let us consider a 5D supersymmetric SO(10) model on $S^1/(Z_2 \times Z_2')$ with a Pati-Salam brane and a SO(10) brane (see [34] for a description of such models). The vector fields (V, Σ) live in the bulk together with a chiral hypermultiplet (Φ_1, Φ_2) in the adjoint of SO(10), while the SM matter, the Higgses and other fields live on the branes. The bulk fields can be classified in terms of their two orbifold parities $(P_1, P_2) = (\pm 1, \pm 1)$. The orbifold boundary conditions are chosen such that the SO(10) adjoints V, Σ, Φ_1, Φ_2 split into their PS adjoint components and the orthogonal component, with orbifold parities defined as follows

(V, Σ)	(Φ_1, Φ_2)	
V_{++}, Σ_{--}	Φ_{1++}, Φ_{2--}	PS adjoints
V_{+-}, Σ_{-+}	Φ_{1+-}, Φ_{2-+}	SO(10)/PS adjoints

.

The massless zero-modes are given by the gauge fields $V_{++}^{(0)}$ and an adjoint field $\Phi_{1++}^{(0)}$. The odd KK states contain fields of the SO(10)/PS adjoint representation, while the even KK states contain those of the PS adjoint.

Clearly, neither the even nor the odd states correspond to full SO(10) (or SU(5)) multiplets. Still, both of them could form magic sets, in which case the threshold effects associated to the KK tower of fields would vanish at the one-loop level. This is indeed the case in the example we are considering, The easiest way to see it is to observe that the (V, Σ) and (Φ_1, Φ_2) multiplets together form an $\mathcal{N} = 4$ SUSY hypermultiplet, which gives no contribution to the beta functions (the contribution of three chiral multiplets Σ, Φ_1, Φ_2 cancels exactly the one of the gauge fields V). Therefore both the even and the odd levels of the KK towers do not spoil unification.

In order to avoid experimental bounds, the zero-mode Φ_{1++} cannot be too light. It should have a mass at some intermediate scale M_Φ, which can be identified with the PS breaking scale. In order to maintain unification it is sufficient to add some fields of mass M_Φ on the PS brane which form a magic field content together with Φ_{1++}, for example

$$(\mathbf{4},\mathbf{1},\mathbf{2}) + (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{3}).$$

2.4.2 Gauge Mediation

In gauge mediated supersymmetry breaking (GMSB), the messenger sector is usually assumed to be made up of full SU(5) multiplets in order not to spoil gauge coupling unification. In the light of the above discussion, it is natural to consider also the case of a messenger sector composed of magic field sets. Gauge mediation with incomplete GUT multiplets was previously studied in [35]. This analysis was however restricted to messengers with SM matter quantum numbers (and their conjugates), which moreover were not required to maintain gauge coupling unification. Here instead we insist that gauge coupling unification remains intact and therefore obtain additional constraints on the low-energy sparticle spectrum, despite many of the conclusions in [35] apply also to this case.

We assume the usual superpotential

$$W = S\overline{\Psi}_i\Psi_i + M\overline{\Psi}_i\Psi_i, \tag{2.16}$$

where $\Psi_i, \overline{\Psi}_i$ form a magic set of fields and S is the spurion with $\langle F_S\rangle \neq 0$. The gaugino masses at the scale μ are given by

$$M_a(\mu) = \frac{\alpha_a(\mu)}{4\pi} b_a^N \frac{F_S}{M}, \tag{2.17}$$

while the scalar masses are

$$\tilde{m}_i^2(\mu) = \sum_a 2\left(\frac{\alpha_a(\mu)}{4\pi}\right)^2 C_a^i b_a^N \left[\frac{\alpha_a^2(Q_0)}{\alpha_a^2(\mu)} - \frac{b_a^N}{b_a^0}\left(1 - \frac{\alpha_a^2(Q_0)}{\alpha_a^2(\mu)}\right)\right]\left|\frac{F_S}{M}\right|^2, \tag{2.18}$$

where C_a^i is the quadratic Casimir, a is the index of the gauge group, i runs over the matter fields, and b_a^N is the contribution from the messengers to the beta function coefficients. On the basis of the above expression, the sum rules for sfermion masses that hold in gauge mediation models [35, 36] are still valid. Using condition (2.3), we obtain a sum rule for gaugino masses valid at all scales:

$$7\frac{M_3}{\alpha_3} - 12\frac{M_2}{\alpha_2} + 5\frac{M_1}{\alpha_1} = 0. \tag{2.19}$$

In contrast to a messenger sector composed of full SU(5) multiplets, the three beta function coefficients b_a^N can all be different when using magic field sets as messengers. This leads to gaugino and scalar mass hierarchies which are typically more pronounced than in the usual scenario. For instance, if the messenger sector is given by

$$Q + \overline{Q} + G,$$

the ratio between gaugino masses is strongly hierarchical

$$M_1 : M_2 : M_3 = 1 : 30 : 200,$$

and also the scalar masses turn out to be quite split

$$m_{\tilde{e}^c}/m_{\tilde{q}} \sim 1/20.$$

For a less peculiar scenario such as

$$(Q + \overline{Q}) + G + (U^c + \overline{U}^c) + (D^c + \overline{D}^c) + W,$$

we get

$$M_1 : M_2 : M_3 = 1 : 5 : 20,$$

$$m_{\tilde{e}^c}/m_{\tilde{q}} \sim 1/15.$$

A rough estimation of a typical SUSY spectrum for the two retarded solutions above, with the selectron mass taken close to the present experimental limit is

	M_1	M_2	M_3	$m_{\tilde{e}^c}$	$m_{\tilde{q}}$
$Q\overline{Q} + G$	25 GeV	750 GeV	5 TeV	100 GeV	2 TeV
$Q\overline{Q} + G + U^c\overline{U}^c + D^c\overline{D}^c + W$	75 GeV	400 GeV	1.5 TeV	100 GeV	1.5 TeV

Although the large hierarchy of gaugino masses signals that these scenarios are more fine-tuned than in usual gauge mediation, it serves at the same time as a strong experimental hint of their possible realization.

	Q	U^c	D^c	L	E^c	W	G	V	$(n,m)_y$
SU(3)$_c$	3	$\overline{3}$	$\overline{3}$	1	1	1	8	3	n
SU(2)$_L$	2	1	1	2	1	3	1	2	m
Y	1/6	-2/3	1/3	-1/2	1	0	0	-5/6	y

Table 2.1 SM quantum numbers associated to a given notation for a SM field.

Field content	b_1^N	b_2^N	b_3^N	r	type
$(6,2)_{-1/6}+\text{c.c.}$	2/5	6	10	∞	fake
$(Q+\overline{Q})+G$	1/5	3	5	-1	retarded
$(U^c+\overline{U}^c)+(D^c+\overline{D}^c)+W$	2	2	2	0	usual
$(D^c+\overline{D}^c)+G+((1,3)_1+\text{c.c.})$	4	4	4	0	usual
$(L+\overline{L})+((6,1)_{1/3}+(1,3)_1+\text{c.c.})$	5	5	5	0	usual
$(Q+\overline{Q})+(D^c+\overline{D}^c)+((8,2)_{1/2}+\text{c.c.})$	27/5	11	15	∞	fake
$W+2\,((8,2)_{1/2}+\text{c.c.})$	48/5	18	24	3	hoax
$W+((6,2)_{-1/6}+\text{c.c.})+((1,1)_2+\text{c.c.})$	26/5	8	10	-1	retarded
$((3,3)_{2/3}+(6,2)_{-1/6}+(6,1)_{4/3}+\text{c.c.})$	18	18	18	0	usual
$2W+((6,2)_{5/6}+\text{c.c.})$	10	10	10	0	usual
$((3,3)_{2/3}+(6,2)_{5/6}+(6,1)_{-2/3}+\text{c.c.})$	18	18	18	0	usual
$((8,1)_1+(\overline{3},1)_{4/3}+\text{c.c.})+(8,3)_0$	16	16	16	0	usual
$((8,1)_1+(6,1)_{1/3}+\text{c.c.})+(8,3)_0$	52/5	16	20	∞	fake

Table 2.2 Simplest irreducible magic sets that can be built from SM representations belonging to SO(10) representations with dimension up to **210** and do not correspond to full SU(5) multiplets or anticipated unification.

2.5 Examples

In this section we show the results of a systematic analysis of magic field contents. Note that merging two or more magic sets still gives a magic set of fields. In particular, adding a magic content with $r=0$ does not modify the type of unification; adding two retarded solutions gives a fake solution, and adding a fake to a retarded solution or to another fake gives a hoax solution. Table 2.2 contains the simplest irreducible magic sets that can be built from SM representations belonging to SO(10) representations with dimension up to **210**. The notation for these representations is explained in Table 2.1. We have not included field sets that form complete SU(5) multiplets. Table 2.3 shows the simplest irreducible magic sets which provide retarded unification. Table 2.4 shows the simplest irreducible magic contents for the Pati-Salam case. Again we write only fields belonging to representations of SO(10) up to **210**.

Field content	b_1^N	b_2^N	b_3^N	r
$\left(Q+\overline{Q}\right)+G$	1/5	3	5	-1
$\left(E^c+\overline{E}^c\right)+2W+2G$	6/5	4	6	-1
$2\left(L+\overline{L}\right)+W+2G$	6/5	4	6	-1
$\left(Q+\overline{Q}\right)+\left(U^c+\overline{U}^c\right)+\left(D^c+\overline{D}^c\right)+W+G$	11/5	5	7	-1
$3\left(D^c+\overline{D}^c\right)+2W+G$	6/5	4	6	-1
$\left(U^c+\overline{U}^c\right)+\left(L+\overline{L}\right)+2W+2G$	11/5	5	7	-1
$\left(Q+\overline{Q}\right)+2\left(D^c+\overline{D}^c\right)+\left(E^c+\overline{E}^c\right)+W+G$	11/5	5	7	-1
$2\left(Q+\overline{Q}\right)+\left(D^c+\overline{D}^c\right)+2\left(E^c+\overline{E}^c\right)+G$	16/5	6	8	-1
$2\left(Q+\overline{Q}\right)+\left(U^c+\overline{U}^c\right)+3\left(D^c+\overline{D}^c\right)$	16/5	6	8	-1
$2\left(Q+\overline{Q}\right)+2\left(U^c+\overline{U}^c\right)+\left(L+\overline{L}\right)+G$	21/5	7	9	-1
$2\left(Q+\overline{Q}\right)+2\left(D^c+\overline{D}^c\right)+G+\left(V+\overline{V}\right)$	31/5	9	11	-1

Table 2.3 Simplest irreducible magic sets which provide retarded unification. We show only fields belonging to representations of SO(10) up to **45**.

Field content	b_4^N	b_L^N	b_R^N	r
$(6,1,3)$	3	0	12	0
$(1,2,2)+((20',1,1)+\text{c.c.})$	8	1	1	∞
$(6,1,1)+((10,1,1)+\text{c.c.})$	7	0	0	∞
$((10,1,1)+\text{c.c.})+(15,2,2)$	22	15	15	∞
$(1,2,2)+2(15,1,1)$	8	1	1	∞
$(6,1,1)+(6,2,2)+((20',1,1)+\text{c.c.})$	13	6	6	∞
$(6,1,1)+(6,1,3)+(1,2,2)$	4	1	13	0
$((4,1,2)+(4,2,1)+\text{c.c.})+(6,1,3)$	7	4	16	0
$(1,3,3)+((10,1,1)+\text{c.c.})+(6,1,3)$	9	6	18	0
$(6,2,2)+((20',1,1)+\text{c.c.})+(15,2,2)$	28	21	21	∞
$(1,2,2)+(6,1,3)+(15,2,2)$	19	16	28	0
$(1,1,3)+(6,1,3)+((20,2,1)+\text{c.c.})$	29	20	14	-3
$(6,1,3)+((4,2,3)+(20,2,1)+\text{c.c.})$	35	32	44	0
$(6,1,3)+((4,3,2)+(20,1,2)+\text{c.c.})$	35	32	44	0
$(6,2,2)+(6,3,1)+(15,1,3)$	19	18	36	1/3
$(1,2,2)+(15,1,1)+((10,2,2)+\text{c.c.})$	28	21	21	∞
$(1,2,2)+2((10,2,2)+\text{c.c.})$	48	41	41	∞

Table 2.4 Simplest irreducible magic contents for the Pati-Salam case that can be built from PS representations belonging to SO(10) representations with dimension up to **210** and do not correspond to full SU(5) multiplets or anticipated unification. We denote the fields as (a,b,c), where a, b, c are representations of $SU(4)_c$, $SU(2)_L$, $SU(2)_R$ respectively.

3

Summary

In this part we systematically analyzed "magic" fields, which are sets of SM chiral superfields that do not form complete SU(5) multiplets, but exactly preserve the one-loop MSSM prediction for α_3, independently of their mass scale. Unlike full SU(5) multiplets, such magic field sets can have an impact on the GUT scale. In particular, we have shown that M_{GUT} can be increased in three ways, through a delayed convergence of the gauge couplings, a fake unified running of the gauge couplings below the GUT scale, or a late unification after an hoax crossing of the gauge couplings at a lower scale. We have also shown several examples of dynamics giving rise to magic field contents below the unification scale.

Magic fields can have several applications. For example they can fix gauge coupling unification in two step breakings of the unified group by compensating the contribution to the beta function of the additional gauge bosons at the intermediate scale. Or they can be used to suppress too large thresholds from KK towers in models in which unification is achieved in extra dimensions. Another possibility is that they play the role of messengers of supersymmetry breaking in GMSB models, which typically leads to a more pronounced hierarchy of gaugino masses. In summary we regard magic fields as a useful tool in GUT model building.

Part II

Tree-level Gauge Mediation

4
Introduction

The Minimal Supersymmetric Standard Model (MSSM) is certainly one of the most popular candidates for physics beyond the electroweak scale. It not only predicts precision gauge coupling unification at a scale large enough to explain the long proton lifetime, but it also protects the Higgs mass from the influence of such huge scales and therefore provides a solution to the technical aspect of the Hierarchy Problem. The reason for both these successes is that the MSSM becomes approximately supersymmetric just above the weak scale, meaning that SUSY is violated by interactions involving MSSM fields only softly and the associated scale lies not far above the weak scale.

The MSSM however does not explain the origin of soft SUSY breaking operators, but merely parametrizes all possible soft terms compatible with the other symmetries. This leads to a plethora of new parameters whose structure is strongly constrained by experiments, in particular flavor physics. Many models have been built in order to address the origin of soft SUSY breaking in the MSSM. What typically matters here are not the details how SUSY broken at first place, but how it is communicated to the MSSM, because this is what mainly determines the structure of the soft terms. In four dimensions there are two popular scenarios, in which SUSY breaking is mediated either by gauge interactions at the loop-level or by gravity. In this part instead we discuss a novel scenario that we called tree-level gauge mediation (TGM) [25, 26]. This mechanism might be the simplest possibility to communicate SUSY breaking to the MSSM: through the tree-level exchange of heavy gauge fields, motivated by grand unified theories. We will show that this gives rise to tree-level sfermion masses that are flavor-universal, thus solving the SUSY Flavor Problem.

This framework is not only simple, but also predictive, leading to a fixed ratio of sfermion masses that is to a large extent model-independent. Of course one might wonder why such an attractive scenario has not been considered before[1]. Two main reasons seem to have prevented this possibility. The first is related to the existence of a mass sum rule which constraints the tree-level spectrum. We will show that these constraints are not as severe as usually stated, but can be satisfied by including both heavy matter charged under the SM gauge group and a new gauge U(1) gauge field, ingredients that are naturally present in grand unified theories. The other objection regards the fact that in contrast to sfermion masses gaugino masses cannot arise at tree-level. Generating them on loop-level instead would lead to large hierarchy between the soft masses and result in very heavy sfermions of $\mathcal{O}\,(10\,\mathrm{TeV})$. In our framework gaugino masses arise at loop-level, but the loop factor can easily be compensated by numerical factors, reducing or even eliminating the hierarchy among soft masses. Therefore TGM is a viable and attractive mechanism to explain the origin of soft terms in the MSSM, leading to distinct predictions that make this scheme testable at the LHC.

We begin with an overview chapter aiming to explain the framework of TGM in a concise way. We first discuss the tree-level origin of sfermion masses and show how the constraints from the mass sum rule are satisfied. Then we turn to one-loop gaugino masses and sketch how the loop factor can be compensated by numerical factors. We finally show how these concepts can be realized in a simple SO(10) model where we illustrate the characteristic features of TGM, regarding both the implementation of the main ideas and its phenomenological consequences. In the following two chapters we take a more general point of view. First we provide complete expressions for tree-level and one-loop soft masses in a generic setup of TGM, which we then use to define some guidelines for model building. Finally we discuss the origin of the μ-term in TGM.

[1]For earlier works in this direction see [37, 38].

5

Overview

5.1 Tree-Level Sfermion Masses

In the spurion formalism [13] soft sfermion masses arise from the effective operator

$$\int d^4\theta \, \frac{Z^\dagger Z Q_i^\dagger Q_j}{M^2}, \tag{5.1}$$

where Z is a SM singlet chiral superfield whose F-term vev breaks supersymmetry, $\langle Z \rangle = F\theta^2$, and Q is a generic MSSM chiral superfield with flavor index i. We assume $F \ll M^2$ such that the above operator gives the dominant contribution to sfermion masses.

The main idea of tree-level gauge mediation is that this operator arises from a renormalizable tree-level exchange of heavy vector superfield as in Fig. 5.1. The

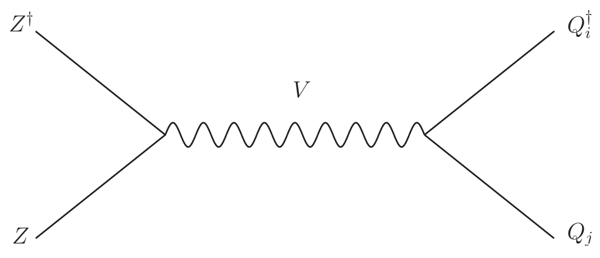

Figure 5.1 Tree-level supergraph inducing sfermion masses.

vector superfield V is a SM singlet and associated to a heavy U(1) factor which is non-anomalous[1] and part of a simple unified group such as SO(10) or E_6. Denoting the mass of the heavy vector by M_V, the corresponding broken generator by X and the associated charges of Z and Q_i by X_Z and X_Q respectively, the above diagram induces the Kähler potential operator

$$-2g^2 X_Z X_Q \delta_{ij} \int d^4\theta \, \frac{Z^\dagger Z Q_i^\dagger Q_j}{M_V^2} \tag{5.2}$$

in the effective theory below $M_V \approx M_{\rm GUT}$[2]. This term gives in turn rise to sfermion masses

$$(\tilde{m}_Q^2)_{ij} = 2g^2 X_Z X_Q \delta_{ij} \left(\frac{F}{M_V}\right)^2. \tag{5.3}$$

We make the following observations:

- The sfermion masses do not actually depend on the gauge coupling (and X-charge normalization), because the vector squared mass M_V^2 is also proportional to g^2 (and two units of X-charges).

- Since they arise from gauge interactions, sfermion masses are flavor-universal. Therefore this mechanism provides a solution to the SUSY Flavor Problem.

- Ratios of different sfermion masses depend only on the corresponding charge ratios and thus provide the main prediction of this mechanism, which is distinct from all other scenarios of SUSY breaking mediation.

- To have positive sfermion masses at tree-level, the embedding of MSSM fields Q_i and the spurion field Z into multiplets of the unified group must be such that the product $X_Z X_Q$ is positive.

- Since the X-generator is traceless over a complete multiplet, there must be some sfermions which acquire negative SUSY breaking masses. This is not a problem since such fields can have large supersymmetric mass terms, to which the SUSY breaking contribution is just a small correction.

The above result for sfermion masses holds in the effective theory below the GUT scale where we can integrate out the additional gauge fields. In the full theory at $M_{\rm GUT}$ instead sfermion masses must arise from a renormalizable operator, which can only be the coupling of the sfermions to the D-term of the heavy gauge field.

[1] The possibility that SUSY breaking is mediated by an anomalous U(1) gauge factor has been considered in [39].

[2] Throughout this chapter we assume that the U(1) factor is broken near the GUT scale, but in principle the breaking scale could also be much smaller.

Indeed a vev for the D-term is induced by the F-term vev of the spurion according to

$$\langle D_X \rangle = -2gX_Z \left(\frac{F}{M_V} \right)^2, \tag{5.4}$$

which in turn generates sfermion masses

$$(\tilde{m}_Q^2)_{ij} = -gX_Q \langle D_X \rangle = 2g^2 X_Z X_Q \delta_{ij} \left(\frac{F}{M_V} \right)^2, \tag{5.5}$$

reproducing the result obtained in the effective theory.

5.2 The Mass Sum Rule

It is sometimes stated that the restrictions imposed by the supertrace formula rule out the possibility of tree-level SUSY breaking, e.g. [40]. We therefore review the potential problems on which these arguments are based on and show how they are solved in our framework.

As we just have seen, sfermion masses arise at tree-level in a renormalizable theory. Therefore the spectrum is constrained by the mass sum rule [41], which states that the supertrace of the squared masses equals the trace over the D-term vevs

$$\mathrm{Str}\, \mathcal{M}^2 = -2gD_a \,\mathrm{Tr}(T_a), \tag{5.6}$$

which holds separately for each set of conserved quantum numbers [10]. If the trace is taken over the full spectrum, the supertrace has to vanish in the absence of anomalies.

Since the experimental lower bounds on MSSM sfermion masses require them to be significantly larger than the MSSM fermion masses, we necessarily need a positive supertrace for the tree-level MSSM spectrum[3]

$$\mathrm{Str}\, \mathcal{M}^2_{\mathrm{MSSM}} > 0. \tag{5.7}$$

In order to end up with a vanishing supertrace over the full spectrum, we necessarily have to add new fields (with non-trivial MSSM quantum numbers) with negative supertrace, such that

$$\mathrm{Str}\, \mathcal{M}^2_{\mathrm{MSSM}} + \mathrm{Str}\, \mathcal{M}^2_{\mathrm{new}} = 0. \tag{5.8}$$

[3]Note that gaugino masses do not contribute since they do not arise at tree-level.

