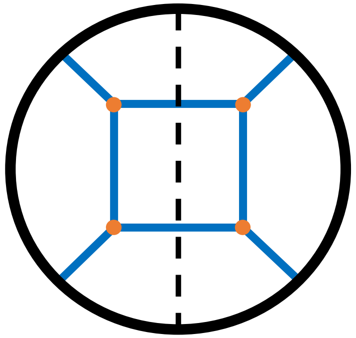
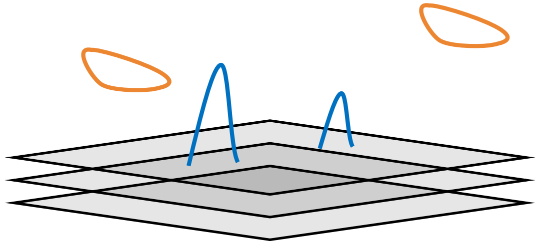
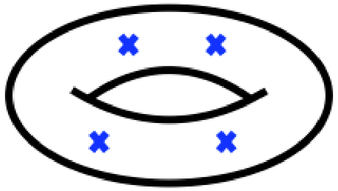


# Scattering Amplitudes and Extra Dimensions in AdS/CFT


Eric Perlmutter

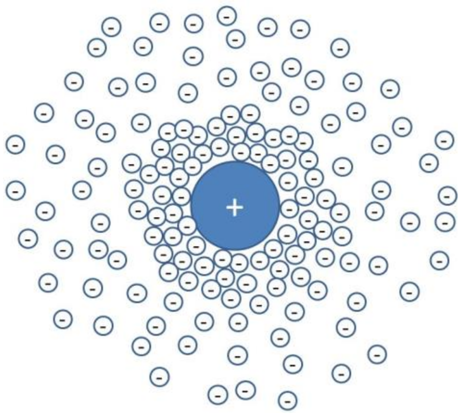
Caltech, Simons Collaboration on Nonperturbative Bootstrap

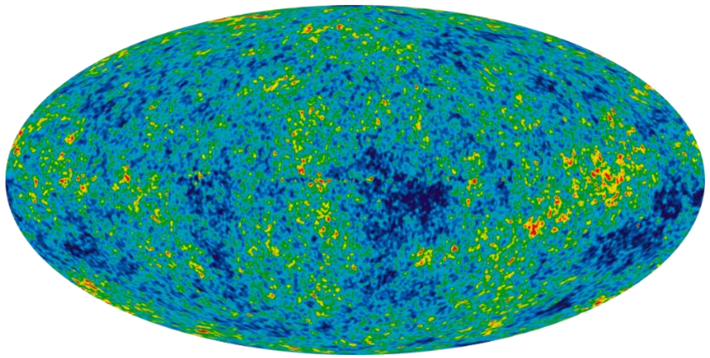



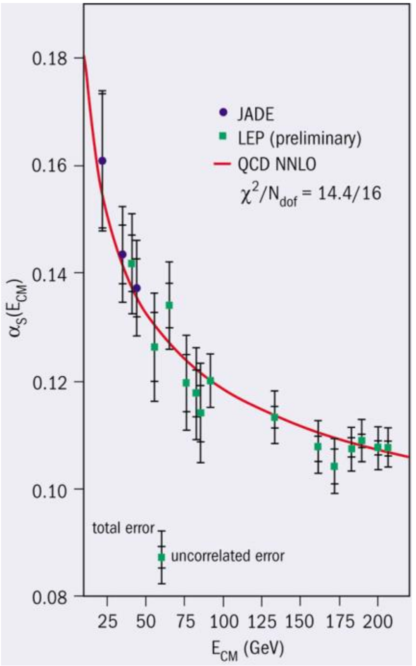


SISSA/ICTP Joint Seminar, 18 September 2019

One of the physical world's most fascinating features is its dependence on scale.

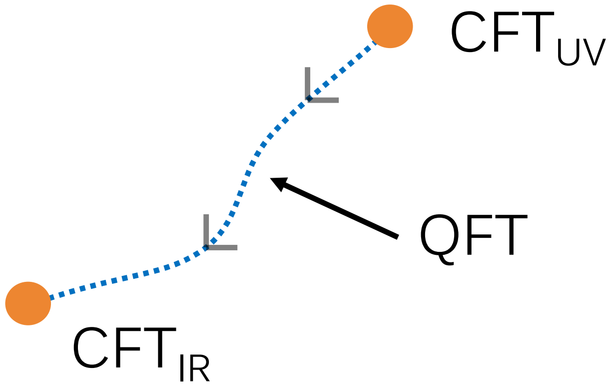







In quantum field theory, this dependence is encoded in the renormalization group.

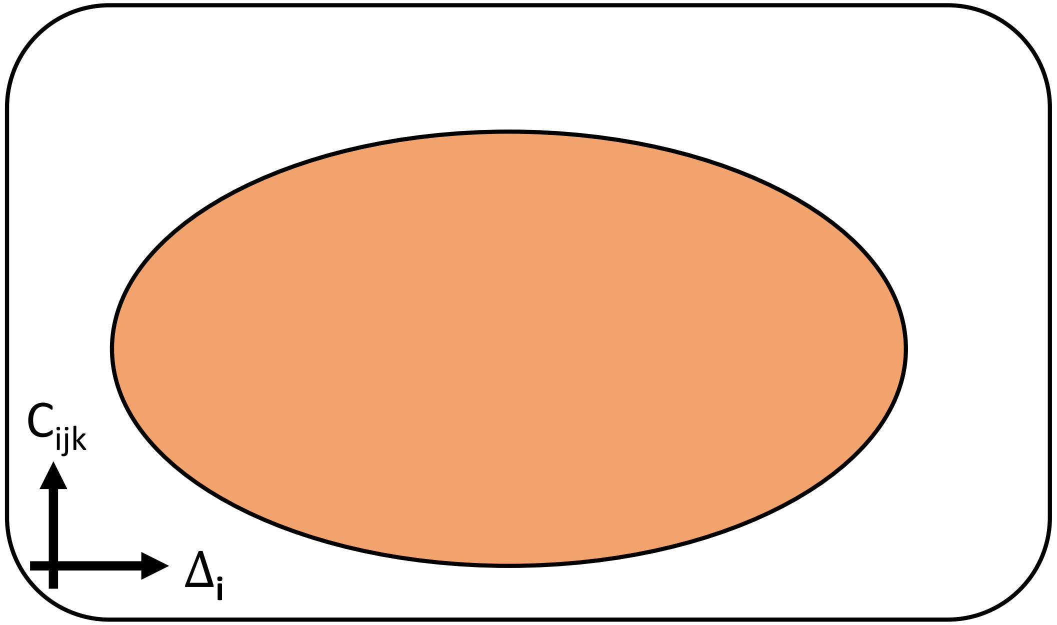
A **conformal field theory** (CFT) is a renormalization group fixed point, and hence essential to the study of quantum field theory.




We are living in a **golden age of CFT**.

There has been a proliferation of new ideas about what, fundamentally, a CFT *is*.

**Conformal bootstrap:** the program of classifying conformal field theories using symmetries and other abstract constraints.