Therefore the mass sum rule requires in general the presence of additional fields beyond the MSSM ones. This is precisely the case in our TGM framework, where the tracelessness of the X generator implies the presence of fields with negative X charge. Their scalar components pick up negative masses from the D-term vev, leading to a negative supertrace. These additional fields are simply the GUT partners of MSSM fields, i.e. they are together in the same multiplet of the unified group. They will get a large supersymmetric mass term so that the SUSY breaking contribution from D-terms is only a small correction, and all scalars in the theory have positive masses. As we will see in the next chapter, this splitting can be obtained without ad hoc model building efforts.

An even stronger implication of the mass sum rule was derived by Dimopoulos and Georgi [10]. It basically states that one cannot obtain a positive supertrace for all MSSM fields using only MSSM D-terms. The reason is that only D-terms associated to the hypercharge and diagonal SU(2) generators can get a vev, but some of the MSSM fields always carry negative charges under these generators and thus get negative supertraces, which in term implies the existence of sfermions lighter than the corresponding fermions. More precisely, one can show that there must be either an up-type squark lighter than the up quark, or a down-type squark lighter than the down quark [10, 40]. The above argument is however invalidated in the presence of a new U(1) gauge factor, under which the all MSSM fields carry positive charge. This is precisely what happens in our setup, and actually defines it.

5.3 Gaugino Masses

Besides the mass sum rule, another objection against tree-level SUSY breaking is the argument that (Majorana) gaugino masses cannot arise at tree-level[4]. Therefore gaugino masses are expected to be suppressed with respect to scalar masses. If this suppression is very large, the lower bounds on gaugino masses imply that sfermions are very heavy, leading to a significant fine-tuning in the determination of the Higgs mass and approaching the regime of split supersymmetry [38, 42]. Indeed, a toy-model version of the basic tree-level mechanism was discussed in that context [38].

In the framework we consider here instead, a large hierarchy between sfermion and gauginos is avoided. Gaugino masses arise at loop-level as in ordinary gauge

[4] A coupling scalar-gaugino-gaugino that could give rise to such a mass term is forbidden by supersymmetry.

mediation, but the loop factor is partially (or fully) compensated by numerical factors of different origin as we are going to discuss briefly in this section.

In order to generate gaugino masses at loop-level, we need heavy fields with SM quantum numbers that couple to the SUSY breaking F-term vev. In our scenario heavy fields charged under the SM are naturally present, and even required as we have seen in the discussion of the mass sum rule. If we couple them to the same SUSY breaking F-term vev we used for the sfermion masses, these fields act as the messengers of gauge mediation and generate gaugino masses at one-loop which are schematically of the form

$$M_a \sim N \frac{g_a^2}{16\pi^2} \frac{F}{M}, \tag{5.9}$$

where M is the mass scale of the messengers and N is the messenger index.

First note that the mass scale of these messengers and the mass scale M_V of the heavy vector inducing sfermion masses could in principle be completely unrelated, thus leaving any freedom to choose the gaugino mass scale. However, for the sake of model-building elegance and predictivity, it is clearly desirable that sfermion and gaugino masses originate from a common single scale, which sets both the mass of the loop messengers and the scale of U(1) breaking, up to coupling constants. Still, these coupling constants can give a sizable enhancement of gaugino masses, in particular if they are related to small Yukawa couplings, as we will see in the next section.

Another source of $\mathcal{O}(1)$ factors that always tends to compensate the loop factor is the cumulative effect of vevs suppressing sfermion masses and messengers contributing to gaugino masses. That is, the tree-level messenger mass suppressing sfermion masses receives contributions from all vevs breaking the U(1), and gaugino masses receive contributions from all heavy fields with SM quantum numbers coupling to SUSY breaking, described by the messenger index. Finally, sfermion masses depend on a ratio of U(1) charges that can also be small.

We will see in the next section that even in simple models these enhancement factors are naturally present and can easily reduce the naive loop hierarchy between sfermion and gaugino masses to a factor 10, leading to sfermions at the TeV scale. Note also that enhancing the one-loop gaugino masses implies enhancing the usual two-loop contribution to sfermion masses. Therefore a small suppression of gaugino masses is actually desirable, since it implies that the novel tree-level contribution to sfermion masses dominates the ordinary two-loop one.

5.4 A Simple SO(10) Model

In this section we discuss an explicit SO(10) model that gives rise to tree-level sfermion masses and one-loop, enhanced gaugino masses along the lines of the previous sections. The main purpose is to illustrate the characteristic features of TGM in a simple setup, regarding both the implementation of the main idea and its phenomenological consequences. We do not intend to build a fully realistic model, and therefore will ignore the details of SO(10) breaking to the SM, the origin of neutrino masses and the generation of the MSSM flavor structure. Instead we would like to demonstrate that the peculiar requirements of TGM can be easily realized, namely i) the large SUSY mass needed for GUT partners of MSSM fields with negative SUSY breaking masses and ii) the presence of enhancement factors compensating the loop suppression of gaugino masses. Furthermore we use this simple model to show that the tree-level result for sfermion masses can easily dominate other contributions to sfermion masses, in particular the ordinary two-loop contribution from gauge mediation and the gravity-mediated one. We will also discuss A-terms, showing that they are mainly generated by the usual RG evolution, although there are other, strongly model-dependent contributions. Since also the implementation of MSSM Higgs sector is pretty model-dependent, we comment only briefly on the possible origin of B_μ and the μ-term. Finally we discuss the main phenomenological and cosmological consequences of the model, which are again representative for the whole class of models based on TGM.

5.4.1 Setup

Our aim is to construct a model in which sfermion masses are generated at tree-level by integrating out heavy vector fields as described in Section 5.1. Such a mechanism requires specific gauge structures and field contents and we will discuss these constraints in extenso in the next chapters. Let us nevertheless briefly motivate the setup we are using in this section.

First of all, the heavy vector field V in Fig. 5.1 must be a SM singlet, as Z is. Therefore we need a gauge group with a least rank 5 and an obvious choice is to identify the broken generator with the SU(5) singlet generator X of SO(10). The SM singlet Z whose F-term breaks supersymmetry must belong to a non-trivial SO(10) multiplet such that Z has a non-vanishing charge under X. The easiest possibility is that Z is the SU(5) singlet component of the spinorial representation **16**. We also need a $\mathbf{16}+\overline{\mathbf{16}}$ to break SO(10) at the GUT scale[5]. In total two $\mathbf{16}+\overline{\mathbf{16}}$ are then required, one getting a vev along the scalar component and the other along the F-term component. Note that gauge invariance prevents us from using the same field for scalar and F-term vev.

What regards the matter fields, the embedding of MSSM fields Q into SO(10) representations must be such that the sign of $X_Z X_Q$ is positive, see Eq. (5.3). In our charge conventions, the decomposition of the **16** of SO(10) under $SU(5)\times U(1)_X$ is given by

$$\mathbf{16} = \mathbf{10}_1 + \overline{\mathbf{5}}_{-3} + \mathbf{1}_5,$$

so $X_Z = 5$. It is clear that the standard embedding of a whole MSSM family into a 16 of SO(10) cannot work, because sfermions embedded in the $\overline{\mathbf{5}}$ of the **16** would have negative squared masses. But also the **10** of SO(10) contains a $\overline{\mathbf{5}}$ according to

$$\mathbf{10} = \overline{\mathbf{5}}_2 + \mathbf{5}_{-2},$$

so this one has positive X-charge. We therefore distribute the MSSM fields in three **16** and three **10** of SO(10), where MSSM fields in a **10** of SU(5) are embedded into the **16** of SO(10) with $X_{10} = 1$, while the fields in a $\overline{\mathbf{5}}$ of SU(5) are embedded into the **10** of SO(10) with $X_{\overline{5}} = 2$. The spare fields in these SO(10) representations are vectorlike under SU(5) and thus can obtain a large supersymmetric mass term. We will see in a moment that this splitting can be obtained very easily.

The mixed embedding of MSSM matter fields determines the embedding of the Higgs fields. In order to obtain the Yukawas from SO(10) invariant operators, the up-type Higgs h_u must at least partially reside in a **10** of SO(10) while the down-type Higgs h_d must be at least partially in a **16** of SO(10). If this embedding would be pure however, both Higgs scalars would acquire negative soft masses. This could be a potential risk for obtaining correct electroweak symmetry breaking, which requires $m^2_{h_u} + m^2_{h_d} + 2|\mu|^2 > 2|B\mu|$ at the GUT scale and below. We therefore allow for the more general possibility that both Higgs fields are linear combinations of the **10** and the **16** and $\overline{\mathbf{16}}$, respectively. The sign of their soft masses is then determined by the mixing angles which we take as free parameters.

Let us summarize the setup of our model. The gauge group is SO(10), and the matter fields (negative R-parity) are embedded in three families of 16_i =

[5]Another representation such as a **45** is needed to break SO(10) to the SM, but this is not relevant for sfermion masses which gets contributions only from the heavy U(1) generator.

$(\overline{5}^{16}, 10^{16}_{\rm SM}, 1^{16})_i$ and $10_i = (5^{10}, \overline{5}^{10}_{\rm SM})_i$, $i = 1, 2, 3$, where we indicated the MSSM matter fields. Supersymmetry and SO(10) breaking to SU(5) is provided by positive R-parity fields $16 = (\overline{5}^{16}, 10^{16}, N)$, $\overline{16} = (5^{16}, \overline{10}^{16}, \overline{N})$, $16' = (\overline{5}'^{16}, 10'^{16}, Z)$, $\overline{16}' = (5'^{16}, \overline{10}'^{16}, \overline{Z})$, with vevs

$$\langle Z \rangle = F\,\theta^2, \qquad \langle \overline{Z} \rangle = 0, \qquad \langle N \rangle = M, \qquad \langle \overline{N} \rangle = M, \tag{5.10}$$

and $\sqrt{F} \ll M \sim M_{\rm GUT}$. The D-term condition forces $|\langle N \rangle| = |\langle \overline{N} \rangle|$ and the phases of all vevs can be taken positive without loss of generality. The MSSM up-type Higgs h_u is a linear combination of doublets in a $10 = (5^{10}, \overline{5}^{10})$ and the $\overline{16}$ of SO(10), while the down-type Higgs h_d is a mixture of the doublets in the 10 and the 16,

$$10 = c_u h_u + c_d h_d + \text{heavy}, \quad 16 = s_d h_d + \text{heavy}, \quad \overline{16} = s_u h_u + \text{heavy}, \tag{5.11}$$

where $c_{u,d} = \cos\theta_{u,d}$, $s_{u,d} = \sin\theta_{u,d}$ and $0 \leq \theta_{u,d} \leq \pi/2$ parametrize the mixing in the up and down Higgs sector.

We have checked that it is possible to generate the required vevs, break SU(5) to the SM, achieve doublet-triplet splitting and Higgs mixing as above, and give mass to all the extra fields with an appropriate superpotential $W_{\rm vev}$ involving additional SO(10) representations, see Appendix B.

5.4.2 Sfermion and Higgsino Masses

Before specifying the superpotential we can already calculate soft scalar masses by integrating out the heavy vector fields, using Eq. (5.3) with $M_V^2 = 2g^2 X_Z^2 M^2$

$$\tilde{m}_Q^2 = \frac{X_Q}{2X_Z}\, m^2, \qquad m \equiv \frac{F}{M}. \tag{5.12}$$

Putting in the X-charges, we obtain SU(5) invariant sfermion masses

$$\begin{aligned} \tilde{m}_q^2 = \tilde{m}_{u^c}^2 = \tilde{m}_{e^c}^2 = \tilde{m}_{10}^2 = \frac{1}{10}\, m^2, \\ \tilde{m}_l^2 = \tilde{m}_{d^c}^2 = \tilde{m}_{\overline{5}}^2 = \frac{1}{5}\, m^2, \end{aligned} \tag{5.13}$$

and Higgsino masses

$$
\begin{aligned}
m_{h_u}^2 &= -\frac{2c_u^2 - 3s_u^2}{10}\, m^2, \\
m_{h_d}^2 &= \frac{2c_d^2 - 3s_d^2}{10}\, m^2,
\end{aligned}
\tag{5.14}
$$

which holds at the scale of the heavy vector messengers $M_V \sim M_{\text{GUT}}$. Therefore all sfermion masses are positive and flavor-universal, with sfermions masses belonging to the 10 and $\overline{5}$ of SU(5) related by

$$
\tilde{m}^2_{q,u^c,e^c} = \frac{1}{2}\tilde{m}^2_{l,d^c} \tag{5.15}
$$

at the GUT scale. Because gaugino masses are typically small, this peculiar relation holds approximately also at low energies.

5.4.3 Superpotential and Yukawas

We now write the most general R-parity conserving superpotential for our field content, except a possible mass term for the 10_i[6]

$$
W = \frac{y_{ij}}{2} 16_i 16_j 10 + h_{ij} 16_i 10_j 16 + h'_{ij} 16_i 10_j 16' + W_{\text{vev}} + W_{\text{NR}}, \tag{5.16}
$$

where $W_{\text{vev}} = W_{\text{vev}}(16, \overline{16}, 10, \ldots)$ does not involve the matter fields and takes care of the vevs, the doublet triplet splitting, and the Higgs mixing. W_{NR} contains non-renormalizable contributions to the superpotential needed in order to account for the measured ratios of down quark and charged lepton masses, an issue that we will ignore in our discussion.

The first two terms reproduce the MSSM Yukawas (at the renormalizable level and at the GUT scale) given by[7]

$$
y^U_{ij} = \cos\theta_u y_{ij}, \qquad y^E_{ij} = y^D_{ij} = \sin\theta_d h_{ij}. \tag{5.17}
$$

Because the 16 gets a vev in its SU(5) singlet component, the second term also gives rise to large SUSY mass terms for the additional matter fields $\overline{5}_i^{16}$ and 5_i^{10}

[6] Such a mass term would imply that the MSSM fields in the $\overline{\mathbf{5}}$ of SU(5) reside in a linear combination of both 10_i and 16_i. We insist instead of a pure embedding in order to avoid possible dangerous flavor-violating effects, see the discussion in Section 7.2.1.

[7] Note that despite the SO(10) structure, the up-quark Yukawa matrix is not correlated to the down-quark and charged lepton Yukawa matrix. This allows to accommodate the stronger mass hierarchy observed in the up quark sector.

that have negative X-charge. Therefore only the MSSM superfield content survives at the electroweak scale, provided that the three singlets in the 16_i also get a mass, e.g. from non-renormalizable interactions with the $\overline{16}$. This shows that the required embedding of the MSSM fields in SO(10) representations can indeed easily be realized.

The third term provides a large SUSY breaking mass term for the heavy $\bar{5}_i^{16}$ and 5_i^{10}, since the $16'$ gets an F-term vev. These fields can therefore act as the messengers of ordinary gauge mediation and induce one-loop gaugino masses.

5.4.4 Gaugino Masses

While sfermion masses arise at tree-level and only depend on the choice of the unified gauge group and the MSSM embedding, gaugino masses arise at one-loop and depend on the superpotential parameters. Because the chiral multiplets $\bar{5}_i^{16}$ and 5_j^{10} get a large supersymmetric mass $h_{ij}M$ and a SUSY breaking mass $h'_{ij}F$, they play the role of three pairs of chiral messengers in ordinary gauge mediation. They generate one-loop gaugino masses, at the GUT scale given by

$$M_a = \frac{\alpha}{4\pi}\,\mathrm{Tr}(h' h^{-1})\,m \equiv M_{1/2}, \quad a = 1, 2, 3, \tag{5.18}$$

where α is the unified gauge coupling and we neglected running effects due the fact that the contribution of each messenger arises at a separate scale different from M_{GUT}.

Let us compare this result to sfermion masses. Particularly interesting is the ratio $\tilde{m}_t/M_2$, because the Wino mass M_2 is bounded to be larger than about 100 GeV, while $\tilde{m}_t$ dominates the radiative corrections to the Higgs mass. Therefore, the ratio $\tilde{m}_t/M_2$ should not be too large in order not to increase the fine-tuning. From

$$\left.\frac{M_2}{\tilde{m}_t}\right|_{M_{\mathrm{GUT}}} = \frac{3\sqrt{10}}{(4\pi)^2}\,\lambda, \qquad \lambda = \frac{g^2\,\mathrm{Tr}(h' h^{-1})}{3} \tag{5.19}$$

we see that the loop factor separating $\tilde{m}_t$ and M_2 is partially compensated by a combination of numerical factors. The naive loop factor $(4\pi)^2 \sim 100$. which would lead to $\tilde{m}_t \gtrsim 10\,\mathrm{TeV}$, is replaced by $(4\pi)^2/(3\sqrt{10}) \sim 10$, leading to $\tilde{m}_t \gtrsim 1\,\mathrm{TeV}$ for $\lambda = 1$. Let us spell out the origin of these enhancement factors: a factor $\sqrt{5}$ is related to the ratio of X charges in Eq. (5.12), another factor $\sqrt{2}$ comes from the two contributions to the heavy vector mass from both the 16 and $\overline{16}$ vevs and the factor 3 corresponds to the number of messengers, here equal to the number of families ($\text{Tr}(h'h^{-1}) = 3$ for $h = h'$). A largish value of the factor λ can then further reduce the hierarchy and even make $M_2 \sim \tilde{m}_t$, if needed. Both $\mathcal{O}(1)$ and large values of λ are in fact not difficult to obtain, depending on the overall size and flavor structure of h and h'[8]. Note however that a large value of λ enhances also the standard two-loop contribution to sfermion masses, as we are going to discuss now.

5.4.5 Other Contributions to Sfermion Masses

Reducing the hierarchy between gaugino and sfermion masses implies reducing the hierarchy between the two-loop contributions to sfermion masses from standard gauge mediation and the tree-level result in Eq. (5.12). To quantify the relative size of these two contributions, let us consider for simplicity the basis in the messenger flavor space in which the matrix h is diagonal and positive, the limit in which h' is also diagonal in that basis, and let us call h_i and h'_i, $i = 1, 2, 3$ their eigenvalues. Neglecting the running between the GUT scale and the mass of the relevant messengers[9], the sfermion masses are given, at the high scale, by

$$\tilde{m}^2_Q = (\tilde{m}^2_Q)_{\text{tree}} + 2\,\eta\, C_2(Q) M^2_{1/2}, \qquad \eta = \frac{\sum (h'_i/h_i)^2}{(\sum_i h'_i/h_i)^2} \geq \frac{1}{3}, \tag{5.20}$$

where $(\tilde{m}^2_Q)_{\text{tree}}$ is the tree-level value given in Eqs. (5.13, 5.14) and $C_2(Q)$ is the total SM quadratic casimir of the sfermion $\tilde{Q}$ (or Higgs Q)

Q	q_i	u^c_i	d^c_i	l_i	e^c_i	h_u	h_d
$C_2(Q)$	21/10	8/5	7/5	9/10	3/5	9/10	9/10

(5.21)

If the contribution of a single messenger dominates gaugino masses, $\eta \approx 1$. In the numerical example we will consider later, the relative size of the two loop contribution to sfermion masses ranges from 2% to 10%.

Additional, subleading contributions to sfermion masses can arise from different

[8] h is related to the down quark Yukawa matrix and has a hierarchical structure, with two eigenvalues certainly small and the third one, related to the bottom Yukawa, also allowed to be small, depending on θ_d and $\tan\beta$.

[9] The relevant messengers are the ones with the largest h'_i/h_i. If the most relevant messenger is the third family one, the effect of the running that we are neglecting is not too large. The third family messenger mass is in fact given by $h_3 M = m_b/(v\cos\beta\sin\theta_d)M$ (m_b is the bottom mass, $v = 174\,\text{GeV}$), not too far (in logarithmic scale) from $M \sim M_{\text{GUT}}$. Still, we expect the messengers to be lighter enough than the GUT scale in such a way that only the SM casimirs (and not the GUT ones) are relevant.

sources. One-loop contributions from an induced $\mathrm{U}(1)_X$ Fayet-Iliopoulos term [43] only arise if h' is non-diagonal in the basis where h is diagonal and $|h'_{ij}| \neq |h'_{ji}|$. Moreover, they are suppressed (typically negligible) because $\mathrm{U}(1)_X$ is broken above the scale of the loop messengers. Another contribution could come from gravity effects. Since in our scenario the messenger scale is expected to be around the GUT scale, the gravity mediated contribution to the spectrum, although subleading, could be relevant for flavor physics, as it could in principle be strongly flavor violating. In order to quantify this effect, let us assume that the gravity contribution to an arbitrary entry of the squared mass matrix of the sfermions in the **10** of SU(5) is given by $\tilde{m}^2_{\text{grav}} = F^2/M^2_{\text{Pl}}$, where $M_{\text{Pl}} = 2.4 \cdot 10^{18}\,\text{GeV}$ is the reduced Planck mass. The conservative bound $\tilde{m}^2_{\text{grav}} < 2 \cdot 10^{-3}\,\tilde{m}^2_{10}$, which guarantees that all FCNC effects are under control, then translates in the following bound on the messenger scale:

$$M < 3 \cdot 10^{16}\,\text{GeV}. \tag{5.22}$$

Note that this bound is roughly an order of magnitude larger than in ordinary gauge mediation, due to the absence of the loop factor in the expression for sfermion masses. If the messenger scale exceeds this bound, we are in a hybrid framework and contributions to soft terms from gravity mediation become relevant for flavor observables [44].

Finally, another potentially relevant source of flavor non-universality might come from one-loop contributions to sfermion masses arising from the superpotential Yukawa interactions in Eq. (5.16), once the necessary mass terms for the components of the 16 and $16'$ are taken into account. Such effects are certainly under control if the matrix h', as h, has a hierarchical structure and is approximately aligned to h.

5.4.6 Other Soft Terms and the μ-Problem

Besides sfermion masses also A-terms can arise at one-loop, due to direct couplings between messengers and observable fields in Eq. (5.16). Assuming for simplicity that the matrices h' and y are diagonal in the same basis in which h is, we have

$$\begin{aligned} A_{l_i, d^c_i} &= -\frac{1}{4\pi^2}\frac{h'_i}{h_i}\left(h_i^2 + h_i'^2\right) m \\ A_{q_i, u^c_i, e^c_i} &= -\frac{1}{(4\pi)^2}\frac{h'_i}{h_i}\left(3(h_i^2 + h_i'^2) + 2y_i^2\right) m \end{aligned} \tag{5.23}$$

at the messenger scale[10]. Within the simplified diagonal flavor structure we are considering, we can compare the A-terms with the gaugino masses in Eq. (5.18), which are in this case proportional to $\sum_i h'_i/h_i$. This means that the largest A-terms at the high scale are comparable or smaller than the gaugino masses, depending on which term dominates the sum. One can therefore expect that the dominant contribution to A-terms at low energy comes from the usual RGE evolution proportional to the gaugino masses. It is desirable to verify this expectation in a complete model, since it is based on a simplified flavor structure of the couplings and neglects other, model-dependent sources of one-loop A-terms, due to the necessary presence of mass terms for the components of the 16 and $16'$. We are planning to perform such an analysis in a future publication.

Next, we comment on the μ-Problem. Relating the μ-term to SUSY breaking is a highly model-dependent issue, due to the various possibilities of implementing supersymmetry breaking and embedding the Higgs fields in SO(10). We will discuss several approaches to the μ-Problem in our framework in Section 7.4. At this point we want only to anticipate a simple possibility in which both the F-term, $\langle Z \rangle = F\,\theta^2$ and μ originate from the same parameter $m \sim$ TeV in the superpotential $W \supset mZ\overline{N}$. Once $\overline{N}$ gets its vev $\langle \overline{N} \rangle = M \sim M_{\text{GUT}}$, Z acquires an F-term $F = mM$, so that m is indeed the parameter introduced in Eq. (5.12). Now Z and $\overline{N}$ are part of the SO(10) multiplets $16'$ and $\overline{16}$ respectively. A μ-term related to the supersymmetry breaking scale $\mu \sim m$ therefore arises if h_u has a component in $\overline{16}$ and h_d has a component in $16'$. Such a situation can be achieved with an appropriate superpotential, see Appendix B. Note that in this solution of the μ-Problem it is the SO(10) structure which relates the μ-term to SUSY breaking.

In contrast to standard gauge mediation there is no μ-B_μ problem here, because the μ-term does not have to be generated at one-loop and therefore does not give rise to B_μ at the same level of suppression. Instead B_μ can be generated at the tree-level, for example as in [38], or it can be induced by the RGE evolution.

5.4.7 Phenomenology

We now briefly discuss the phenomenological aspects of our model which are characteristic for tree-level gauge mediation. We concentrate on the case where λ is not too large, because otherwise we would merely mimic the phenomenology of ordinary gauge mediation. The two main predictions at the high scale are then:

[10] We define the sign of the A-terms according to $\mathcal{L} \supset -\sum_Q A_Q \tilde{Q}(\partial W(\tilde{Q}))/(\partial Q)$.

- SU(5) invariant sfermion masses with fixed ratio for sfermions embedded in the **10** and the $\overline{\mathbf{5}}$ of SU(5)

- Light gauginos, for $\lambda \approx 1$ roughly a factor 10 lighter than sfermions.

Because gaugino masses are small, these predictions should not be affected much by the running and are expected to hold approximately also at the low scale. Therefore the peculiar integer ratio of sfermion masses should be traceable in the low-energy spectrum, thus providing a clear and testable prediction of our mechanism. Moreover gauginos should be lighter than sfermions also at low scale, and thus the lightest gaugino is also the lightest ordinary SUSY particle[11]. Because gauginos run like the gauge couplings, we therefore expect the Bino to be the NLSP.

We illustrate these properties with a typical example of a low-energy spectrum that can be obtained from our model. We neglect the small effect of the intermediate scale $\overline{5}_i^{16}$ and 5_j^{10} and use the MSSM RGE equations, as implemented in `Suspect 2.41` [45], with boundary conditions at high energy as in Eqs. (5.13, 5.14, 5.20), the A-terms set to zero, and $\eta = 1$. We assume the messenger mass to coincide with the GUT scale, $M = M_{\rm GUT}$. The overall normalization of the unified gaugino masses $M_{1/2}$ can be considered as a free parameter due to the presence of the factor $\mathrm{Tr}(h' h^{-1})$ in Eq. (5.18), or equivalently of the factor λ in Eq. (5.19). As the size of the parameters μ and B_μ is model-dependent, we consider them as free parameters as well and recover them as usual in terms of M_Z and $\tan\beta$. Under the above assumptions, the parameters that specify the model are: m, θ_u, θ_d, $M_{1/2}$, $\tan\beta$ and the sign of μ. Table 5.2 shows the low-energy spectrum corresponding to $\theta_u = 0$, $\theta_d = \pi/6$, $\tan\beta = 30$ and $\mathrm{sign}(\mu) = +$. The common gaugino mass is $M_{1/2} = 150\,\mathrm{GeV}$, near the minimal value allowed at present by chargino direct searches. The value of m is near the minimal value allowed to obtain $m_h > 114\,\mathrm{GeV}$. These parameters correspond to $\lambda = 2.5$, indicating that two-loop contributions to sfermion masses from ordinary gauge mediation are small, ranging from 2% to 10%.

In the spectrum of Figure 5.2 one can recognize the characteristic features discussed above: gauginos are light and the lightest neutralino is dominantly Bino. Sfermions are roughly a factor 10 heavier than gauginos and clearly split into two groups corresponding to their SU(5) embedding. From the table one can see that their mass ratio is close to the high-scale value of $\sqrt{2}$.

[11]The LSP is the gravitino as in ordinary gauge mediation, provided that the messenger mass is consistent with Eq. (5.22).

Higgs:	m_{h^0}	114
	m_{H^0}	1543
	m_A	1543
	$m_{H^\pm}$	1545
Gluinos:	$M_{\tilde{g}}$	448
Neutralinos:	$m_{\chi_1^0}$	62
	$m_{\chi_2^0}$	124
	$m_{\chi_3^0}$	1414
	$m_{\chi_4^0}$	1415
Charginos:	$m_{\chi_1^\pm}$	124
	$m_{\chi_2^\pm}$	1416
Squarks:	$m_{\tilde{u}_L}$	1092
	$m_{\tilde{u}_R}$	1027
	$m_{\tilde{d}_L}$	1095
	$m_{\tilde{d}_R}$	1494
	$m_{\tilde{t}_1}$	1007
	$m_{\tilde{t}_2}$	1038
	$m_{\tilde{b}_1}$	1069
	$m_{\tilde{b}_2}$	1435
Sleptons:	$m_{\tilde{e}_L}$	1420
	$m_{\tilde{e}_R}$	1091
	$m_{\tilde{\tau}_1}$	992
	$m_{\tilde{\tau}_2}$	1387
	$m_{\tilde{\nu}_e}$	1418
	$m_{\tilde{\nu}_\tau}$	1382

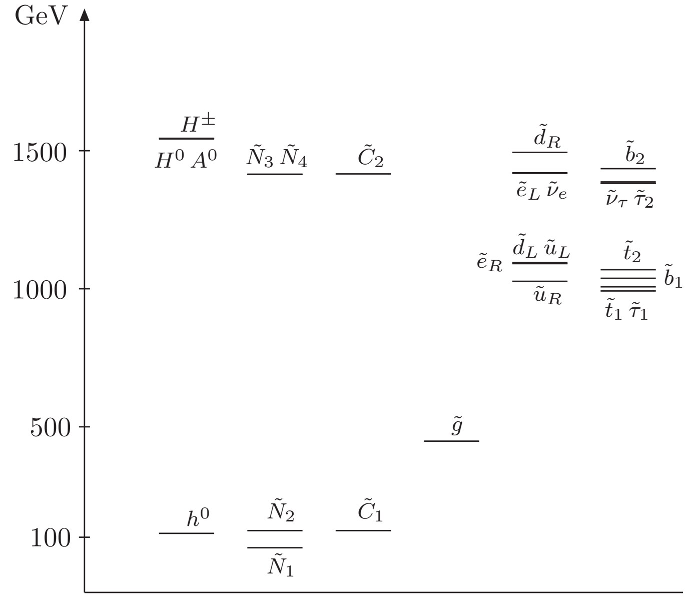

Figure 5.2 An example of spectrum, corresponding to $m = 3.2\,\text{TeV}$, $M_{1/2} = 150\,\text{GeV}$, $\theta_u = 0$, $\theta_d = \pi/6$, $\tan\beta = 30$ and $\text{sign}(\mu) = +$, $A = 0$, $\eta = 1$. All masses are in GeV, the first two families have an approximately equal mass.

This shows that there is some confidence to test tree-level gauge mediation at the LHC, provided that sfermion masses could be measured with sufficient accuracy. We are planning to study the possible LHC signals of our framework in a future publication.

5.4.8 Cosmology

We close this chapter with a brief discussion of the cosmological consequences of our model. As in ordinary gauge mediation the LSP is the gravitino, provided that the messenger mass is consistent with Eq. (5.22). In fact, the supersymmetry breaking parameter is given by

$$\sqrt{F} \approx 0.8 \cdot 10^{10}\,\text{GeV} \left(\frac{\tilde{m}_{10}}{\text{TeV}} \frac{M}{2 \cdot 10^{16}\,\text{GeV}} \right)^{1/2}, \tag{5.24}$$

which determines the gravitino mass

$$m_{3/2} = \frac{F}{\sqrt{3}M_{\mathrm{P}}} \approx 15\,\mathrm{GeV}\left(\frac{\tilde{m}_{10}}{\mathrm{TeV}}\,\frac{M}{2\cdot 10^{16}\,\mathrm{GeV}}\right), \tag{5.25}$$

where $\tilde{m}_{10}$ is the tree-level mass of the sfermions in the **10** of SU(5) at the GUT scale. Note that F and the gravitino mass are smaller than in loop gauge mediation, for a given messenger scale M, because of the absence of a loop factor in Eq. (5.24).

For a stable (on cosmological timescales) gravitino with a mass as large as in Eq. (5.25), a dilution mechanism such as inflation is necessary in order not to overclose the universe. The upper bound on the reheating temperature T_R depends on the gravitino and the gaugino masses [46]. The thermal contribution to the gravitino energy density, for a reheating temperature around $10^9\,\mathrm{GeV}$ is given by

$$\Omega_{\tilde{G}}^{\mathrm{TP}} h^2 \approx 6\times 10^{-2}\left(\frac{T_{RH}}{10^9\,\mathrm{GeV}}\right)\left(\frac{15\,\mathrm{GeV}}{m_{3/2}}\right)\left(\frac{M_{1/2}}{150\,\mathrm{GeV}}\right)^2 . \tag{5.26}$$

For the example spectrum in Table 5.2, the bound $\Omega_{\tilde{G}}^{\mathrm{TP}} h^2 \leq \Omega_{\mathrm{DM}} h^2 = 0.11$ translates in $T_R < 2\cdot 10^9\,\mathrm{GeV}$.

We also have to take care of the decays of the NLSP into the gravitino, which might spoil big bang nucleosynthesis (BBN) unless it is fast enough. The fate of BBN depends on what the NLSP is. In the bulk of the parameter space we expect the NLSP to be the lightest neutralino (or a stau if λ is large). In the example in Table 5.2, the NLSP is essentially a Bino. For $m_{3/2} \sim 15\,\mathrm{GeV}$, the decay of a Bino NLSP through its coupling to the Goldstino component of the gravitino is way too slow, as one would need $m_{3/2} < 100\,\mathrm{MeV}$ in order not to spoil BBN [47]. A Bino NLSP therefore requires a much faster decay channel, which could be provided for example by a tiny amount of R-parity violation [48]. Such a possibility is also consistent with thermal leptogenesis and gravitino dark matter.

The other possibility is that the NLSP is a stau. In this case, all the BBN constraints can be satisfied if the lifetime of the stau is $\tau_{\tilde{\tau}} \approx 48\pi m_{3/2}^2 M_{\mathrm{P}}^2/m_{\tilde{\tau}}^5 \lesssim 6\cdot 10^3\,s$ [49]. This possibility however requires sizable gaugino masses and therefore large $\lambda = \mathcal{O}(100)$. For such large values of λ, the ordinary gauge mediation contribution to sfermion masses dominate over the tree-level one, and the spectrum merely resembles the predictions of standard gauge mediation.

We therefore find the first possibility much more appealing, with interesting cosmological consequences that are quite different from those in usual gauge mediation. We plan to study the general cosmological implications of models with TGM together with their phenomenology in a future research paper.

6

Soft Terms in Tree-level Gauge Mediation

In tree-level gauge mediation sfermion masses arise from a tree-level exchange of heavy vector superfields, while gauginos masses are generated via one-loop diagrams involving heavy chiral fields, enhanced by numerical factors that partially compensate the loop factor. In this chapter we identify the conditions under which this scenario is viable, in the general context of a generic, renormalizable, $N = 1$ globally supersymmetric gauge theory in four dimensions. We first calculate the soft terms in the low-energy effective theory obtained from integrating out heavy vector superfields at tree-level. This includes the general expression for sfermion soft masses which we discuss also in the full, renormalizable theory. Then we turn to one-loop contributions to soft terms, focussing mainly on gaugino masses. In particular we calculate the contributions from both vector and chiral fields, and show that the latter can include various enhancement factors that can compensate the loop suppression.

6.1 General Setup

We consider a general supersymmetric gauge theory described by the Lagrangian obtained from the canonical Kählerpotential $K = \Phi^\dagger e^{2gV}\Phi$, canonical gauge kinetic terms and a generic superpotential $W(\Phi)$ that is a function of the chiral superfields $\Phi \equiv (\Phi_1 \ldots \Phi_n)$. The gauge group G is broken by the scalar component vev

$\phi_0 = \langle\phi\rangle$ to the subgroup H at a scale $M_V \sim g|\phi_0| \gg M_Z$, at which the theory is approximately supersymmetric[1]. Correspondingly, the vector superfields split into light and heavy ones, associated to the orthonormalized generators T^l_a and T^h_b respectively:

$$V = V^l_a T^l_a + V^h_b T^h_b, \qquad a = 1 \dots N_l, \qquad b = 1 \dots N_h.$$

The mass matrix for the heavy vector superfields is given by

$$(M^2_{V0})_{ab} = g^2 \phi^\dagger_0 \{T^h_a, T^h_b\} \phi_0. \tag{6.1}$$

We choose the basis of heavy generators T^h_a in such a way that the above mass matrix is diagonal,

$$(M^2_{V0})_{ab} = M^2_{V_a} \delta_{ab}. \tag{6.2}$$

The heavy vector superfields become massive by eating up a corresponding number of Goldstone chiral superfields. It is then convenient to split the chiral superfields as follows

$$\Phi = \phi_0 + \Phi' + \Phi^G, \qquad \Phi^G = \sqrt{2}\, g \frac{\Phi^{\mathrm{G}}_a}{M_{V_a}} T^h_a \phi_0, \qquad \Phi' = \Phi'_i b_i, \tag{6.3}$$

where Φ^G_a, $a = 1 \dots N_h$ are the Goldstone superfields associated to the generators T^h_a and $b_i = (b^i_1 \dots b^i_n)$, $i = 1 \dots n - N_h$ is an orthonormal basis in the space of the "physical" chiral fields Φ', defined by $b^\dagger_i T_a \phi_0 = 0$. In the supersymmetric limit, ϕ_0 is orthogonal to Φ^G and Φ^G does not mix with the physical superfields. The physical components of the massive vector superfield V_a are[2] v^μ_a, λ_a, ψ^G_a, $\mathrm{Re}(\phi^G_a)/\sqrt{2}$, all with mass M_{V_a}. The imaginary part of ϕ^G_a, the Goldstone boson, becomes as usual the longitudinal component of the massive gauge boson v^μ_a and the spinors ψ^G_a and λ_a pair up in a Dirac mass term. This spectrum can be split by supersymmetry breaking corrections, as we will see in Section 6.3.1.

The supersymmetric mass matrix for the physical chiral superfields Φ'_i is given by

$$M^0_{ij} = \frac{\partial^2 W}{\partial\Phi'_i \partial\Phi'_j}(\phi_0). \tag{6.4}$$

[1] In the phenomenological applications we have in mind, H contains the SM gauge group G_{SM}, G is a simple grand unified group like SO(10) or E_6, and the breaking scale is of the order of the GUT scale.

[2] We follow the conventions of Wess and Bagger [50] throughout this thesis.

Again, we choose the basis b_i in such a way that the above mass matrix is diagonal and positive,

$$M^0_{ij} = M_i \delta_{ij}, \quad M_i \geq 0. \tag{6.5}$$

The scalar and fermion components of Φ' can be split by supersymmetry breaking corrections, which can also induce a mixing with the scalar and fermion components of the heavy vector superfields.

Supersymmetry is supposed to be broken at a much lower scale than M_V, where some of the fields Φ' get an F-term vev,

$$\langle \Phi' \rangle = F_0 \theta^2, \qquad M_Z^2 \ll |F_0| \ll M_V^2.$$

Using gauge invariance of the superpotential, one can derive the condition

$$F_0^\dagger T_a \phi_0 = 0, \tag{6.6}$$

which implies that the Goldstone superfields Φ^G do not get F-term vevs (the D-term condition implies that in the supersymmetric limit they do not get scalar vev either). The F-term vevs give the leading contribution to the F-term and D-term conditions at the scale M_V,

$$\langle F_i \rangle = -\partial_i W(\phi_0) = 0 + \mathcal{O}\left(|F_0|\right), \tag{6.7}$$

$$\langle D_a \rangle = -g \phi_0^\dagger T_a \phi_0 = 0 + \mathcal{O}\left(|F_0/M_V|^2\right), \tag{6.8}$$

for each i, a. The F-terms can indeed induce a non-vanishing vev for the D-terms D^h_a of the heavy vector superfields. The stationary condition for the scalar potential V, $\partial V / \partial \phi_i = 0$, together with the gauge invariance of the superpotential give

$$\left\langle D^h_a \right\rangle = -2g \frac{F_0^\dagger T^h_a F_0}{M^2_{V_a}}, \tag{6.9}$$

with the light D-terms still vanishing. Clearly, only generators T^h_a that are singlets under the unbroken group H can contribute to such D-term vevs. In turn, the D-terms above give rise to tree-level soft masses for the scalar components ϕ'_i of the chiral superfields Φ'_i

$$V \supset \frac{1}{2}D^2 \supset -g\,\phi'^\dagger T^h_a \phi' \langle D^h_a \rangle = (\tilde{m}^2_{ij})_D \phi'^\dagger_i \phi'_j, \tag{6.10}$$

$$(\tilde{m}^2_{ij})_D = 2g^2 (T^h_a)_{ij} \frac{F_0^\dagger T^h_a F_0}{M^2_{V_a}}, \tag{6.11}$$

provided that both F_0 and the scalars ϕ' are charged under the (broken) gauge interaction associated to T^h_a and provided that T^h_a is a singlet under H.

The tree-level mass spectrum of this theory must necessarily satisfy the supertrace formula $\mathrm{Str}(\mathcal{M}^2) = 0$. In the case of the soft terms in Eq. (6.11), this simply follows from $\mathrm{Tr}\, T^h_a = 0$. In particular, the tracelessness condition implies that positive soft masses are accompanied by negative ones in Eq. (6.10). This is a potential phenomenological problem, which has long been considered as an obstacle to models in which supersymmetry breaking terms arise from renormalizable tree-level operators, as it is the case here. However, this problem can easily be addressed by adding a large positive supersymmetric mass term for the chiral superfields whose scalar components pick up negative SUSY breaking masses.

6.2 Tree-Level Soft Terms and Sfermion Masses

We will now recover the complete list of tree-level soft terms including the above result for sfermion masses in the effective theory below M_V, where the heavy vector and the Goldstone chiral superfields have been integrated out[3]. In this theory, the chiral degrees of freedom are the Φ', the gauge group is H and it is unbroken in the limit where we neglect electroweak symmetry breaking. As a consequence, there is no D-term contribution to supersymmetry breaking. Instead, scalar masses arise in this context from F-terms vevs through an effective Kähler operator, as we will show now.

Vector superfields can be integrated out by solving the equations of motion [51, 52, 53]

$$\partial K / \partial V^h_a = 0.$$

In Appendix A we illustrate the details of such a procedure in a general case, but for the present purposes, we are only interested in the terms in the effective Lagrangian which are the dominant sources of soft supersymmetry breaking. Those are con-

[3]Similar methods were used in [51] for different goals.

tained in the effective tree-level contribution to the Kähler potential in Eq. (A.10):

$$\delta K^0_{\rm eff} = -\frac{g^2}{M^2_{V_a}}(\Phi'^\dagger T^h_a \Phi')(\Phi'^\dagger T^h_a \Phi'), \tag{6.12}$$

where we recall that Φ' has no vev in its scalar component. The operator in Eq. (6.12) can be seen to arise from the diagram on the left in Fig. 6.1. The only

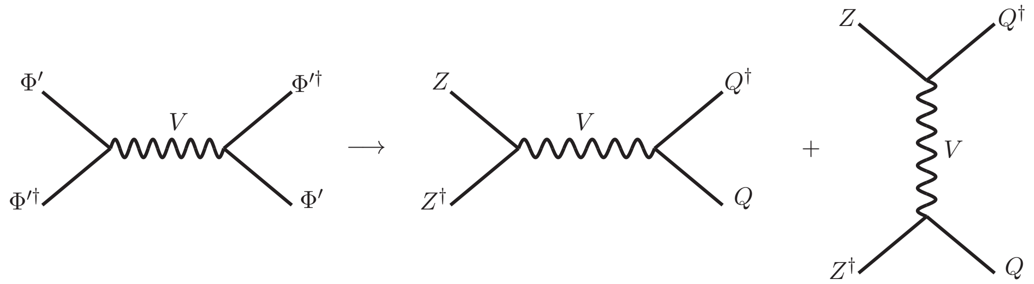

Figure 6.1 Tree level gauge mediation supergraph generating the operator in Eq. (6.12) by integrating out heavy vector superfields.

possible source of supersymmetry breaking in the effective theory are the F-term vevs of the chiral superfields Φ'. We recall that such F-term vevs must belong to non-trivial representations of the full group G, in order to play a role in TGM. The only terms in the Lagrangian containing such F-term vevs, at the tree level and up to second order in F_0, $F_0^\dagger$, and $1/M_{V_a}$, arise from the superpotential and from the operator in Eq. (6.12):

$$\begin{aligned}
-\mathcal{L}^{\rm tree}_{\rm soft} &= -F_{0i}\frac{\partial \hat{W}}{\partial \Phi'_i} - 2g^2\frac{(F_0^\dagger T^h_a \psi')(\phi'^\dagger T^h_a \psi')}{M^2_{V_a}} + \text{h.c.} \\
&\quad + 2g^2\frac{(F_0^\dagger T^h_a F_0)(\phi'^\dagger T^h_a \phi')}{M^2_{V_a}} + 2g^2\frac{(\phi^\dagger T^h_a F_0)(F_0^\dagger T^h_a \phi')}{M^2_{V_a}} - F_0^\dagger F_0\,,
\end{aligned} \tag{6.13}$$

where $\hat{W}$ is the superpotential in the effective theory,

$$\hat{W}(\Phi') = W(\phi_0 + \Phi') \qquad (\Phi^G = 0). \tag{6.14}$$

Let us analyze the different terms in Eq. (6.13). The first term in the second line reproduces the contribution to the soft scalar masses in Eq. (6.11). The second term also contributes[4] to soft scalar masses, but is only relevant to superfields that are

[4]This contribution can be obtained in the context of the full theory by using the unitary gauge

gauge partners of the Goldstino superfield (and have the same quantum numbers under H as some of the generators of G). Taking them together, we get

$$\tilde{m}^2_{ij} = 2g^2 \left[(T^h_a)_{ij} \frac{F_0^\dagger T^h_a F_0}{M^2_{V_a}} + \frac{(T^h_a F_0)^*_i (T^h_a F_0)_j}{M^2_{V_a}} \right]. \tag{6.15}$$

Note that the soft terms do not actually depend on the gauge coupling or on the normalization of the generators T, as $M^2_{V_a}$ is also proportional to $g^2 T^2$.

The second term in the first line of Eq. (6.13) is a Yukawa interaction with coupling $\lambda = \mathcal{O}\left(|F_0|/M_V^2\right)$, usually absent in models of supersymmetry breaking. It formally reintroduces quadratic divergences in the theory. However, even in the case in which the fields involved in the Yukawa interactions are light (so that we might worry about destabilizing their mass hierarchy), such quadratic divergences have the same parametric dependence on the supersymmetry breaking scale $|F_0|$ and the "cutoff" scale M_V as the usual logarithmic divergences induced by soft supersymmetry breaking terms, except that they are not log-enhanced. In fact, let us consider for example the radiative contribution δm^2 to the mass term $m^2 \phi'^* \phi'$ of the scalar ϕ' entering the Yukawa interaction. We then have

$$\delta m^2 \sim \frac{\lambda^2}{(4\pi)^2} M_V^2 \sim \frac{1}{(4\pi)^2} \frac{|F_0|^2}{M_V^2}. \tag{6.16}$$

These Yukawa couplings are very small, of the order

$$\lambda \sim m_{\text{soft}}/M_V \sim m_{\text{soft}}/M_{\text{GUT}} \sim 10^{-13}.$$

From a phenomenological point of view, such tiny Yukawa couplings might play a role in neutrino physics, where they could represent naturally small Dirac neutrino Yukawa couplings [54].

Finally, the first term in Eq. (6.13) represents a potential direct coupling of light fields to SUSY breaking. In order not to destabilize the hierarchy, the SUSY breaking sector should be "hidden" from the light fields, in the sense that its effects are only indirect, mediated by heavy fields and thus suppressed by M_V. In the phenomenological applications we have in mind, the light spectrum will contain the MSSM chiral superfields, as part of a light, "observable" sector. The latter will be charged under the residual gauge group $H \supseteq G_{\text{SM}}$. On the other hand,

or in Wess-Zumino gauge from the F-term contribution to the scalar potential using Eq. (6.27) below.

the SUSY breaking superfields do not feel the residual gauge interactions. In the effective theory the SUSY breaking sector is therefore hidden from the observable sector what regards gauge interactions. In order for the SUSY breaking sector to be hidden also with respect to superpotential interactions, it is sufficient to make sure that the first term in Eq. (6.13) does not induce a direct coupling between the two sectors. To be more precise, we can write the chiral superfields Φ' of the effective theory as

$$\Phi' = (Z, Q, \Phi^h). \tag{6.17}$$

The superfield Z is the only one getting an F-term vev, $\langle Z \rangle = |F_0|\theta^2$. Its fermion component is the Goldstino and therefore Z is a massless eigenstate of the mass matrix M^0 in Eq. (6.4). The remaining mass eigenstates are divided in two groups, the heavy ones, Φ_i^h with masses $M_i^h \gg |F_0|$, and the light or observable ones Q_i, with masses $M_i^Q \lesssim |F_0|$. In order to hide supersymmetry breaking from the observable sector with respect to superpotential interactions, we require that

$$\frac{\partial^2 \hat{W}}{\partial Z \partial Q_j}(Z, Q, \Phi^h = 0) = 0. \tag{6.18}$$

We can then see supersymmetry breaking as arising in a hidden sector and then communicated to the observable sector by the diagrams on the right-hand side of Fig. 6.1. This can perhaps be considered as the simplest way to communicate supersymmetry breaking: through the tree-level renormalizable exchange of a heavy gauge messenger. Since heavy gauge messengers at a scale not far from the Planck scale are automatically provided by grand unified theories, this possibility is not only simple but also well motivated. The reason whyinput it has not been pursued in the past is an apparent obstacle arising from the supertrace theorem that, as mentioned, can be easily evaded by providing heavy, supersymmetric masses to some of the superfields. Such mass terms can naturally arise in the context of grand unified theories, as we will see in Chapter 7.

We end this chapter with some comments on integrating out heavy chiral superfields and the corresponding possible tree-level contributions to soft terms. The heavy vector superfields may not be the only fields living at the scale M_V, as chiral superfields could have mass terms of similar size or get it after gauge symmetry breaking. Such chiral fields should also be integrated out in order to write down the effective theory below the scale M_V in a consistent way.

In general, we want to integrate out all the heavy chiral superfields Φ^h. Since their masses M_i^h are assumed to be much larger than the supersymmetry breaking scale, it is still be possible to write the effective theory in a manifestly supersymmetric way. In order to integrate them out, let us write the superpotential as

$$\hat{W} = -|F_0|Z + \frac{M_i^Q}{2}Q_i^2 + \frac{M_i^h}{2}(\Phi_i^h)^2 + W_3(Z, Q, \Phi^h), \tag{6.19}$$

where W_3 is at least trilinear in its argument. The equations of motion $(\partial\hat{W})/(\partial\Phi_i^h) = 0$ give

$$\Phi_i^h = -\frac{1}{M_i^h}\frac{\partial W_3}{\partial \Phi_i^h}(Z, Q) + \mathcal{O}\left(\frac{1}{M_h^2}\right). \tag{6.20}$$

The effective superpotential for the light fields Z and Q is therefore

$$W_{\text{eff}}(Z, Q) = \hat{W}(Z, Q) - \frac{1}{2M_i^h}\sum_i\left(\frac{\partial W_3}{\partial \Phi_i^h}(Z, Q)\right)^2 + \mathcal{O}\left(\frac{1}{M_h^2}\right). \tag{6.21}$$

A contribution to the effective Kähler is also induced

$$\delta K_\Phi = \frac{1}{(M_i^h)^2}\sum_i\left|\frac{\partial W_3}{\partial \Phi_i^h}(Z, Q)\right|^2 + \mathcal{O}\left(\frac{1}{M_h^3}\right). \tag{6.22}$$

The effective contributions to the superpotential and to the Kähler in Eqs. (6.21, 6.22) may give rise to "chiral-mediated" tree-level A-terms and (negative) additional contributions to soft scalar masses respectively. The latter should be subleading with respect to the (positive) vector mediated contributions in Eq. (6.15), at least in the case of the MSSM sfermions. Such tree-level contributions could only arise in the presence of trilinear superpotential couplings in the form $ZQ\Phi^h$. In the following we will restrict to the case in which such a coupling is absent.

$$\frac{\partial^3\hat{W}}{\partial Z\partial Q\partial\Phi^h}(0) = 0, \tag{6.23}$$

so that the chiral-mediated tree-level contributions vanish. This is often the case, as illustrated in the model in Section 5.4.

6.3 One-Loop Soft Terms and Gaugino Masses

We now want to calculate gaugino masses arising at one-loop in the full theory above M_V. There are two types of diagrams contributing to gaugino masses, depending on whether the degrees of freedom running in the loop are components of heavy

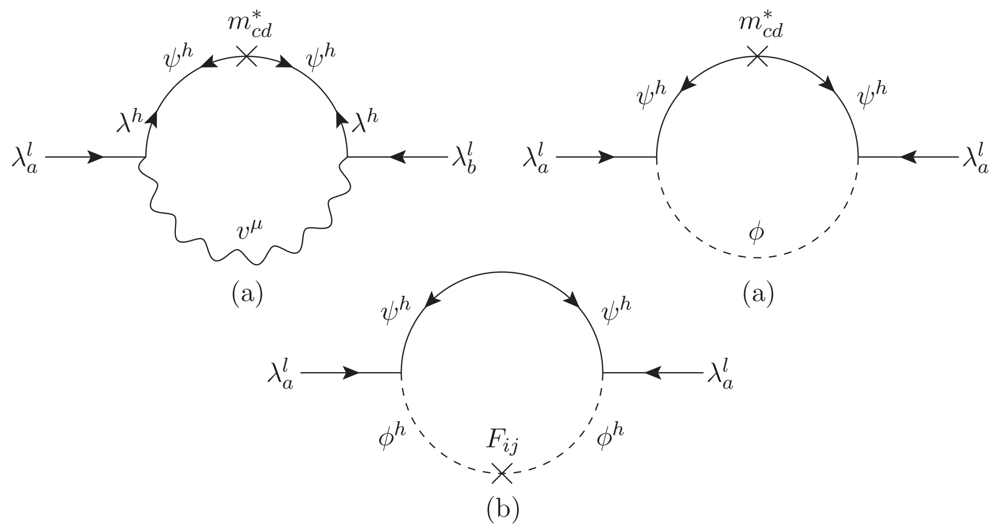

Figure 6.2 One-loop contributions to light gaugino masses from the exchange of heavy vector (a) and chiral (b) degrees of freedom.

vector superfields (including the Goldstone superfields), as in Fig. 6.2a, or physical chiral superfields, as in Fig. 6.2b. Correspondingly, we will distinguish a "vector" and a "chiral" contribution to the light gaugino masses,

$$M^g_{ab} = (M^g_{ab})_V + (M^g_{ab})_\Phi. \tag{6.24}$$

Supersymmetry breaking is transmitted to the light gauge sector due to a tree-level splitting among the components of the heavy vector and chiral superfields respectively. We now analyze the two contributions in Eq. (6.24) and write the known results [55] in a form general enough for the subsequent discussion of their quantitative importance compared to tree-level soft scalar masses.

6.3.1 Vector Contribution to Gaugino Masses

In the supersymmetric limit, the fields v^μ_a, λ_a, ψ^G_a, $\mathrm{Re}(\phi^G_a)/\sqrt{2}$ form a massive vector multiplet with mass M_{V_a}. Once supersymmetry is broken, this spectrum is split by corrections to the fermion and scalar masses, which may also mix them with the components of the physical chiral superfields. Here we are interested in the supersymmetry breaking fermion mass term in the form $-m_{ab}\psi^G_a\psi^G_b/2$, which is the source of the vector contribution to gaugino masses from the diagrams in

Fig. 6.2a. The mass term

$$m_{ab} = \frac{\partial^2 W}{\partial \Phi^G_a \partial \Phi^G_b}(\phi_0) \tag{6.25}$$

vanishes in the supersymmetric limit because of the gauge invariance of W. The situation is different in the presence of supersymmetry breaking, when the gauge invariance of W gives

$$m_{ab} = g^2 \frac{F_0^\dagger \{T^h_a, T^h_b\}\phi_0}{M_{V_a} M_{V_b}}. \tag{6.26}$$

Note also the more general expression for the mixed supersymmetry breaking terms

$$\frac{\partial^2 W}{\partial \Phi_i \partial \Phi^G_a}(\phi_0) = \sqrt{2} g \frac{F^\dagger_{0j}(T^h_a)_{ji}}{M_{V_a}}. \tag{6.27}$$

Before calculating the gaugino masses induced by m_{ab}, let us note that the heavy vector representation is in general reducible, under the unbroken gauge group H, to a set of irreducible components, each with a definite mass. Let us call $\hat{M}_{V_R}|_1$ this mass in the representation R and denote

$$g^2 \phi_0^\dagger \{T^h_a, T^h_b\} F_0 = m^*_{ab} \hat{M}^2_{V_R}|_1 \equiv \delta_{ab} M^2_{V_R}|_{\theta^2}, \tag{6.28}$$

if T^h_a, T^h_b belong to the representation R. We choose this notation in order to indicate that $\hat{M}^2_{V_R}|_1$ and $\hat{M}^2_{V_R}|_{\theta^2}$ are just the scalar and F-term components of the superfield function

$$\hat{M}^2_{V_R} \equiv \frac{\partial^2 K}{\partial V_R \partial V_{\overline{R}}}(\langle \Phi \rangle, V = 0),$$

where V_R is the vector superfield in the irrep R of H.

In the limit $|F_0| \ll M^2_V$, the supersymmetry breaking source m_{ab} can be treated as a perturbation in the one-loop computation of gaugino masses. At the leading order in m_{ab}, the diagram in Fig. 6.2a generates a contribution to light gaugino masses given by

$$(M^g_{ab})_V = -2 \frac{\alpha}{4\pi} \sum_R I(R)\, m^*_{ab} = -2\, \delta_{ab} \frac{\alpha}{4\pi} \sum_R I(R)\, \frac{M^2_{V_R}|_{\theta^2}}{M^2_{V_R}|_1}, \tag{6.29}$$

where $I(R)$ is the Dynkin index of the representation $R : T \to R(T)$ of the generator T. The above contribution to gaugino masses arises at the scale M_V where the heavy vectors live.

Let us now discuss the relevance of the above contribution to gaugino masses. First, let us note that in order for $(M^g_{ab})_V$ to be non-vanishing we need the following two conditions to be verified at the same time

$$\phi_0^\dagger \{T_a^h, T_b^h\} F_0 \neq 0 \quad \text{for some } a, b, \qquad \phi_0^\dagger T_a^h F_0 = 0 \quad \text{for all } a, \tag{6.30}$$

as it can be seen from Eqs. (6.28) and (6.6). In particular, we need at least one irreducible (under the full group G) chiral superfield multiplet to get vev in both its scalar and F components. At the same time, we need

$$F_0^\dagger T_a^h F_0 \neq 0 \quad \text{for some } a, \tag{6.31}$$

in order for the tree-level contribution to scalar masses to be generated. The conditions in Eqs. (6.30, 6.31) often force the vector contribution to gaugino masses to vanish. On top of that, $(M^g_{ab})_V$ is always suppressed by a loop factor $g^2/(4\pi)^2$ compared to the typical scalar mass in Eq. (6.15). As we will see in a moment, the chiral contribution to gaugino masses can be significantly larger than the vector contribution, thus reducing or even eliminating the loop suppression with respect to soft scalar masses. In this case, the vector contribution to gaugino masses typically ends up to be subdominant.

6.3.2 Chiral Contribution to Gaugino Masses

The chiral contribution to gaugino masses arises from the one-loop diagram in Fig. 6.2b, as in ordinary loop gauge mediation. The scalar and fermion components of the chiral superfields entering the loop are split by a SUSY breaking scalar mass term $-(F_{ij}\phi_i^h \phi_j^h + \text{h.c.})/2$. The scalar mass F_{ij} is given by

$$F_{ij} = -\frac{\partial^3 \hat{W}}{\partial \Phi_i^h \partial \Phi_j^h \partial Z}(0)|F_0|, \tag{6.32}$$

which adds to the supersymmetric scalar mass term $-M_i^2 |\phi_i'|^2$, see Eq. (6.5).

The physical chiral superfield representation under the unbroken gauge group H is in general reducible to a set of irreducible components, each with a definite mass. Let us call $\hat{M}_R^h|_1$ the mass in the representation R and denote

$$\frac{\partial^3 \hat{W}}{\partial \Phi_i^h \partial \Phi_j^h \partial Z}(0)|F_0| = -F_{ij} \equiv \delta_{ij} \hat{M}_R^h|_{\theta^2}. \tag{6.33}$$

Again $\hat{M}_R^h|_1$ and $\hat{M}_R^h|_{\theta^2}$ are just the scalar and F-term components of the superfield function

$$\hat{M}^h_R \equiv \frac{\partial^2 \hat{W}}{\partial \Phi^h_R \partial \Phi^h_{\overline{R}}}(\langle \Phi \rangle),$$

where Φ^h_R is the heavy chiral superfield in the irrep R of H.

At the leading order in F_{ij}, the diagram in Fig. 6.2b generates a contribution to light gaugino masses given by

$$(M^g_{ab})_\Phi = \delta_{ab} \frac{\alpha}{4\pi} \sum_R I(R) \frac{\hat{M}^h_R|_{\theta^2}}{\hat{M}^h_R|_1}. \tag{6.34}$$

Each of the contributions in the sum in the RHS of Eq. (6.34) arises at the scale $\hat{M}^h_R|_1$ at which the corresponding chiral superfield lives.

Let us now discuss the size of the typical chiral contribution to gaugino masses M_g and compare it with the typical size of the tree-level scalar soft masses $\tilde{m}^2$ in Eq. (6.15). Let us consider for simplicity the case in which the scalar masses are due to the exchange of a single heavy vector and the irreducible (under H) components of the physical chiral superfields have definite charges Q_R under the corresponding generators. As for the dynamics giving rise to gaugino masses, let us assume that there are no bare mass terms in the superpotential. Then both $\hat{M}^h_R|_1 = \lambda_{RS}\phi_{0S}$ and $\hat{M}^h_R|_{\theta^2} = \lambda_{RS}F_{0S}$ arise from the same trilinear term in $W(\Phi)$. Under the above assumptions, we have

$$\tilde{m}^2 = \frac{\sum_R (Q_R/Q)|F_{0R}|^2}{\sum_R (Q_R/Q)^2 |\phi_{0R}|^2}, \qquad M_g = \frac{\alpha}{4\pi} \sum_R I(R) \frac{\sum_S \lambda_{RS} F_{0S}}{\sum_S \lambda_{RS} \phi_{0S}}, \tag{6.35}$$

where Q is the charge of the scalar acquiring the mass $\tilde{m}$. While the loop factor $g^2/(4\pi)^2$ suppresses M_g compared to $\tilde{m}$ by a $\mathcal{O}(100)$ factor, the expressions in Eqs. (6.35) can easily cause an enhancement of $\tilde{m}/M_g$ reducing or even eliminating the loop hierarchy:

- In the context of grand unified theories the number of heavy vectors contributing to the soft scalar masses is typically small (one in the case of SO(10)), while gaugino masses can may get a contribution from several chiral messengers.

- Sfermion and gaugino masses depend on different group factors. Sfermions can get a mild suppression if $Q_R/Q > 1$.

- The heavy vector masses whose exchange generates $\tilde{m}$ collect all the vevs breaking the corresponding charge Q. The scalar mass $\tilde{m}$ is therefore suppressed by all such vevs. On the other hand, gaugino masses are only suppressed by the vevs related to supersymmetry breaking by superpotential interactions λ_{RS}. Unless some of them have $Q = 0$, the vevs suppressing gaugino masses will be a subset of the vevs suppressing scalar masses, thus leading to an enhancement of gaugino masses. In the presence of an hierarchy between the vevs related to supersymmetry breaking and some of the other, Q-breaking vevs, this enhancement can be quite large.

- Different couplings λ_{RS} can appear in the numerator and denominator of the expression $(\sum_S \lambda_{RS} F_{0S})/(\sum_S \lambda_{RS}\phi_{0S})$. This is likely to be the case as a consequence of the relation $\sum_S Q_S(F^*_{0S}\phi_{0S}) = F_0^\dagger T^h \phi_0 = 0$, which can be satisfied without cancellations only in the case in which the fields charged under Q do not have vevs in both the F and scalar components. If the couplings appearing in the numerator and the denominator are hierarchical, gaugino masses can be sizeably enhanced.

The study of simple models shows that the enhancement factors above can naturally arise, see Sections 5.4.4 and 7.2.1.

6.3.3 Contributions to other Soft Terms

Besides gaugino masses, which can be seen to arise from one-loop corrections to the gauge kinetic function, a number of soft terms can be generated or get a contribution from the one-loop corrections to the Kähler. The latter can be computed using the general results in [56], which give

$$\delta K_{\text{1-loop}} = -\frac{1}{32\pi^2}\,\mathrm{Tr}\left[M_\Phi^\dagger M_\Phi\left(\log\frac{M_\Phi^\dagger M_\Phi}{\Lambda^2} - 1\right)\right] + \frac{1}{16\pi^2}\,\mathrm{Tr}\left[M_V^2\left(\log\frac{M_V^2}{\Lambda^2} - 1\right)\right], \tag{6.36}$$

where

$$(M_\Phi)_{ij} = \frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}(\Phi), \qquad (M_V^2)_{ab} = \frac{\partial^2 K}{\partial V_a \partial V_b}(\Phi, V=0) \tag{6.37}$$

are functions of the chiral superfields, K is the canonical Kählerpotential $K = \Phi^\dagger e^{2gV}\Phi$ and the indices run over the heavy vector and chiral superfields. The first term comes from chiral superfields running in the loop, the second term from vector fields. As in the case of gaugino masses, the soft terms might get a contribution from both.

Because these contribution to one-loop soft terms are highly model dependent, we just collect their general form in terms of $\delta K_{\text{1-loop}}$. Let us therefore expand $\delta K_{\text{1-loop}}$ in terms of powers of Q and Z around ϕ_0. The relevant terms are

$$\begin{aligned}\delta K_{\text{1-loop}} = &\alpha^{(1)}_{ij} Z Q^\dagger_i Q_j + \frac{\beta^{(1)}_{ij}}{2} Z^\dagger Q_i Q_j + \text{h.c.} \\ &+ \alpha^{(2)}_{ij} Z^\dagger Z Q_i{}^\dagger Q_j \\ &+ \frac{\beta^{(2)}_{ij}}{2} Z^\dagger Z Q_i Q_j + \text{h.c.} + \ldots ,\end{aligned} \tag{6.38}$$

where $\alpha^{(1)}$, $\alpha^{(2)}$, are hermitian, $\beta^{(1)}$, $\beta^{(2)}$ symmetric and all are dimensionful. We have omitted $Z^\dagger Q_i$ terms, which are well-known to destabilize the hierarchy [57]. Their absence can be ensured for example by requiring that there are no light chiral fields with the same quantum numbers as Z.

The first term $\alpha^{(1)}$ gives rise to trilinear A-terms

$$\mathcal{L}^A_{\text{1-loop}} = -A_{ij} \tilde{Q}_i \frac{\partial \hat{W}}{\partial Q_j}(\tilde{Q}), \quad \text{with} \quad A_{ij} = |F_0| \alpha^{(1)} \tag{6.39}$$

(and to a two loop contribution to scalar soft masses), where $\tilde{Q}$ is the scalar component of Q. The second term $\beta^{(1)}$ generates a contribution to the μ-term in the superpotential[5]

$$W^\mu_{\text{1-loop}} = \frac{\mu_{ij}}{2} Q_i Q_j, \quad \text{with} \quad \mu_{ij} = |F_0| \beta^{(1)}, \tag{6.40}$$

and the fourth term $\beta^{(2)}$ a contribution to the B_μ-term

$$\mathcal{L}^{B_\mu}_{\text{1-loop}} = -\frac{(B_\mu)_{ij}}{2} q_i q_j, \quad \text{with} \quad (B_\mu)_{ij} = -|F_0|^2 \beta^{(2)}. \tag{6.41}$$

Finally, $\alpha^{(2)}$ gives a one-loop contributions to soft scalar masses

$$\delta \tilde{m}^2_{ij} = -|F_0|^2 \alpha^{(2)}_{ij} \tag{6.42}$$

that add to the tree-level contributions in Eq. (6.15).

Additional one-loop contributions to soft scalar masses can come from an induced Fayet-Iliopoulos term [43] associated for example to the heavy H-singlet generators, in particular to those involved in the mediation of supersymmetry breaking at the tree level. Such terms vanish if the heavy chiral mass matrix and the matrix

[5] A detailed discussion of the μ-term can be found in Section 7.4.

of their couplings to the spurion Z are diagonal in the same basis (in which case the condition in Eq. (6.23) is also automatically satisfied), or if the latter matrix of couplings is hermitian in one basis in which the mass matrix is diagonal [21].

This completes the list of the soft terms arising at the one-loop level. Two-loop corrections to soft scalar masses can also arise, as in ordinary gauge mediation, and are sizable in the presence of an enhancement of one-loop gaugino masses, see Section 5.4.

7

Model Building

We now discuss the possibility to obtain a phenomenologically viable model from the general formalism introduced in the previous chapter. We will see that clear model building guidelines emerge from this analysis, leading, in economical schemes, to peculiar predictions for the pattern of MSSM sfermion masses.

7.1 General Guidelines

In a phenomenologically viable model, the unbroken gauge group H should contain the SM group, $G_{\text{SM}} \subseteq H$, and the light superfield content should contain the MSSM spectrum, $(q_i, u^c_i, d^c_i, l_i, e^c_i) \subseteq Q$, in standard notation, where $i = 1, 2, 3$ is the family index. We assume that the full gauge group G is a simple, grand unified group, motivated by the successful predictions of the SM fermion gauge quantum numbers and the QCD coupling α_3 in the MSSM. The candidates for the unified group G in a four-dimensional theory are SU(N), $N \geq 5$, SO($4n$+2), $n \geq 2$, and the exceptional group E_6 [58]. In the following we will focus on the smallest representatives of each class, SU(5), SO(10) and E_6.

We want the MSSM sfermions to get a positive mass around the TeV scale through tree-level gauge mediation. The general form of such mass terms is given in Eq. (6.15). They arise from two contributions, corresponding to the two diagrams on the right in Fig. 6.1. In order for the second contribution to play a role for sfermion masses, the corresponding chiral superfields should live in the same unified multiplet as the supersymmetry breaking source Z. This will not be the case for

SO(10)	16			$\overline{16}$			10		45				54		
SU(5)	1	10	$\overline{5}$	1	$\overline{10}$	5	5	$\overline{5}$	1	10	$\overline{10}$	24	24	15	$\overline{15}$
X	5	1	-3	-5	-1	3	-2	2	0	-4	4	0	0	-4	4

Table 7.1 Quantum numbers of the non-trivial SO(10) representations with dimension $d < 120$ under the SO(10) generator X.

the models we want to consider as a consequence, for example, of a matter parity telling the supersymmetry breaking multiplet from the matter ones[1]. The MSSM sfermions then get their tree-level soft masses from the first term in Eq. (6.15) only. In order for $F_0^\dagger T_a^h F_0$ to be non-vanishing, the heavy generator T_a^h must be a SM singlet, since F_0 is. We therefore need a group G with rank 5 at least, which means that SU(5) cannot give rise to tree-level gauge mediation. In the following we concentrate mainly on SO(10) and discuss very briefly E_6.

7.2 SO(10) Embedding

In SO(10) there is exactly one (up to a sign) orthonormalized heavy SM-singlet generator, $T_h = X/\sqrt{40}$, where $X = 5(B-L) - 4Y$ is the SU(5) invariant SO(10) generator. The quantum numbers of the SO(10) representations with dimension $d < 120$ under X are given in Table 7.1. The values of the X quantum numbers are crucial because the soft terms turn out to be proportional to those charges. From Eq. (6.15) we obtain in fact

$$\tilde{m}_f^2 = \frac{X_f (F_0^\dagger X F_0)}{\phi_0^\dagger X^2 \phi_0} \quad \text{at the scale} \quad M_V = \frac{g^2}{20}\phi_0^\dagger X^2 \phi_0, \tag{7.1}$$

where X_f is the X-charge of the sfermion $\tilde{f}$ and M_V is the mass of the vector superfield associated to the generator X. In order to calculate the spectrum of tree-level sfermion masses, we just need to specify the embedding of the three MSSM families into SO(10), which we will do through their SU(5) embedding into three light $\overline{5}_i^l + 10_i^l$, $i = 1, 2, 3$.

We use two constraints to determine the embedding of the $\overline{5}_i^l + 10_i^l$ into SO(10) representations. The first one is related to a nice feature Eq. (7.1): the soft terms

[1]This second contribution might however contribute to the Higgs masses, if some of the gauge generators have the same quantum numbers as the Higgses. This is not the case in SO(10), the group on which we focus in this chapter, but could be possible in E_6.

are family-universal, provided that the three families of each of the MSSM matter multiplets are embedded in the same type of SO(10) representation. This is what we want to assume in order to solve the SUSY flavor problem. Second, we want the MSSM sfermion soft masses in Eq. (7.1) to be positive in order to avoid spontaneous symmetry breaking of color, electric charge, or lepton number at the scale $\tilde{m}$. Clearly, the standard embedding of a whole family into a **16** of SO(10) would not work, as it would lead to negative masses for the sfermions in either the $\overline{\mathbf{5}}$ or the **10** of SU(5). Because of the tracelessness of the X generator in SO(10), every SO(10) multiplet will contain SU(5) representations with negative X-charge, and thus scalars that pick up negative masses from TGM. This apparent obstacle can be easily overcome by splitting the SO(10) representation containing the MSSM multiplet through SO(10) breaking, in such a way that the extra fields with negative soft masses acquire a large supersymmetric mass term. The negative soft mass will then represent a negligible supersymmetry breaking correction to that large positive mass. It turns out that such a splitting is actually expected to arise, as will see, a fact that reinforces the logical consistency of this framework.

We are now ready to discuss the embeddings of the three $\overline{5}^l_i$ and 10^l_i of SU(5) containing the light MSSM families in SO(10). As $\phi_0^\dagger X^2 \phi_0$ is positive, the possible choices depend on the sign of $F_0^\dagger X F_0$. We limit ourselves to the SO(10) representations with $d < 120$, as in Table 7.1. There are then only two possibilities:

- $F_0^\dagger X F_0 > 0$. In this case we need to embed the $\overline{5}^l_i$'s and 10^l_i's into SO(10) representations containing $\overline{\mathbf{5}}$ and **10** of SU(5) with positive charges under X. From Table 7.1 we see that the only possibility is to use three $16_i = (1^{16}_i, 10^{16}_i, \overline{5}^{16}_i)$ and three $10_i = (5^{10}_i, \overline{5}^{10}_i)$, where we have explicitly indicated the SU(5) decomposition, and to embed the 10^l_i's into the 16_i's, $10^l_i \equiv 10^{16}_i$, and the $\overline{5}^l_i$'s into the 10_i's, $\overline{5}^l_i \equiv \overline{5}^{10}_i$. The spare components $\overline{5}^{16}_i$, 5^{10}_i get negative soft masses and need to acquire a large supersymmetric mass term.

- $F_0^\dagger X F_0 < 0$. In this case we need the $\overline{5}^l_i$'s and 10^l_i's to have negative charges under X. The only possibility is then to use three 16_i's as before and three $45_i = (1^{45}_i, 10^{45}_i, \overline{10}^{45}_i, 24^{45}_i)$, with $\overline{5}^l_i \equiv \overline{5}^{16}_i$ and $10^l_i \equiv 10^{45}_i$. The spare components 10^{16}_i, $\overline{10}^{45}_i$, get negative soft masses and need to acquire a large supersymmetric mass term.

Note that in both cases the chiral content of the theory is still given by three **16** of SO(10). We have implicitly discarded the possibility of mixed embeddings in which for example the $\overline{5}_i$'s of SU(5) are a superposition of the $\overline{5}_i$'s in the 10_i's and 16_i's of SO(10). While this possibility is in principle not excluded, it would introduce a dependence of the sfermion soft masses on mixing parameters that are in general flavor violating, thus possibly spoiling the flavor universality result.

Without specifying anything else, the two possibilities above give already rise to two definite predictions for the ratios of sfermion soft masses at the scale M_V:

$$(\tilde{m}^2_l)_{ij} = (\tilde{m}^2_{d^c})_{ij} = m^2_{\overline{5}}\delta_{ij},$$
$$(\tilde{m}^2_q)_{ij} = (\tilde{m}^2_{u^c})_{ij} = (\tilde{m}^2_{d^c})_{ij} = m^2_{10}\delta_{ij},$$

$$m^2_{\overline{5}} = 2m^2_{10} \quad \text{if} \quad F^\dagger_0 X F_0 > 0,$$
$$m^2_{\overline{5}} = \frac{3}{4}m^2_{10} \quad \text{if} \quad F^\dagger_0 X F_0 < 0. \tag{7.2}$$

To summarize, these predictions are based on the following hypotheses: "minimal" unified gauge group SO(10), embedding of the MSSM families in the SO(10) representations with dimension $d < 120$ not containing the Goldstino, and absence of mixed embeddings to automatically preserve flavor-universality. The ratios $m_{\overline{5}}/m_{10}$ in Eq. (7.2) are the main predictions of TGM and peculiar enough to make this scheme of SUSY breaking testable at the LHC.

What regards the source of supersymmetry breaking, $\langle Z \rangle = |F_0|\theta^2$, we need Z to have a non-vanishing charge under X. If we limit ourselves again to representations with $d < 120$, the only possibility is that Z has a component in the "right-handed neutrino" direction of a **16** or a $\overline{\mathbf{16}}$. With the sign conventions we adopted, a component in a **16** gives a positive contribution to $F^\dagger_0 X F_0$, while a component in a $\overline{\mathbf{16}}$ gives a negative contribution.

We now want to show that the two embeddings of the light MSSM families described above can be obtained in a natural way. It will turn out that the SO(10) breaking vevs of a $\mathbf{16} + \overline{\mathbf{16}}$, essential to break SO(10) to the SM (unless representations with $d \geq 126$ are used to reduce the rank) just provide the needed splitting, i.e. they make heavy precisely the components of the SO(10) representations that get a negative soft supersymmetry breaking mass. In the following, we first discuss the $16_i + 10_i$ embedding in a general, top-bottom perspective and then turn to the possibility of the $16_i + 45_i$ embedding.

7.2.1 The Embedding into $16_i + 10_i$

Let us consider the embedding associated to the case $F_0^\dagger X F_0 > 0$. We assume the existence of a matter parity symmetry that tells matter superfields from Higgs superfields. Let 16, $\overline{16}$ be the SO(10) multiplets breaking SO(10) to SU(5). The most general renormalizable superpotential involving 16, $\overline{16}$, 16_i, 10_i, $i = 1, 2, 3$, and invariant under a matter parity under which the SO(10) Higgs fields 16, $\overline{16}$ are even and the the matter fields are odd is

$$W = h_{ij} 16_i 10_j 16 + \frac{\mu_{ij}}{2} 10_i 10_j + W_{\text{vev}}, \tag{7.3}$$

where W_{vev} takes care of providing a vev to the 16, $\overline{16}$ in the SM-singlet direction[2] and does not depend on the matter fields (but can involve additional even fields). The term $h_{ij} 16_i 10_j 16$ is just what is needed to split the SU(5) components of the $16_i = (1_i^{16}, 10_i^{16}, \overline{5}_i^{16})$ and of the $10_i = (5_i^{10}, \overline{5}_i^{10})$, making heavy precisely the unwanted components $\overline{5}_i^{16}$ and 5_j^{10}. Once 16 acquires a vev in its singlet neutrino component, $\langle 16 \rangle = M_S$, a mass term is generated for those components,

$$M_{ij} \overline{5}_i^{16} 5_j^{10}, \qquad M_{ij} = h_{ij} M_S. \tag{7.4}$$

It is remarkable that the components acquiring a large mass are precisely those that get a negative soft mass term. On the other hand, this is only true in the limit in which the μ_{ij} mass term in Eq. (7.3) can be neglected. In the presence of a non-negligible μ_{ij} the full mass term would be

$$(\overline{5}_i^{16} M_{ij} + \overline{5}_i^{10} \mu_{ij}) 5_j^{10}, \tag{7.5}$$

which would give rise to a mixed embedding of the light $\overline{5}_i^l$'s in the 16_i's and 10_i's. In order to stick to our assumptions, which exclude the possibility of mixed embeddings, such a μ_{ij} term should be absent. This can be easily forced by means of an appropriate symmetry. Let us however relax for a moment that assumption in order to quantify the deviation from universality associated to a small, but non-negligible μ_{ij}. The MSSM sfermions in the $\overline{5}$ of SU(5) receive in this case two contributions to their soft mass, a positive one associated to the components in the 10_i's, proportional to $X(\overline{5}^{10}) = 2$, and a negative one associated to the components in the 16_i's, proportional to $X(\overline{5}^{16}) = -3$. The soft mass matrix for the light sfermions in the

[2] The simplest possibility is $W_{\text{vev}} = X(\overline{16}16 - M_S^2)$, where X is an SO(10) singlet.

$\overline{5}$ of SU(5) can be easily calculated in the limit in which the μ_{ij} mass term can be treated as a perturbation. In this limit, the light MSSM fields in the $\overline{5}$ of SU(5) are in fact

$$\overline{5}^l_i \approx \overline{5}^{10}_i - (\mu M^{-1})^*_{ij}\overline{5}^{16}_j \tag{7.6}$$

and their soft scalar mass matrix at the scale M_V is

$$(\tilde{m}^2_{\overline{5}})_{ij} \approx \frac{2}{5}\tilde{m}^2\left(\delta_{ij} - \frac{5}{2}\left(\mu^* M^{*-1} M^{T-1}\mu^T\right)_{ij}\right), \tag{7.7}$$

where $\tilde{m}^2$ is defined below. The mixed embedding induced by the mass term μ_{ij} leads to flavor-violating soft-terms. Setting $\mu_{ij} = 0$ allows to preserve the flavor blindness of the soft terms and to satisfy the FCNC constraints without the need of assumptions on the structure of the flavor matrices h_{ij} and μ_{ij}. We therefore assume that μ_{ij} is vanishing or negligible. We then have $\overline{5}^l_i = \overline{5}^{10}_i$, $10^l_i = 10^{16}_i$, with the extra components $\overline{5}^{16}_i$ and 5^{10}_i obtaining a large supersymmetric mass term $M_{ij}\overline{5}^{16}_i 5^{10}_i$, as desired. The soft masses for the light sfermions are

$$\begin{aligned}
(\tilde{m}^2_l)_{ij} = (\tilde{m}^2_{d^c})_{ij} &= \frac{2}{5}\tilde{m}^2\delta_{ij}, \\
(\tilde{m}^2_q)_{ij} = (\tilde{m}^2_{u^c})_{ij} = (\tilde{m}^2_{d^c})_{ij} &= \frac{1}{5}\tilde{m}^2\delta_{ij}, \\
\tilde{m}^2 = 5\frac{(F^\dagger_0 X F_0)}{\phi^\dagger_0 X^2 \phi_0} &> 0,
\end{aligned} \tag{7.8}$$

as anticipated in Eq. (7.2). The reason for taking out the factor $5 = X(1^{16})$ will become clear later.

We now need to identify the embedding of the MSSM Higgs superfields and reproduce their MSSM superpotential interactions, in particular the MSSM Yukawas. It is useful to discuss the Yukawa interaction in SU(5) language. The up quark Yukawa interactions arise from the SU(5) operator

$$\frac{\lambda^U_{ij}}{2}10^l_i 10^l_j 5_H, \tag{7.9}$$

where 5_H contains the MSSM up-type Higgs h_u. Because of $10^l_i = 10^{16}_i$, the operator in Eq. (7.9) can arise at the renormalizable level from a SO(10) invariant operator only if 5_H has a component in a 10_H of SO(10), $10_H = (5^{10}_H, \overline{5}^{10}_H)$, with

$$5^{10}_H = \cos\theta_u 5_H + \ldots, \quad 0 \leq \theta_u \leq \pi/2, \tag{7.10}$$

where $\cos^2\theta_u$ measures the size of the 5_H component from 10 representations of SO(10). The operator in Eq. (7.9) will then emerge as

$$\frac{y^H_{ij}}{2} 16_i 16_j 10_H = \frac{\lambda^U_{ij}}{2} 10^l_i 10^l_j 5_H + \ldots, \quad \text{with} \quad \lambda^U_{ij} = \cos\theta_u y^H_{ij}. \tag{7.11}$$

The down quark and charged lepton Yukawa interactions arise at the renormalizable level[3] from the SU(5) operator

$$\lambda^D_{ij} 10^l_i \,\overline{5}^l_j \,\overline{5}_H, \tag{7.12}$$

where $\overline{5}_H$ contains the MSSM down-type Higgs h_d. Because we have $10^l_i = 10^{16}_i$ and $\overline{5}^l_i = \overline{5}^{10}_i$, the operator in Eq. (7.12) can arise at the renormalizable level from a SO(10) invariant operator only if $\overline{5}_H$ has a component into a 16_H of SO(10), $16_H = (1^{16}_H, 10^{16}_H, \overline{5}^{16}_H)$, with

$$\overline{5}^{16}_H = \sin\theta_d 5_H + \ldots, \quad 0 \leq \theta_d \leq \pi/2, \tag{7.13}$$

where $\sin^2\theta_d$ measures the size of the $\overline{5}_H$ component from 16 representations of SO(10). The operator in Eq. (7.12) will then emerge as

$$h^H_{ij} 16_i 10_j 16_H = \lambda^D_{ij} 10^l_i \,\overline{5}^l_j \,\overline{5}_H + \ldots, \quad \text{with} \quad \lambda^D_{ij} = \sin\theta_d h^H_{ij}. \tag{7.14}$$

It is economical to identify the 16_H with 16, the field whose vev breaks SO(10) to SU(5), in which case $h^H = h$ and the mass of the heavy extra components $\overline{5}^{16}_i$ and 5^{10}_i in Eq. (7.4) turns out to be proportional to the corresponding light fermion masses[4] (up to non-renormalizable corrections needed to fix the light fermion mass ratios).

Having introduced the MSSM Higgs fields, let us now discuss their soft mass terms. To summarize the previous discussion, the up (down) Higgs superfield h_u (h_d) can be embedded in either 10's or 16's ($\overline{16}$'s) of SO(10), in both cases through the embedding into a 5_H ($\overline{5}_H$) of SU(5). We have denoted by $\cos^2\theta_u$ ($\cos^2\theta_d$) the overall size of the h_u (h_d) component in the 10's. The overall size of the component in the 16's ($\overline{16}$'s) is then measured by $\sin^2\theta_u$ ($\sin^2\theta_d$). Correspondingly, the Higgs

[3]SU(5)-invariant renormalizable Yukawa interactions lead to wrong mass relations for the two lighter families of down quarks and charged leptons, which may be fixed by including non-renormalizable operators. We ignore this issue in the following and only consider the renormalizable part of the superpotential.

[4]This property can give rise to a predictive model of leptogenesis in the context of type-II see-saw models [59, 60].

soft masses get two contributions from the first term in Eq. (7.1) proportional to two different X charges:

$$
\begin{aligned}
m_{h_u}^2 &= \frac{-2c_u^2 + 3s_u^2}{5}\tilde{m}^2, \\
m_{h_d}^2 &= \frac{2c_d^2 - 3s_d^2}{5}\tilde{m}^2,
\end{aligned}
\tag{7.15}
$$

so that

$$
\begin{aligned}
-\frac{2}{5}\tilde{m}^2 \leq m_{h_u}^2 \leq \frac{3}{5}\tilde{m}^2, \\
-\frac{3}{5}\tilde{m}^2 \leq m_{h_d}^2 \leq \frac{2}{5}\tilde{m}^2.
\end{aligned}
\tag{7.16}
$$

Let us now consider gaugino masses. A general discussion of all possible contributions to gaugino masses in the presence of an arbitrary number of SO(10) representation with $d < 120$ would be too involved. We therefore consider just a few examples meant to generalize the specific model presented in Section 5.4 and to illustrate the general properties discussed in Section 6.3.

We begin by elucidating the structure of supersymmetry breaking. With the representation content of Table 7.1, supersymmetry breaking can be associated to the F-term vevs of superfields in $\mathbf{16}$, $\overline{\mathbf{16}}$, $\mathbf{45}$, $\mathbf{54}$ representations, because only these contain SM singlets. However, only the $\mathbf{16}$, $\overline{\mathbf{16}}$, whose singlets have non-vanishing X-charges, can contribute to tree-level soft masses. Let us call 16^H_α, $\overline{16}^H_\alpha$ the matter parity even superfields in the $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations of SO(10). In a generic basis, we can parametrize the vevs of their singlet components as

$$
\left\langle 1_\alpha^{16_H} \right\rangle = M_\alpha + F_\alpha \theta^2, \qquad \left\langle 1_\alpha^{\overline{16}_H} \right\rangle = \overline{M}_\alpha + \overline{F}_\alpha \theta^2. \tag{7.17}
$$

The D-term condition for the X generator requires

$$
\sum_\alpha |M_\alpha|^2 \approx \sum_\alpha |\overline{M}_\alpha|^2, \tag{7.18}
$$

while gauge invariance gives

$$
\sum_\alpha M_\alpha^* F_\alpha = \sum_\alpha \overline{M}_\alpha^* \overline{F}_\alpha. \tag{7.19}
$$

Sfermion masses are proportional to (cf. Eq. (7.8))

$$\tilde{m}^2 = \frac{\sum_\alpha (|F_\alpha|^2 - |\overline{F}_\alpha|^2)}{\sum_\alpha (|M_\alpha|^2 + |\overline{M}_\alpha|^2)}, \tag{7.20}$$

where $\sum_\alpha |F_\alpha|^2 > \sum_\alpha |\overline{F}_\alpha|^2$ by definition in the case we are considering. Note that $\tilde{m}^2$ is suppressed by *all* vevs contributing to X breaking.

Let us now comment on the vector contribution to gaugino masses. First we assume that the $\overline{16}$'s do not break supersymmetry. Without loss of generality we can then assume that supersymmetry breaking is only associated to $16' \equiv 16^H_1$. The gauge invariance condition then gives $M_1 = 0$, i.e. a vev for both the F-term and scalar components is not allowed. Since the F-term and scalar components belong to different irreducible representations, no vector contribution to gaugino masses is generated by the 16's. A vector contribution can still be generated by the F-term vev of a **45**, for example, for which the gauge invariance condition does now prevent a vev in both the scalar and F-term component. Or, it can be generated by the F-terms of the 16's if some of the $\overline{16}$ also breaks supersymmetry and cancels the contribution of the 16 to Eq. (7.19).

Now we analyze the chiral contribution to gaugino masses. The massive components $\overline{5}^{16}_i$ and 5^{10}_j of the matter superfields will act as chiral messengers if they are coupled to supersymmetry breaking. As before we consider the case in which the $\overline{16}$'s do not break supersymmetry, and supersymmetry breaking is provided by the F-term vev F of the singlet component of the $16'$ and is felt by the chiral messengers through the $h'_{ij} 16_i 10_j 16'$ interaction. Let $16 \equiv 16^H_2$ be the field whose vev gives mass to the $\overline{5}^{16}_i$, 5^{10}_j through the $h_{ij} 16_i 10_j 16$ interaction, as in Eq. (7.3). And let us assume that additional 16^H_α's and $\overline{16}^H_\alpha$'s get vevs in their scalar components. The chiral messengers $\overline{5}^{16}_i$, 5^{10}_j have therefore a supersymmetric mass $M_{ij} = h_{ij} M$ and their scalar components get a supersymmetry breaking term mass term $F_{ij} = h'_{ij} F$. The induced one-loop chiral contribution to gaugino masses is then

$$M_g = \frac{g^2}{(4\pi)^2} \,\mathrm{Tr}(h' h^{-1}) \frac{F}{M}. \tag{7.21}$$

The tree-level soft mass of the stop (belonging to the **10** of SU(5)) is

$$\tilde{m}^2_t = \frac{1}{5} \frac{|F|^2}{|M|^2 + \sum_\alpha |M_\alpha|^2 + |\overline{M}|^2 + \sum_\alpha |\overline{M}_\alpha|^2}. \tag{7.22}$$

We can then compare stop and gaugino masses (before radiative corrections). Their ratio is particularly interesting, as the gaugino mass M_g is at present bounded to be heavier than about 100 GeV, while $\tilde{m}_t$ enters the radiative corrections to the Higgs mass. Therefore, the ratio $\tilde{m}_t/M_g$ should not be too large in order not to increase the fine-tuning and not to push the stops and the other sfermions out of the LHC reach. From the previous equations we find

$$\frac{M_g}{\tilde{m}_t} = \frac{3\sqrt{5k}}{(4\pi)^2}\,\lambda, \tag{7.23}$$

with

$$\begin{aligned} \lambda &= \frac{g^2\,\mathrm{Tr}(h'h^{-1})}{3}, \\ k &= \frac{|M|^2 + \sum_\alpha |M_\alpha|^2 + |\overline{M}|^2 + \sum_\alpha |\overline{M}_\alpha|^2}{|M|^2} \geq 2. \end{aligned} \tag{7.24}$$

Eq. (7.23) illustrates all the enhancement factors discussed in Section 6.3 that can compensate the loop suppression of gaugino masses. The factor 3 corresponds to the number of chiral messenger families ($\mathrm{Tr}(h'h^{-1}) = 3$ for $h = h'$) contributing to gaugino masses, to be compared to the single vector messenger generating sfermion masses at the tree level. The factor $\sqrt{5}$ comes from the ratio of charges $X(1^{16})/X(10^{16}) = 5$ suppressing the stop mass in Eq. (6.35). The factor $k \geq 2$ is the ratio of the sum of vevs suppressing sfermion masses and the vev suppressing gaugino masses. Note that in the presence of hierarchies of vevs, the factor k can be large. Finally, λ represents a combination of couplings that can further enhance (or suppress) gaugino masses. All in all, we see that the loop factor separating $\tilde{m}_t$ and M_g is partially compensated by a combination of numerical factors: $(4\pi)^2 \sim 100$ (leading to $\tilde{m}_t \gtrsim 10$ TeV for $\lambda = 1$) becomes at least $(4\pi)^2/(3\sqrt{10}) \sim 10$ (leading to $\tilde{m}_t \gtrsim 1$ TeV for $\lambda = 1$). A largish value of the factors k or λ can then further reduce the hierarchy and even make $M_g \sim \tilde{m}_t$, if needed.

7.2.2 The Embedding into $16_i + 45_i$

Let us now consider the second type of embedding identified above, corresponding to $F_0^\dagger X F_0 < 0$. The most general renormalizable superpotential involving 16, $\overline{16}$ and 16_i, 45_i, $i = 1, 2, 3$ and invariant under matter parity is

$$W = h_{ij} 16_i 45_j \overline{16} + \frac{\mu_{ij}}{2} 45_i 45_j + W_{\text{vev}}. \tag{7.25}$$

The term $h_{ij}16_i45_j\overline{16}$ is just what needed to split the SU(5) components of the $16_i = (1_i^{16}, 10_i^{16}, \bar{5}_i^{16})$ and of the $45_i = (1_i^{45}, 10_i^{45}, \overline{10}_i^{45}, 24_i^{45})$ and make heavy the unwanted components 10_i^{16} and $\overline{10}_j^{45}$. Once 16 acquires a vev M, a mass term is generated for those components,

$$M_{ij}10_i^{16}\overline{10}_j^{45}, \qquad M_{ij} = h_{ij}M. \tag{7.26}$$

It is remarkable that also in this case the components acquiring a large mass are precisely those that get a negative soft mass term. On the other hand, this is only true in the limit in which the μ_{ij} mass term in Eq. (7.25) can be neglected. In order to abide to our pure embedding assumption, we will neglect such a term. Let us note, however, that such a term should arise at some level in order to make the 24_i^{45}'s components heavy. Note that the 24_i's do not affect gauge coupling unification at one-loop and can therefore be considerably lighter than the GUT scale, consistently with the required smallness of μ_{ij}. The soft masses for the light sfermions are now

$$\begin{aligned}
(\tilde{m}_l^2)_{ij} = (\tilde{m}_{d^c}^2)_{ij} &= \frac{3}{5}\tilde{m}^2\delta_{ij}, \\
(\tilde{m}_q^2)_{ij} = (\tilde{m}_{u^c}^2)_{ij} = (\tilde{m}_{d^c}^2)_{ij} &= \frac{4}{5}\tilde{m}^2\delta_{ij}, \\
\tilde{m}^2 = -5\frac{(F_0^\dagger X F_0)}{\phi_0^\dagger X^2 \phi_0} &> 0.
\end{aligned} \tag{7.27}$$

Unfortunately, the embedding we are discussing cannot be implemented with renormalizable interactions and $d < 120$ representations only. The problem is obtaining the Yukawa interactions. Let us consider the up quark Yukawas, arising as we saw from the SU(5) operator in Eq. (7.9). Given its size, we expect at least the top Yukawa coupling to arise at the renormalizable level. As in the present case $10_i^l = 10_i^{45}$, the operator in Eq. (7.9) can arise at the renormalizable level from a SO(10) invariant operator only if 5_H has a component in a SO(10) representation coupling to 45_i45_j. And the lowest dimensional possibility containing the **5** of SU(5) is the **210**. For this reason, we do not pursue this possibility further here, although models with large representations are of course not excluded.

7.3 E_6 Embedding

We close this section with a few considerations about the possibility to identify the unified group with E_6. Such a possibility looks particularly appealing in the light of the discussion of the SO(10) case above. We have seen in fact that the most straightforward possibility to realize tree-level gauge mediation in SO(10) requires the matter superfield content to include three $16_i + 10_i$, $i = 1, 2, 3$. This is precisely what E_6 predicts. The fundamental of E_6, a representation of dimension 27, decomposes under SO(10) as

$$\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}. \tag{7.28}$$

The matter content needed by the $16_i + 10_i$ embedding can therefore be provided in the context of E_6 by three matter 27_i, and the 16_H and 10_H needed to accommodate the Higgs fields can also be provided by a single Higgs 27_H. All Yukawas can then in principle follow from the single E_6 interaction

$$\lambda_{ij} 27_i 27_j 27_H. \tag{7.29}$$

We postpone the analysis of this promising possibility to further study.

7.4 The μ-Problem

In this section, we discuss a few approaches to the μ-Problem in the context of tree-level gauge mediation. The problem is to relate the scale of the supersymmetric mass $W \supset \mu\, h_u h_d$, to the supersymmetry breaking scale in the observable sector, which in our case is given by $\tilde{m} \sim |F_0|/M_V$. We discuss in the following three possible connections. One is peculiar of tree-level gauge mediation, the other two have been considered in other contexts, but have specific implementations in tree-level gauge mediation. We classify them according to the dimension D of the SO(10) operator from which the μ-term arises. Note that we are not addressing the origin of the smallness of $\tilde{m}$ and μ compared to the Planck scale, just their connection. The three options we consider are:

- $D = 3$: μ comes from the operator $\mu\, h_u h_d \subset W$. It is the supersymmetry breaking scale that is derived from μ, and not viceversa: $F_0 \sim \mu M$, where

$M = \mathcal{O}(M_V)$, and $\tilde{m} \sim F_0/M \sim \mu$.

- $D = 4$: μ comes from the operator $\lambda S h_u h_d \subset W$. The light SM singlet S gets a vev from a potential whose only scale is $\tilde{m}$, so that $\mu \sim \lambda \langle S \rangle \sim \tilde{m}$.

- $D = 5$: μ comes from the operator $a(Z^\dagger/M) h_u h_d \subset K$, so that $\mu = aF_0/M$.

7.4.1 $D = 3$

This possibility was already mentionend in Section 5.4, here we will discuss a concrete implementation. Let us consider the $16_i + 10_i$ embedding. As discussed in Section 7.2.1, h_u is a superposition of the up-type Higgs components in the $\overline{16}$'s and 10's (with $R_P = 1$) in the model. Analogously, h_d will be a superposition of the down-type Higgs components in the ($R_P = 1$) 16's and 10's. The only possible $D = 3$ origin of the μ-term in the context of the full SO(10) theory are then $\mathcal{O}(\text{TeV})$ mass terms for the above $\overline{16}$'s, 16's, and 10's. As mentioned above, we do not address the origin of such a small parameter in the superpotential, as we do not address the smallness of the supersymmetry breaking scale, but such small scales could for example be explained by a dynamical mechanism. We want however to relate such mass parameters, in particular the coefficient of a $\overline{16}16$ mass term, to the supersymmetry breaking scale. This is actually pretty easy, as the embedding we are considering provides all the necessary ingredients and the result arises simply from their combination. We have seen that the model needs a 16, $\overline{16}$ pair to get a vev in the SM singlet direction of the scalar component, in order to break SO(10) to the SM. Moreover, an independent $16'$, $\overline{16}'$ pair is required to break supersymmetry trough the F-term vev of the SM singlet component in the $16'$. The simplest way to achieve such a pattern is through a superpotential like

$$W_1 = \lambda_1 Z(\overline{16}16 - M^2) + m16'\overline{16} + \lambda_2 X 16\overline{16}', \tag{7.30}$$

where X, Z are SO(10) singlets and $M \sim M_{\text{GUT}}$. This is a generalization of a U(1) toy model in [38]. Finally, we have seen that the light Higgses may have a component in 16, $16'$, $\overline{16}$, $\overline{16}'$. Let α' be the coefficient of the h_d component in the $16'$ and α the coefficient of the h_u component in the $\overline{16}$. Then a μ-parameter is generated in the form

$$\mu = \alpha' \alpha\, m \tag{7.31}$$

from the $m16'\overline{16}$ term in Eq. (7.30). The parameter m is therefore required to be in the window $100\,\text{GeV}/(\alpha'\alpha) \lesssim m \lesssim \text{TeV}/(\alpha'\alpha)$. In the limit $\mu = 0$, supersymmetry is unbroken and 16, $\overline{16}$ acquire a vev that we assume to be in the SM singlet component $\langle 1^{16} \rangle = \langle 1^{\overline{16}} \rangle = M$. A non-vanishing μ instead triggers supersymmetry breaking and induces an F-term vev for the singlet component of the $16'$, $\langle 1^{16'} \rangle = F\theta^2$, with $F = mM$. We therefore have

$$\tilde{m} \sim \frac{F}{M} = m = \frac{\mu}{\alpha'\alpha}, \tag{7.32}$$

thus providing the desired connection between μ and the supersymmetry breaking scale. Tree-level gauge mediation plays a crucial role not only in providing the ingredients (and no need to stir), but also because it is the very SO(10) structure providing the heavy vector messengers to relate in a single irreducible representation (the 16') supersymmetry breaking (the F-term vev of its SM singlet component) and the down Higgs entering the μ-term (the lepton doublet-type component of the 16'). In Appendix B we provide an existence proof of a (perturbative) superpotential that i) implements the mechanism above, thus breaking supersymmetry and SO(10) to SU(5), ii) further breaks SU(5) to the SM, iii) makes all the fields that are not part of the MSSM spectrum heavy, in particular achieves doublet-triplet splitting.

7.4.2 $D = 4$

This is an implementation of the NMSSM solution of the μ-Problem (see e.g. [61] and references therein). As we will see, the realization of such a solution in the context of tree-level gauge mediation avoids some of the problems encountered in ordinary gauge mediation.

In order to use the NMSSM solution of the μ-Problem, an explicit term $\mu\, h_u h_d$ should be forbidden, for example by a symmetry. The light fields Q should include a SM singlet S, coupling to the Higgses through the superpotential interaction $\lambda S h_u h_d$ and S should develop a non-zero vev. The μ-parameter will then be generated, $\mu = \lambda \langle S \rangle$. In the absence of terms linear or quadratic in S in the superpotential, the scale of a vev for S can only be provided by the supersymmetry breaking terms in the soft Lagrangian, $\langle S \rangle \sim \tilde{m}$, in which case $\mu = \lambda \langle S \rangle \sim \lambda \tilde{m}$, as desired.

In order to generate a non-zero vev for S, one would like to have a negative soft mass for S at the weak scale, along with a stabilization mechanism for large values of the fields. In ordinary gauge mediation this is not easy to achieve. While the stabilization can be simply provided by a S^3 term in W, as in the NMSSM (or by a quartic term in Z' extensions of the MSSM [62]), the soft mass term of S vanishes at the messenger scale because S is typically a complete gauge singlet. A non-vanishing negative mass term is generated by the RGE running but it is typically too small. Another problem is that the Higgs spectrum can turn out to be non-viable [63]. A sizable soft mass can still be generated by coupling S to additional heavy fields. Such possibilities can be implemented in our setup by promoting S to an SO(10) singlet and coupling it to the Higgses through a $S\,\overline{16}\,16$ or $S\,10\,10$ coupling to the SO(10) representations containing the Higgs fields.

Tree-level gauge mediation offers a different avenue. A sizable, negative soft mass term for S can easily be generated by embedding S in a $\overline{16}$ of SO(10) (this is the only choice within the fields in Table 7.1). On the other hand, the stabilization of the potential for S is not straightforward. A sizable S^3 term is not expected to arise, as it should involve a SO(10) operator with three $\overline{16}$. However, the S^3 term can be replaced by a term involving a second light singlet N,

$$W = \lambda S h_u h_d + \kappa S^2 N. \tag{7.33}$$

The latter can come from a $\overline{16}\,\overline{16}\,126$ coupling, if N is in the 126 singlet, or from a $\overline{16}\,\overline{16}\,16_1 16_2/\Lambda$ coupling, where N is the 16_1 singlet and 16_2 gets a vev.

The scalar potential for $V(h_u, h_d, S, N)$ can be written as

$$V = V_{\text{MSSM}} + |\kappa S^2|^2 + m_S^2 |S|^2 + |\lambda h_u h_d + 2\kappa S N|^2 + M_N^2 |N|^2, \tag{7.34}$$

where V_{MSSM} is the MSSM scalar potential with $\mu \to \lambda S$, $m_S^2 = -\tilde{m}^2$, and $m_N^2 = 2\tilde{m}^2$ or $\tilde{m}^2$ depending on whether N comes from a 126 or a 16. We have neglected the A-terms, which play a role in explicitly breaking R-symmetries that could lead to massless states. The potential above has a minimum with a sizable $\langle S \rangle$, and a μ parameter whose size is controlled by λ.

7.4.3 $D = 5$

Finally, We discuss the possibility to generate the μ parameter through a $D = 5$ correction to the Kähler in the form $a(Z^\dagger/M) h_u h_d$, as in the Giudice-Masiero mechanism [20]. The F-term vev $|F_0|$ of Z would give in this case $\mu = a|F_0|/M$.

We show first that the operator above cannot arise at the tree level from integrating out heavy vector or chiral superfields. The corrections to the Kähler obtained by integrating out heavy vector superfields are given in Eq. (A.10). All terms are at least of second order in $1/M_V$ and no trilinear term is present. Moreover, no sizable trilinear term can be obtained through the vev of Φ', as by definition the scalar components of Φ' do not get a vev (and an F-term vev would give an additional F_0/M_V suppression). A similar conclusion can be obtained for the corrections one obtains by integrating out chiral superfields Φ_i^h with mass $M \gg \sqrt{|F_0|}$. We have seen in Section 6.2 that the equations of motion allow to express Φ_i^h in terms of the light fields as in Eq. (6.20). Since W_3 contains terms at least trilinear in the fields, the expression for Φ_i^h is at least quadratic in the light fields. When plugging Eq. (6.20) in the canonical Kähler for Φ_i^h one gets again terms that contain at least four light fields, with none of them getting a vev in the scalar component. Therefore, no operator $Z^\dagger h_u h_d$ can be generated at the tree-level by integrating out heavy fields.

Let us now consider the possibility that the $D = 5$ operator above is obtained at the one-loop level. This possibility raises two issues. First, μ would be suppressed compared to, say, the stop mass $\tilde{m}_t$ by a loop factor $\mathcal{O}\left(10^{-2}\right)$. As for the case of gaugino masses vs. sfermion masses, such a large hierarchy would lead to sfermions beyond the reach of the LHC and a significant fine-tuning. However this problem can be overcome in the same way as for the gaugino masses. Indeed we will see in an explicit model that μ and $M_{1/2}$ get a similar enhancement factor. The second issue is the well known μ-B_μ problem. B_μ is a dimension two parameter generated, as μ, at the one-loop level. Therefore, we expect an order of magnitude separation between $\sqrt{B_\mu}$ and μ: $\sqrt{B_\mu}/\mu \sim 4\pi$. This is however tolerable in a scheme in which $\tilde{m}_t \sim \sqrt{B_\mu} \sim 4\pi\mu \sim 4\pi M_{1/2}$, with $\tilde{m}_t \sim \sqrt{B_\mu} \sim$ TeV and $\mu \sim M_{1/2} \sim$ 100 GeV. The explicit model will show that the above pattern can be achieved in the large $\tan\beta$ regime. In turn, the large $\tan\beta$ regime raises a new issue. The minimization of the MSSM potential shows in fact that large $\tan\beta$ corresponds to small $B_\mu/(m_{h_u}^2 + m_{h_d}^2 + 2|\mu|^2)$, while in the situation we want to reproduce, $\tilde{m}_t \sim \sqrt{B_\mu}$, we expect $B_\mu/(m_{h_u}^2 + m_{h_d}^2 + 2|\mu|^2) \sim 1$. In order to make $\tan\beta$ large we therefore need to cancel the contribution to B_μ we get at one-loop with an additional contribution, at least in the specific example we consider. Such a cancellation may not be required in different implementations of the one-loop $D = 5$ origin of the μ parameter. That is why we believe it is worth illustrating the example below despite the cancellation that needs to be invoked.

Let us consider as before a model involving the following $R_P = 1$ fields: 16, $\overline{16}$,

$16'$, $\overline{16}'$, 10, with $\langle 1^{16} \rangle = \langle 1^{\overline{16}} \rangle = M$, $\langle 1^{16'} \rangle = F\theta^2$, $\langle 1^{\overline{16}'} \rangle = 0$. Let us denote the coefficients of the h_u and h_d components in the above SO(10) representations as follows: $16 \supset s_d \alpha_d h_d$, $16' \supset s_d \alpha'_d h_d$, $10 \supset c_d h_d$, $\overline{16} \supset s_u \alpha_u h_u$, $\overline{16}' \supset s_u \alpha'_u h_u$, $10 \supset c_u h_u$, where $|\alpha_d|^2 + |\alpha'_d|^2 = 1$, $|\alpha_u|^2 + |\alpha'_u|^2 = 1$, $c_d = \cos\theta_d$, $s_d = \sin\theta_d$, etc. The notation is in agreement with the definition of θ_u, θ_d in Section 7.2.1. The μ and B_μ parameters, as the gaugino masses, get a vector and a chiral one-loop contribution, see Eqs. (6.36, 6.38, 6.40, 6.41). The vector contribution turns out to be

$$|(\mu)_V| = \frac{3}{2}\frac{g^2}{(4\pi)^2} s_u s_d |\alpha'_d \alpha_u| \left|\frac{F}{M}\right|, \tag{7.35}$$

$$(B_\mu)_V = \frac{3}{4}\frac{g^2}{(4\pi)^2} s_u s_d |\alpha'_d \alpha_u| \left|\frac{F}{M}\right|^2. \tag{7.36}$$

As in the case of gaugino masses, the vector contribution to μ is suppressed with respect to the sfermion masses by a full loop factor. We therefore need a larger chiral contribution in order to reduce the hierarchy between μ and $\tilde{m}_t$. Let us then consider the one-loop chiral contribution associated to the superpotential

$$h_{ij} 16_i 10_j 16 + h'_{ij} 16_i 10_j 16'. \tag{7.37}$$

That is easily found to be vanishing because of a Peccei-Quinn (PQ) symmetry of the superpotential. Such a PQ symmetry can however be broken by adding a term

$$\frac{M^1_{ij}}{2} 1^{16}_i 1^{16}_j \tag{7.38}$$

to the above superpotential, coming for example from the non-renormalizable SO(10) operator $(\alpha_{ij}/\Lambda)(\overline{16}16_i)(\overline{16}16_j)$ after $\overline{16}$ gets its vev (note that $\Lambda \gg M$ would give $M^1_{ij} \ll M$). The singlet mass term in Eq. (7.38) is nothing but the right-handed neutrino Majorana mass term entering the see-saw formula for light neutrino masses. Note however that no light neutrino mass is generated here, as the light lepton doublets do not have Yukawa interactions with the "right-handed neutrinos", 1^{16}_i. Once the PQ symmetry is broken by the mass term in Eq. (7.38), the μ and B_μ

parameters get a chiral one-loop contribution given by

$$|(\mu)_\Phi| = \frac{\lambda_t\lambda_b}{(4\pi)^2} f\left(\frac{\sqrt{(M^1M^{1*})_{33}}}{|h_{33}M|}\right) \frac{|M^1_{33}|}{\sqrt{(M^1M^{1*})_{33}}} \left|\frac{h'_{33}F}{h_{33}M}\right|, \tag{7.39}$$

$$(B_\mu)_\Phi = \frac{\lambda_t\lambda_b}{(4\pi)^2} g\left(\frac{\sqrt{(M^1M^{1*})_{33}}}{|h_{33}M|}\right) \frac{|M^1_{33}|}{\sqrt{(M^1M^{1*})_{33}}} \left|\frac{h'_{33}F}{h_{33}M}\right|^2, \tag{7.40}$$

where λ_t, λ_b are the top and bottom Yukawa couplings respectively and the functions f, g are given by

$$f(x) = \frac{1 - x^2 + x^2 \log x^2}{(x^2-1)^2} x, \qquad g(x) = \frac{x^4 - 2x^2 \log x^2 - 1}{(x^2-1)^3} x. \tag{7.41}$$

We have assumed the Yukawa couplings h_{ij}, h'_{ij} to be hierarchical in the basis in which the down Yukawa matrix is diagonal.

We can see from Eq. (7.40) that the one-loop chiral contribution to μ is comparable to the corresponding contribution to $M_{1/2}$ if i) $\lambda_b \sim 1$, which corresponds to the large $\tan\beta$ regime (remember that the bottom mass is given by $m_b = \lambda_b \cos\beta v$, where $v = 174\,\text{GeV}$); ii) $|h'_{33}/h_3| \gtrsim |h'_{ii}/h_i|$, $i = 1, 2$; iii) $|M_{33}| \gtrsim |M_{3i}|$; iv) $|h_{33}M| \sim |M_{33}|$. If the above conditions are satisfied, $\mu \sim M_{1/2}$ and both parameters can easily be enhanced, as explained in Section 6.3.2, for example because $|h'_{33}/h_{33}| \gg 1$. The only non-trivial condition is the large $\tan\beta$ one. In fact $\tan\beta$ is determined by B_μ through the minimization of the MSSM potential, which gives

$$\sin 2\beta = \left.\frac{2B_\mu}{m^2_{h_u} + m^2_{h_d} + 2|\mu|^2}\right|_{M_Z}. \tag{7.42}$$

Therefore large $\tan\beta$, i.e. small $\sin 2\beta$, requires a small B_μ. This is in contrast with the situation we want to reproduce, $\tilde{m}_t \sim \sqrt{B_\mu}$. The RGE evolution of B_μ from the scale at which it is generated ($|h_{33}M|$) down to the electroweak scale can reduce the value of B_μ but not enough to make it as small as we need. A significant RGE contribution would in fact require $M_{1/2} \gtrsim \tilde{m}_t$, in contrast with the $\tilde{m}_t \sim 4\pi M_{1/2}$ we are trying to reproduce. We are then forced to invoke a cancellation between the one-loop contribution to B_μ in Eq. (7.40) and an additional contribution. For example, a tree-level contribution to B_μ can be obtained as in Appendix B or in [38].

8
Summary and Outlook

In this chapter we have discussed what might be the simplest way to communicate supersymmetry breaking, namely through the tree-level, renormalizable exchange of superheavy gauge messengers, which naturally arise in the context of grand unified theories. We have shown that such a scheme is not only viable and motivated, but also attractive, leading to flavor-universal sfermion masses, and finally testable, by making a prediction of sfermion mass ratios that is to a large extent model-independent.

Sfermion masses arise at tree-level from a supersymmetry breaking source that is part of a non-trivial GUT multiplet. This is most conveniently seen in the effective theory in which heavy vector superfields associated to broken U(1) generators are integrated out at tree-level. This gives rise to flavor-universal sfermion masses whose sign is determined by their charge under the additional U(1) generators. Because these generators are embedded into a simple grand unified group, they are traceless over full GUT multiplets, and thus induce both positive and negative soft masses. This is connected to the mass sum rule, which has long been considered as an obstacle to tree-level supersymmetry breaking. The presence of negative soft masses is however not problematic, because the associated fields can simply have large (positive) supersymmetric mass terms. This requires a splitting of GUT multiplets containing the light MSSM fields, but this can be easily achieved at least in the case of SO(10) GUTs.

Gaugino masses do not arise at the tree level, but can be generated at one-loop, as in ordinary gauge mediation. The loop factor suppression of gaugino compared to sfermion masses is however compensated by numerical factors. We have demonstrated that such enhancement factors naturally arise, and can easily reduce the hierarchy of sfermion and gaugino masses to a about a factor of 10, which leads to sfermions at the TeV scale that are in the LHC reach and a fine-tuning of the Higgs mass that is not larger than usual.

We discussed the realization of these basic ideas both in a general setup and an explicit SO(10) model. In a general context we calculated the structure of soft terms arising from integrating out heavy vector and chiral superfields both at tree-level and one-loop, concentrating on the expressions for soft sfermion and gaugino masses. This allowed us to define the general guidelines to obtain phenomenologically viable models which we discussed to great extent for the case of SO(10) as unified group. We studied various ways to generate the μ-term in this framework, in particular the possibility that a tree-level μ is already present in the high-energy theory and connected to SUSY breaking by the SO(10) structure.

The implementation of the general concepts was performed in a simple SO(10) model, where we also briefly discussed the phenomenological and cosmological consequences. One of the main predictions is that sfermion masses are SU(5) invariant with a peculiar ratio of sfermion masses that are embedded in different SU(5) representations. This relation holds at the high scale, but because gaugino masses are typically small, the ratio should be traceable also in the low-energy spectrum and thus might be measurable at the LHC. We are planning to study the phenomenological consequences including collider signatures as well as the cosmological implications in much more detail within a future publication.

Another direction of further research might be related to the scale of U(1) breaking. In this chapter we concentrated on the case where the heavy vector lives at the GUT scale, but another interesting possibility could be that SUSY is broken the TeV scale and communicated to the observable sector by a light Z' gauge field. Gaugino masses would then require also chiral matter at the TeV scale, and since there are many models which consider such extensions of the MSSM for other reasons, it would be very interesting to see whether such a low-scale implementation of TGM can be realized.

Part III

Appendices

A

Integrating out Vector Superfields

In this appendix, we derive the effective theory obtained from integrating heavy vector superfields at the tree-level in unitary gauge in the context of a generic, non-abelian, $N = 1$ globally supersymmetric theory with renormalizable Kähler K and gauge-kinetic function (the superpotential W is allowed to be non-renormalizable). The general prescription has been studied in [51, 52, 53, 64]. In particular, it has been shown in [53] that the usual expansion in the number of derivatives n_∂ can be made consistent with supersymmetry by generalizing n_∂ to the parameter

$$n = n_\partial + \frac{1}{2} n_\psi + n_F, \tag{A.1}$$

where $n_\psi/2$ is the number of fermion bilinears and n_F the number of auxiliary fields from chiral superfields. With such a definition, a chiral superfield Φ has $n = 0$ and $d\theta$ integrations and supercovariant derivatives have $n = 1/2$. Such an expansion makes sense when supersymmetry breaking takes place at a scale much smaller than the heavy superfield mass M, and in particular when the F-terms and fermion bilinears from heavy superfields being integrated out are much smaller than M.

In the presence of vector superfields one should further assume that the D-terms and gaugino bilinears are small and modify Eq. (A.1) in order to account for the number n_λ of gauginos and the number n_D of vector auxiliary fields. We claim that the correct generalization is

$$n = n_\partial + \frac{1}{2} n_\psi + n_F + \frac{3}{2} n_\lambda + 2 n_D, \tag{A.2}$$

which implies that a vector superfield V has $n = 0$. Note that the double weight of

D-terms compared to F-terms is consistent with Eq. (6.9). With such a definition, the initial Lagrangian has $n = 2$, except for the gauge kinetic term which has $n = 4$.

Chiral and vector superfields can now be integrated out at the tree level by using the supersymmetric equations of motion

$$\frac{\partial W}{\partial \Phi} = 0 \quad \text{and} \quad \frac{\partial K}{\partial V} = 0, \tag{A.3}$$

which neglects terms with $n \geq 3$ when integrating out chiral superfields and $n \geq 4$ when integrating vector superfields (and missing terms originating from the gauge-kinetic term having $n \geq 6$).

From a physical point of view, we are interested not only in the expansion in n but also, and especially, in the expansion in the power m of $1/M$. It is therefore important to remark that using Eqs. (A.3) amounts to neglecting terms with $m \geq 3$ when integrating chiral superfields and $m \geq 6$ when integrating out vector superfields.

We are now ready to present our results for the effective theory obtained integrating out the heavy vector superfields in a generic supersymmetric gauge theory as above. We are interested in operators with dimension up to 6 ($m \leq 2$) in the effective theory. We can then use the equation $\partial K/\partial V = 0$. Neglecting higher orders in m, the latter equation can be rewritten as

$$V_a^h (M_V^2)_{ab} = -\frac{1}{2} \frac{\partial K_2}{\partial V_b^h} (\Phi', V^l), \tag{A.4}$$

where Φ' is defined in Eq. (6.3), $K_2(\Phi', V) = \Phi'^\dagger e^{2gV} \Phi'$, the indices run over the broken generators, and M_V^2 is a function of the light vector superfields:

$$(M_V^2)_{ab} = \frac{1}{2} \frac{\partial^2}{\partial V_a^h \partial V_b^h} \left(\phi_0^\dagger e^{2gV} \phi_0 \right) \Big|_{V^h=0} = (M_{V0}^2)_{ab} + (M_{V2}^2)_{ab}, \tag{A.5}$$

$$(M_{V0}^2)_{ab} = g^2 \phi_0^* \{T_a^h, T_b^h\} \phi_0, \tag{A.6}$$

$$(M_{V2}^2)_{ab} = \frac{g^4}{3} \phi_0^* T_a^h V^l V^l T_b^h \phi_0 + (a \leftrightarrow b). \tag{A.7}$$

In order to solve Eq. (A.4) for V_a^h, we need to invert the field-dependent matrix M_V^2. In Wess-Zumino gauge for the light vector superfields, we get

$$(M_V^2)^{-1}_{ab} = (M_{V0}^2)^{-1}_{ab} - (M_{V0}^2)^{-1}_{ac} (M_{V2}^2)_{cd} (M_{V0}^2)^{-1}_{db}. \tag{A.8}$$

Finally, we obtain for the effective contribution to the Kähler potential

$$\delta K_{\rm eff} = -(M_V^2)_{ab} V_a^h V_b^h = \delta K_{\rm eff}^0 + \delta K_{\rm eff}^1 + \delta K_{\rm eff}^2, \tag{A.9}$$

where

$$\delta K_{\rm eff}^0 = -g^2 (M_{V0}^2)^{-1}_{ab} (\Phi'^\dagger T_a^h \Phi')(\Phi'^\dagger T_b^h \Phi'), \tag{A.10}$$

$$\delta K_{\rm eff}^1 = -2g^3 (M_{V0}^2)^{-1}_{ab} (\Phi'^\dagger T_a^h \Phi')(\Phi'^\dagger \{V^l, T_b^h\} \Phi'), \tag{A.11}$$

$$\begin{aligned}\delta K_{\rm eff}^2 = &-\frac{4}{3} g^4 (M_{V0}^2)^{-1}_{ab} (\Phi'^\dagger T_a^h \Phi') \Phi'^\dagger (T_b^h V^l V^l + V^l T_b^h V^l + V^l V^l T_b^h) \Phi' \\ &- g^4 (M_{V0}^2)^{-1}_{ab} (\Phi'^\dagger \{T_a^h, V^l\} \Phi')(\Phi'^\dagger \{T_b^h, V^l\} \Phi') \\ &- \frac{1}{3} g^4 (M_{V0}^2)^{-1}_{ab} (\Phi'^\dagger [T_a^h, V^l] \Phi')(\Phi'^\dagger [T_b^h, V^l] \Phi').\end{aligned} \tag{A.12}$$

In recovering Eq. (A.12) we have used the identity

$$f_{\alpha ab} (M_{V0}^2)_{bc} = -f_{\alpha cb} (M_{V0}^2)_{ba}, \tag{A.13}$$

where f_{abc} are the structure constants of the gauge group, the Latin indices refer to broken generators and the Greek one refers to an unbroken one.

We are interested to soft supersymmetry breaking terms arising from Eq. (A.9) when some of the auxiliary fields get a vev. The relevant terms should contain up to two F-terms and one D-term, cf. Eq. (6.9). The only relevant terms are therefore those in (A.10).

B

Example of SO(10) Superpotential

In this appendix we provide an example of a superpotential which accounts for supersymmetry breaking and SO(10) breaking to the SM. Moreover it takes care of the correct light field content (in particular doublet-triplet splitting) and generates the μ-term of the right order. We do not consider this superpotential particularly simple or realistic, we are just aiming to provide a proof of existence.

SO(10) will broken to the SM at a scale $M \sim M_{\rm GUT}$. Below this scale only the MSSM fields survive, in particular the Higgs triplets are made heavy via a generalization of the Dimopoulos-Wilczek mechanism [60, 65]. The μ-term is present in the theory in the form of a $D = 3$ operator present at the GUT scale and triggers supersymmetry breaking. B_μ is generated at the tree-level and is naturally of the same order as the sfermion masses.

Superpotential

The superpotential we use is

$$W = W_Y + W_1 + W_2 + W_3 + W_4, \tag{B.1}$$

where

$$\begin{aligned}
W_Y &= y_{ij} 16_i 16_j 10 + h_{ij} 16_i 10_j 16 + h'_{ij} 16_i 10_j 16', && \text{(B.2)}\\
W_1 &= \lambda_1\, Z(\overline{16}16 - M^2) + m\, \overline{16}16' + \lambda_2\, X\, \overline{16}'16, && \text{(B.3)}\\
W_2 &= \overline{16}''(\lambda_3\, 45 + \lambda_4\, U)16 + \overline{16}(\lambda_5\, 45 + \lambda_6\, U')16'' \\
&\quad + M_{45}\, 45\, 45 + \lambda_7\, 54\, 45\, 45 + M_{54}\, 54\, 54, && \text{(B.4)}\\
W_3 &= \lambda_8\, 1616'120 + \lambda_9\, \overline{16}\,\overline{16}'120 + M_{120}\, 120\, 120, && \text{(B.5)}\\
W_4 &= \lambda_{10}\, 10'\, 45\, 10 + \lambda_{11}\, \overline{16}\,\overline{16}''10 + M_{10} 10' 10' \\
&\quad + \lambda_{12}\, \overline{16}\,\overline{16}'10 + \lambda_{13}\, 1616''10 + \lambda_{14}\, Z\, 10\, 10. && \text{(B.6)}
\end{aligned}$$

Here we denote the fields according to their SO(10) representation, except the SO(10) singlet fields Z, X, U, U'. The mass parameter m is of the order of the TeV scale (we do not discuss the origin of such a small parameter here), while all other mass parameters are near the GUT scale

$$\text{TeV} \sim m \ll M \sim M_{45} \sim M_{54} \sim M_{10} \sim M_{120} \sim M_{\text{GUT}}.$$

Let us discuss the role of the different contributions to the superpotential and anticipate the vacuum structure and the spectrum. W_1 is responsible for supersymmetry breaking and the breaking of SO(10) to SU(5): as we are going to show below, this part of the superpotential generates $\mathcal{O}\,(M_{\text{GUT}})$ vevs for the scalar components of 16 and $\overline{16}$ along the SU(5) singlet direction $\langle S \rangle \sim M + \mathcal{O}(m^2/M)$ and $\langle \overline{S} \rangle \sim M + \mathcal{O}(m^2/M)$, and a supersymmetry breaking vev for the F-term component of $16'$ along the SU(5) singlet direction $\langle F_{S'} \rangle \sim mM$. It also provides small supersymmetry breaking vevs for the F-term component of X $\langle F_X \rangle \sim m^2$ and for the D-term of the vector superfield corresponding to the $U(1)_X$ generator of SO(10) $\langle D_X \rangle \sim M(\langle S \rangle - \langle \overline{S} \rangle) \sim m^2$. This D-term vev will generate sfermion masses along the lines of Section 7.2.1. This part of the superpotential is a generalization of a toy model in [38].

W_Y contains the MSSM Yukawa couplings and provides supersymmetry breaking masses for heavy chiral superfields that will generate gaugino masses at one-loop as in ordinary gauge mediation. The MSSM matter is embedded in both the 16_i and the 10_i, as explained in Section 7.2.1. The MSSM Higgs fields are linear combinations of different fields and have components in different representations,

$$h_u \subset 10, \overline{16} \qquad h_d \subset 10, 16, 16', 120.$$

Therefore the first term in W_Y contains the up-type Yukawas, while the second and third terms provide down-type and charged lepton Yukawas. The second term gives a large mass to the additional matter fields $5_i^{10} \subset 10_i$ and $\overline{5}_i^{16} \subset 16_i$ as well. These fields couple also to the F-term vev in the $16'$ and therefore act as one-loop messengers of supersymmetry breaking. While this gives a subleading contribution to sfermion masses, it is the only source of gaugino masses in this model.

The role of W_2 is to break of SU(5) to the standard model gauge group. It provides a large vev for the 45 along the $B-L$ direction $\langle 45_{B-L} \rangle \sim M$ as needed for the Dimopoulos-Wilczek mechanism. Also U, U' and the SM singlet in the 54 take large vevs. W_3 merely gives large masses to components in the $16'$ and $\overline{16}'$. Note that since the 120 does not contain SU(5) singlets, the neutrino component in the $16'$ stays massless as it should, being the dominant component of the Goldstino superfield. W_4 takes care of the Higgs sector: it keeps the MSSM Higgs doublets light and gives a large mass to the corresponding triplets. Its last term provides the B_μ term because Z gets a small supersymmetry breaking vev and both h_u and h_d have components in the 10. The μ term is contained in W_1, because h_d has a component also in the $16'$.

Vacuum Structure

We are interested in a vacuum that does not break the SM gauge group[1]. Thus only that part of the superpotential which involve SU(5) singlets is relevant for the determination of the ground state. We denote the singlets in $(16, \overline{16}, 16', \overline{16}', 16'', \overline{16}'')$ by $(S, \overline{S}, S', \overline{S}', S'', \overline{S}'')$ (which is different than the notation used in the main text)

[1]We were not able to exclude the possible existence of SM breaking vacua with lower energy. However, the SM conserving vacuum that we will discuss below is at least a local minimum, since all masses in the theory are positive. If this vacuum is not the global minimum, it might be either sufficiently longlived or made the global minimum by adding new fields and interactions to the superpotential.

and the singlets in the 45, 54 by B, T, V, where B, T are the properly normalized fields corresponding to the $B-L$ and T_{3R} generators in SO(10). The relevant part of the superpotential is

$$
\begin{aligned}
W &= \lambda_1\, Z(\overline{S}S - M^2) + m\,\overline{S}S' + \lambda_2\, X\,\overline{S}'S \\
&+ \overline{S}''\left(-\frac{\lambda_3}{2}T + \frac{\lambda_3}{2}\sqrt{\frac{3}{2}}B + \lambda_4\, U\right)S + \overline{S}\left(-\frac{\lambda_5}{2}T + \frac{\lambda_5}{2}\sqrt{\frac{3}{2}}B + \lambda_6\, U\right)S'' \\
&+ M_{45}(B^2+T^2) + M_{54}V^2 + \lambda_7\, V\left(\frac{1}{2}\sqrt{\frac{3}{5}}T^2 - \frac{1}{\sqrt{15}}B^2\right). \qquad \text{(B.7)}
\end{aligned}
$$

The F-term and D_X-term equations show that SUSY is broken ($F_{S'} \neq 0$) and that all vevs are determined except V, B, T, for which there exist three solutions, all yielding $F_T = F_V = F_B = 0$. This tree-level degeneracy is lifted by one-loop corrections which select the solution with $T = 0, B \neq 0, V \neq 0$. One can check that the vevs are given by

$$
\begin{gathered}
S' = \overline{S}' = S'' = \overline{S}'' = X = Z = T = 0, \\
S = M - \frac{m^2}{4M}\left(\frac{1}{\lambda_1^2} - \frac{1}{50g^2}\right), \qquad \overline{S} = M - \frac{m^2}{4M}\left(\frac{1}{\lambda_1^2} + \frac{1}{50g^2}\right), \\
U = -\frac{3\sqrt{5}}{2}\frac{\lambda_5}{\lambda_6\,\lambda_7}\sqrt{M_{45}M_{54}}, \qquad U' = -\frac{3\sqrt{5}}{2}\frac{\lambda_3}{\lambda_4\,\lambda_7}\sqrt{M_{45}M_{54}}, \qquad \text{(B.8)} \\
V = \frac{\sqrt{15}M_{45}}{\lambda_7}, \qquad B = \frac{\sqrt{30}}{\lambda_7}\sqrt{M_{45}M_{54}}, \\
F_{S'} = -mM, \qquad F_Z = \frac{m^2}{2\lambda_1}, \qquad D_X = -\frac{m^2}{10g}.
\end{gathered}
$$

Spectrum and soft terms

In order to identify the light fields (with respect to M_{GUT}), we can set $m = 0$ and consider the supersymmetric limit. Most fields are at the GUT scale, while the light fields are the MSSM ones, the Goldstino superfield S', and the right-handed neutrinos in the 16_i, which can easily be made heavy through a non-renormalizable superpotential operator $(\overline{16}16_i)(\overline{16}16_j)$. The MSSM matter fields are embedded in the 10_i^{16} and in the $\overline{5}_i^{10}$, as desired. The Higgs doublets are embedded into the $\overline{16}, 10$

and $10, 120, 16', 16$ according to

$$h_u = \frac{1}{N_u}\left(\overline{L}_{\overline{16}} + 3\sqrt{5}\frac{\lambda_5}{\lambda_7\,\lambda_{13}}\frac{\sqrt{M_{45}M_{54}}}{M}\overline{L}_{10}\right)$$

$$h_d = \frac{1}{N_d}\left(L_{10} - \frac{\lambda_{12}}{\lambda_9}L_{120} + 2\frac{\lambda_{12}}{\lambda_8\,\lambda_9}\frac{M_{120}}{M}L_{16'} + \frac{1}{3\sqrt{5}}\frac{\lambda_7\,\lambda_{11}}{\lambda_3}\frac{M}{\sqrt{M_{45}M_{54}}}L_{16}\right) \tag{B.9}$$

with normalization factors N_u and N_d, where L_R, $\overline{L}_R$ denote the SM component with the quantum numbers of h_d, h_d in the SO(10) representation R.

By switching on m, the soft supersymmetry breaking terms and the μ-term are generated. The μ-term is already present in the high-energy Lagrangian and is of order m, the vev of D_X generates sfermion and Higgs masses of order m^2 and the vev of F_X gives rise to a B_μ term of order m^2. The heavy fields 5_i^{10} and $\overline{5}_i^{16}$ act as messengers of SUSY breaking to the gauginos who get masses of order $m^2/(16\pi^2)$. The Goldstino will be mainly the fermion in S' but gets also small contributions from the gaugino corresponding to the $U(1)_X$ generator and the fermion in Z. The corresponding scalar will get a mass of order m^2.

Bibliography

[1] I. Newton, *Philosophiae Naturalis Principia Mathematica*, 1687.

[2] J.C.Maxwell, *A Dynamical Theory of the Electromagnetic Field*, Philosophical Transactions of the Royal Society of London 155, 459-512 (1865).

[3] S. L. Glashow, "Partial Symmetries Of Weak Interactions," Nucl. Phys. **22** (1961) 579.

[4] S. Weinberg, "A Model Of Leptons," Phys. Rev. Lett. **19** (1967) 1264;

A. Salam and J. C. Ward, "Electromagnetic and weak interactions," Phys. Lett. **13** (1964) 168.

[5] S. Weinberg, "The Making of the standard model," Eur. Phys. J. C **34** (2004) 5 [arXiv:hep-ph/0401010].

[6] H. Georgi and S. L. Glashow, "Unity Of All Elementary Particle Forces," Phys. Rev. Lett. **32**, 438 (1974).

[7] H. Georgi, Particles and Fields, Proceedings of the APS Div. of Particles and Fields, ed C. Carlson, p. 575 (1975);

H. Fritzsch and P. Minkowski, "Unified Interactions Of Leptons And Hadrons," Annals Phys. **93** (1975) 193.

[8] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, ed. P. van Nieuwenhuizen and D.Z. Freedman, North-Holland, Amsterdam, 1979, p. 315.

[9] W.J. Marciano, p. 190, Eighth Workshop on Grand Unification, ed. K. Wali, World Scientific Publishing Co., Singapore (1987).

[10] S. Dimopoulos and H. Georgi, "Softly Broken Supersymmetry And SU(5)," Nucl. Phys. B **193** (1981) 150.

[11] P. Langacker and M. x. Luo, "Implications of precision electroweak experiments for M_t, ρ_0, $\sin^2\theta_W$ and grand unification," Phys. Rev. D **44** (1991) 817.

[12] P. Ramond, "Dual Theory for Free Fermions," Phys. Rev. D **3** (1971) 2415;

A. Neveu and J. H. Schwarz, "Factorizable dual model of pions," Nucl. Phys. B **31** (1971) 86;

Yu. A. Golfand and E. P. Likhtman, "Extension of the Algebra of Poincare Group Generators and Violation of p Invariance," JETP Lett. **13** (1971) 323 [Pisma Zh. Eksp. Teor. Fiz. **13** (1971) 452];

D. V. Volkov and V. P. Akulov, "Is the Neutrino a Goldstone Particle?," Phys. Lett. B **46** (1973) 109;

J. Wess and B. Zumino, "Supergauge Transformations in Four-Dimensions," Nucl. Phys. B **70** (1974) 39;

A. Salam and J. A. Strathdee, "Supergauge Transformations," Nucl. Phys. B **76** (1974) 477.

[13] L. Girardello and M. T. Grisaru, "Soft Breaking Of Supersymmetry," Nucl. Phys. B **194** (1982) 65.

[14] H. E. Haber and G. L. Kane, "The Search For Supersymmetry: Probing Physics Beyond The Standard Model," Phys. Rept. **117** (1985) 75.

[15] G. R. Farrar and P. Fayet, "Phenomenology Of The Production, Decay, And Detection Of New Hadronic States Associated With Supersymmetry," Phys. Lett. B **76** (1978) 575.

[16] J. Ellis and D.V. Nanopoulos, "Flavor Changing Neutral Interactions In Broken Supersymmetric Theories," Phys. Lett. B **110**, 44 (1982);

R. Barbieri and R. Gatto, "Conservation Laws For Neutral Currents In Spontaneously Broken Supersymmetric Theories," Phys. Lett. B **110**, 211 (1982);

B.A. Campbell, "Supersymmetry And Neutral Flavor Nonconservation," Phys. Rev. D **28**, 209 (1983).

S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, "Effects Of Supergravity Induced Electroweak Breaking On Rare B Decays And Mixings," Nucl. Phys. B **353**, 591 (1991);

F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, "A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model," Nucl. Phys. B **477**, 321 (1996) [hep-ph/9604387].

[17] J. E. Kim and H. P. Nilles, "The Mu Problem And The Strong CP Problem," Phys. Lett. B **138** (1984) 150.

[18] H. P. Nilles, "Dynamically Broken Supergravity And The Hierarchy Problem," Phys. Lett. B **115** (1982) 193;

A.H. Chamseddine, R. Arnowitt and P. Nath, "Locally Supersymmetric Grand Unification," Phys. Rev. Lett. **49**, 970 (1982);

R. Barbieri, S. Ferrara and C. A. Savoy, "Gauge Models With Spontaneously Broken Local Supersymmetry," Phys. Lett. B **119**, 343 (1982);

L.E. Ibáñez, "Locally Supersymmetric SU(5) Grand Unification," Phys. Lett. B **118**, 73 (1982);

L.J. Hall, J.D. Lykken and S. Weinberg, "Supergravity As The Messenger Of Supersymmetry Breaking," Phys. Rev. D **27**, 2359 (1983);

N. Ohta, "Grand Unified Theories Based On Local Supersymmetry," Prog. Theor. Phys. **70**, 542 (1983).

[19] M. Dine, W. Fischler and M. Srednicki, "Supersymmetric Technicolor," Nucl. Phys. B **189**, 575 (1981);

M. Dine and W. Fischler, "A Phenomenological Model Of Particle Physics Based On Supersymmetry," Phys. Lett. B **110**, 227 (1982);

M. Dine and A. E. Nelson, "Dynamical supersymmetry breaking at low-energies," Phys. Rev. D **48**, 1277 (1993) [arXiv:hep-ph/9303230];

M. Dine, A. E. Nelson and Y. Shirman, "Low-Energy Dynamical Supersymmetry Breaking Simplified," Phys. Rev. D **51**, 1362 (1995) [arXiv:hep-ph/9408384].

[20] G. F. Giudice and A. Masiero, "A Natural Solution to the mu Problem in Supergravity Theories," Phys. Lett. B **206** (1988) 480.

[21] G. R. Dvali, G. F. Giudice and A. Pomarol, "The μ-Problem in Theories with Gauge-Mediated Supersymmetry Breaking," Nucl. Phys. B **478** (1996) 31 [arXiv:hep-ph/9603238].

[22] H. Murayama and A. Pierce, "Not even decoupling can save minimal supersymmetric SU(5)," Phys. Rev. D **65** (2002) 055009 [arXiv:hep-ph/0108104].

[23] K. S. Babu, J. C. Pati and F. Wilczek, "Fermion masses, neutrino oscillations, and proton decay in the light of SuperKamiokande," Nucl. Phys. B **566** (2000) 33 [arXiv:hep-ph/9812538];

G. Altarelli, F. Feruglio and I. Masina, "From minimal to realistic supersymmetric SU(5) grand unification," JHEP **0011** (2000) 040 [arXiv:hep-ph/0007254];

R. Dermisek, A. Mafi and S. Raby, "SUSY GUTs under siege: Proton decay," Phys. Rev. D **63** (2001) 035001 [arXiv:hep-ph/0007213].

[24] L. Calibbi, L. Ferretti, A. Romanino and R. Ziegler, "Gauge coupling unification, the GUT scale, and magic fields," Phys. Lett. B **672** (2009) 152 [arXiv:0812.0342 [hep-ph]].

[25] M. Nardecchia, A. Romanino and R. Ziegler, "Tree Level Gauge Mediation," JHEP **0911** (2009) 112 [arXiv:0909.3058 [hep-ph]].

[26] M. Nardecchia, A. Romanino and R. Ziegler, "General Aspects of Tree Level Gauge Mediation," JHEP **1003** (2010) 024 [arXiv:0912.5482 [hep-ph]].

[27] I. Antoniadis, J. R. Ellis, S. Kelley and D. V. Nanopoulos, "The Price of deriving the Standard Model from string," Phys. Lett. B **272** (1991) 31;

K. R. Dienes and A. E. Faraggi, "Making ends meet: String unification and low-energy data," Phys. Rev. Lett. **75**, 2646 (1995) [arXiv:hep-th/9505018];

K. R. Dienes, "String Theory and the Path to Unification: A Review of Recent Developments," Phys. Rept. **287**, 447 (1997) [arXiv:hep-th/9602045];

B. C. Allanach and S. F. King, "String Gauge Unification and Infra-red Fixed Points in MSSM+X Models," Nucl. Phys. B **473**, 3 (1996) [arXiv:hep-ph/9601391];

J. Giedt, "Optical unification," Mod. Phys. Lett. A **18**, 1625 (2003) [arXiv:hep-ph/0205224].

[28] S. P. Martin and P. Ramond, "Raising The Unification Scale In Supersymmetry," Phys. Rev. D **51** (1995) 6515 [arXiv:hep-ph/9501244].

[29] L. E. Ibanez, "Low-Energy Traces Of Planck Mass Susy Unification," Phys. Lett. B **126** (1983) 196 [Erratum-ibid. B **130** (1983) 463];

J. E. Bjorkman and D. R. T. Jones, “The Unification Mass, Sin**2-Theta-W And M (B) / M (Tau) In Nonminimal Supersymmetric SU(5),” Nucl. Phys. B **259**, 533 (1985);

T. G. Rizzo, “Desert guts and new light degrees of freedom,” Phys. Rev. D **45**, 3903 (1992);

G. F. Giudice and A. Romanino, “Split supersymmetry,” Nucl. Phys. B **699** (2004) 65 [Erratum-ibid. B **706** (2005) 65] [arXiv:hep-ph/0406088].

[30] B. Dutta, Y. Mimura and R. N. Mohapatra, “Proton Decay and Flavor Violating Thresholds in SO(10) Models,” Phys. Rev. Lett. **100** (2008) 181801 [arXiv:0712.1206 [hep-ph]].

[31] M. Bando, J. Sato and T. Takahashi, “Possible candidates for SUSY SO(10) model with an intermediate scale,” Phys. Rev. D **52** (1995) 3076 [arXiv:hep-ph/9411201].

[32] L. Calibbi, L. Ferretti, A. Romanino and R. Ziegler, “Consequences of a unified, anarchical model of fermion masses and JHEP **0903** (2009) 031 [arXiv:0812.0087 [hep-ph]].

[33] G. Altarelli and F. Feruglio, “SU(5) grand unification in extra dimensions and proton decay,” Phys. Lett. B **511** (2001) 257 [arXiv:hep-ph/0102301];

L. J. Hall and Y. Nomura, “Gauge unification in higher dimensions,” Phys. Rev. D **64**, 055003 (2001) [arXiv:hep-ph/0103125];

A. Hebecker and J. March-Russell, “A minimal S(1)/(Z(2) x Z'(2)) orbifold GUT,” Nucl. Phys. B **613** (2001) 3 [arXiv:hep-ph/0106166].

[34] R. Dermisek and A. Mafi, “SO(10) grand unification in five dimensions: Proton decay and the mu problem,” Phys. Rev. D **65** (2002) 055002 [arXiv:hep-ph/0108139];

H. D. Kim and S. Raby, “Unification in 5D SO(10),” JHEP **0301** (2003) 056 [arXiv:hep-ph/0212348];

M. L. Alciati, F. Feruglio, Y. Lin and A. Varagnolo, “Fermion masses and proton decay in a minimal five-dimensional SO(10) model,” JHEP **0611** (2006) 039 [arXiv:hep-ph/0603086].

[35] S. P. Martin, “Generalized messengers of supersymmetry breaking and the sparticle mass spectrum,” Phys. Rev. D **55** (1997) 3177 [arXiv:hep-ph/9608224].

[36] A. G. Cohen, T. S. Roy and M. Schmaltz, "Hidden sector renormalization of MSSM scalar masses," JHEP **0702** (2007) 027 [arXiv:hep-ph/0612100].

[37] R. Barbieri, S. Ferrara and D. V. Nanopoulos, "From High-Energy Supersymmetry Breaking To Low-Energy Physics Through Decoupling," Phys. Lett. B **116** (1982) 16.

[38] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, "Aspects of split supersymmetry," Nucl. Phys. B **709** (2005) 3 [arXiv:hep-ph/0409232].

[39] G. R. Dvali and A. Pomarol, "Anomalous U(1) as a mediator of supersymmetry breaking," Phys. Rev. Lett. **77** (1996) 3728 [arXiv:hep-ph/9607383].

[40] S. Weinberg, "The quantum theory of fields. Vol. 3: Supersymmetry," *Cambridge, UK: Univ. Pr. (2000) 419 p*

[41] S. Ferrara, L. Girardello and F. Palumbo, "A General Mass Formula In Broken Supersymmetry," Phys. Rev. D **20** (1979) 403.

[42] N. Arkani-Hamed and S. Dimopoulos, "Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC," JHEP **0506** (2005) 073 [arXiv:hep-th/0405159];

G. F. Giudice and A. Romanino, "Split supersymmetry," Nucl. Phys. B **699** (2004) 65 [Erratum-ibid. B **706** (2005) 65] [arXiv:hep-ph/0406088].

[43] S. Dimopoulos and G. F. Giudice, "Multi-messenger theories of gauge-mediated supersymmetry breaking," Phys. Lett. B **393** (1997) 72 [arXiv:hep-ph/9609344].

[44] G. Hiller, Y. Hochberg and Y. Nir, "Flavor Changing Processes in Supersymmetric Models with Hybrid Gauge- and Gravity-Mediation," JHEP **0903** (2009) 115 [arXiv:0812.0511 [hep-ph]].

[45] A. Djouadi, J. L. Kneur and G. Moultaka, "SuSpect: A Fortran code for the supersymmetric and Higgs particle spectrum in the MSSM," Comput. Phys. Commun. **176** (2007) 426 [arXiv:hep-ph/0211331],

[46] M. Bolz, A. Brandenburg and W. Buchmuller, "Thermal Production of Gravitinos," Nucl. Phys. B **606** (2001) 518 [Erratum-ibid. B **790** (2008) 336] [arXiv:hep-ph/0012052];

V. S. Rychkov and A. Strumia, "Thermal production of gravitinos," Phys. Rev. D **75**, 075011 (2007) [arXiv:hep-ph/0701104].

[47] J. L. Feng, S. Su and F. Takayama, "Supergravity with a gravitino LSP," Phys. Rev. D **70** (2004) 075019 [arXiv:hep-ph/0404231].

[48] W. Buchmuller, L. Covi, K. Hamaguchi, A. Ibarra and T. Yanagida, "Gravitino dark matter in R-parity breaking vacua," JHEP **0703** (2007) 037 [arXiv:hep-ph/0702184].

[49] M. Pospelov, J. Pradler and F. D. Steffen, "Constraints on Supersymmetric Models from Catalytic Primordial Nucleosynthesis of Beryllium," JCAP **0811** (2008) 020 [arXiv:0807.4287 [hep-ph]].

[50] J. Wess and J. Bagger, "Supersymmetry and supergravity," *Princeton, USA: Univ. Pr. (1992) 259 p*

[51] N. Seiberg, T. Volansky and B. Wecht, "Semi-direct Gauge Mediation," JHEP **0811** (2008) 004 [arXiv:0809.4437 [hep-ph]].

[52] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking In Four-Dimensions And Its Phenomenological Implications," Nucl. Phys. B **256** (1985) 557;

A. Pomarol and S. Dimopoulos, "Superfield derivation of the low-energy effective theory of softly broken supersymmetry," Nucl. Phys. B **453** (1995) 83 [arXiv:hep-ph/9505302];

R. Rattazzi, "A Note on the effective soft SUSY breaking Lagrangian below the GUT scale," Phys. Lett. B **375**, 181 (1996) [arXiv:hep-ph/9507315];

N. Arkani-Hamed, M. Dine and S. P. Martin, "Dynamical supersymmetry breaking in models with a Green-Schwarz mechanism," Phys. Lett. B **431**, 329 (1998) [arXiv:hep-ph/9803432].

[53] L. Brizi, M. Gomez-Reino and C. A. Scrucca, "Globally and locally supersymmetric effective theories for light fields," Nucl. Phys. B **820** (2009) 193 [arXiv:0904.0370 [hep-th]].

[54] N. Arkani-Hamed, L. J. Hall, H. Murayama, D. Tucker-Smith and N. Weiner, "Small neutrino masses from supersymmetry breaking," Phys. Rev. D **64** (2001) 115011 [arXiv:hep-ph/0006312];

D. A. Demir, L. L. Everett and P. Langacker, "Dirac Neutrino Masses from Generalized Supersymmetry Breaking," Phys. Rev. Lett. **100**, 091804 (2008) [arXiv:0712.1341 [hep-ph]].

[55] J. Hisano, H. Murayama and T. Goto, "Threshold correction on gaugino masses at grand unification scale," Phys. Rev. D **49**, 1446 (1994).

[56] I. L. Buchbinder, S. Kuzenko and Z. Yarevskaya, "Supersymmetric effective potential: Superfield approach," Nucl. Phys. B **411** (1994) 665;

B. de Wit, M. T. Grisaru and M. Rocek, "Nonholomorphic Corrections to the One-Loop N=2 Super Yang-Mills Action," Phys. Lett. B **374** (1996) 297 [arXiv:hep-th/9601115];

A. Pickering and P. C. West, "The One Loop Effective Super-Potential and Non-Holomorphicity," Phys. Lett. B **383** (1996) 54 [arXiv:hep-th/9604147];

M. T. Grisaru, M. Rocek and R. von Unge, "Effective Kähler Potentials," Phys. Lett. B **383** (1996) 415 [arXiv:hep-th/9605149];

A. Brignole, "One-loop Kaehler potential in non-renormalizable theories," Nucl. Phys. B **579** (2000) 101 [arXiv:hep-th/0001121].

[57] J. Polchinski and L. Susskind, "Breaking Of Supersymmetry At Intermediate-Energy," Phys. Rev. D **26** (1982) 3661.

[58] R. Slansky, "Group Theory For Unified Model Building," Phys. Rept. **79** (1981) 1.

[59] M. Frigerio, P. Hosteins, S. Lavignac and A. Romanino, "A new, direct link between the baryon asymmetry and neutrino masses," Nucl. Phys. B **806** (2009) 84 [arXiv:0804.0801 [hep-ph]].

[60] L. Calibbi, M. Frigerio, S. Lavignac and A. Romanino, "Flavour violation in supersymmetric SO(10) unification with a type II seesaw mechanism," arXiv:0910.0377 [hep-ph].

[61] U. Ellwanger, C. Hugonie and A. M. Teixeira, "The Next-to-Minimal Supersymmetric Standard Model," arXiv:0910.1785 [hep-ph];

M. Maniatis, "The NMSSM reviewed," arXiv:0906.0777 [hep-ph].

[62] P. Langacker, N. Polonsky and J. Wang, "A low-energy solution to the mu-problem in gauge mediation," Phys. Rev. D **60** (1999) 115005 [arXiv:hep-ph/9905252].

[63] A. de Gouvea, A. Friedland and H. Murayama, "Next-to-minimal supersymmetric standard model with the gauge mediation of supersymmetry breaking," Phys. Rev. D **57** (1998) 5676 [arXiv:hep-ph/9711264].

[64] K. A. Intriligator and N. Seiberg, "Lectures on supersymmetric gauge theories and electric-magnetic duality," Nucl. Phys. Proc. Suppl. **45BC** (1996) 1 [arXiv:hep-th/9509066];

K. A. Intriligator and N. Seiberg, "Lectures on Supersymmetry Breaking," Class. Quant. Grav. **24** (2007) S741 [arXiv:hep-ph/0702069].

[65] S. Dimopoulos and F. Wilczek, NSF-ITP-82-07 (unpublished); M. Srednicki, "Supersymmetric Grand Unified Theories And The Early Universe," Nucl. Phys. B **202** (1982) 327.