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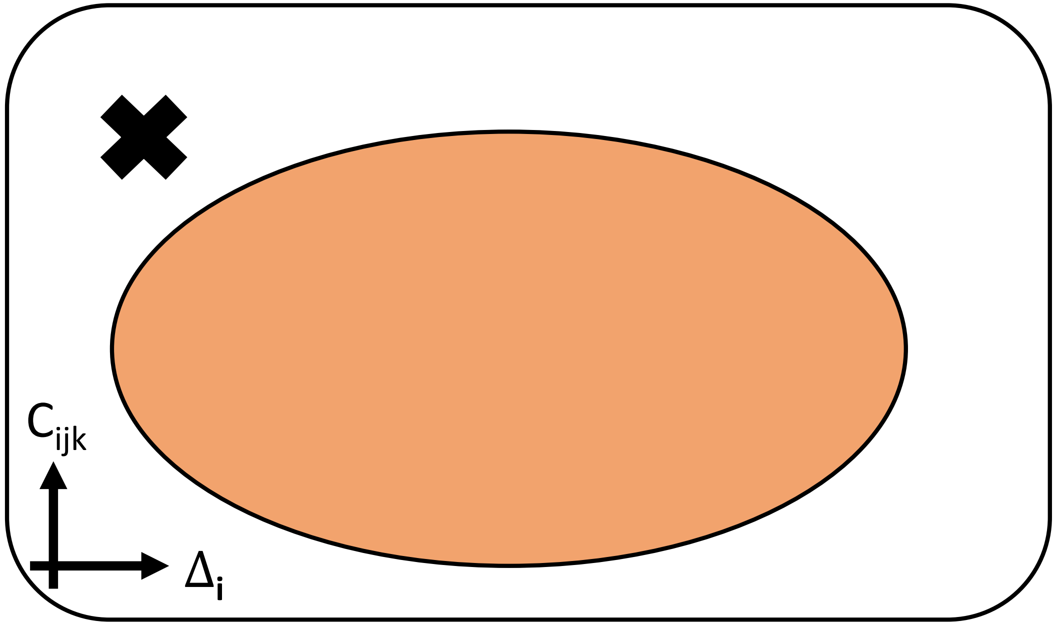

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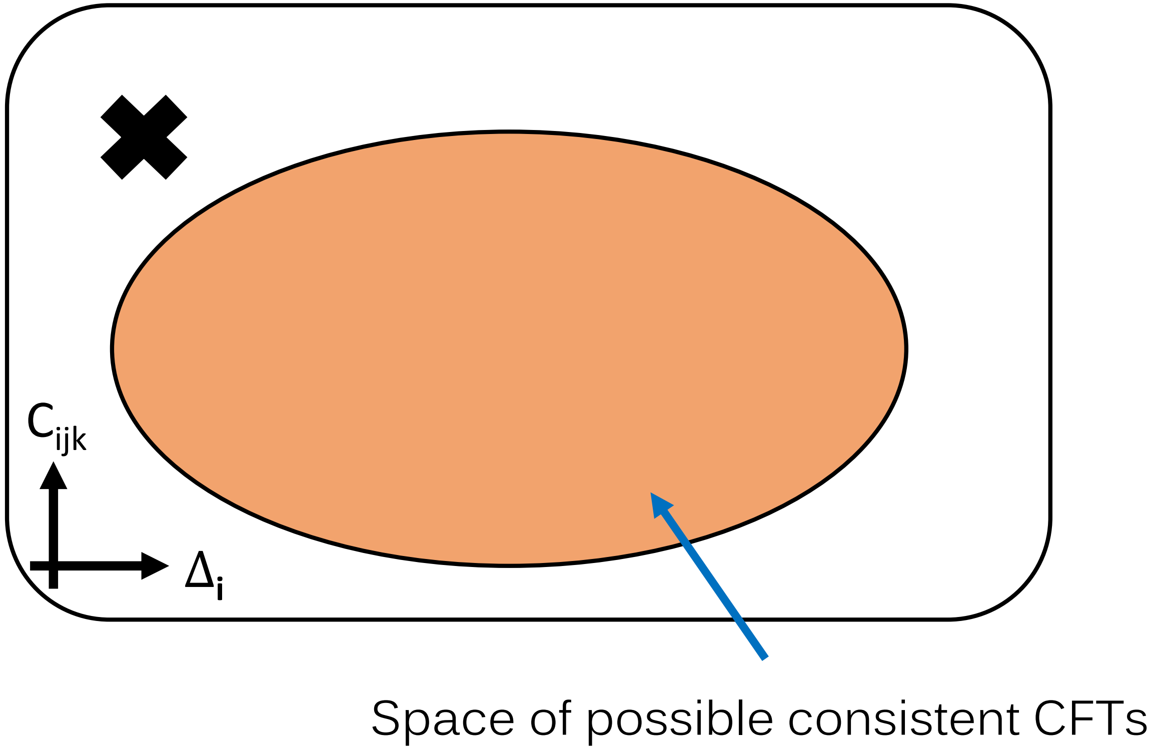

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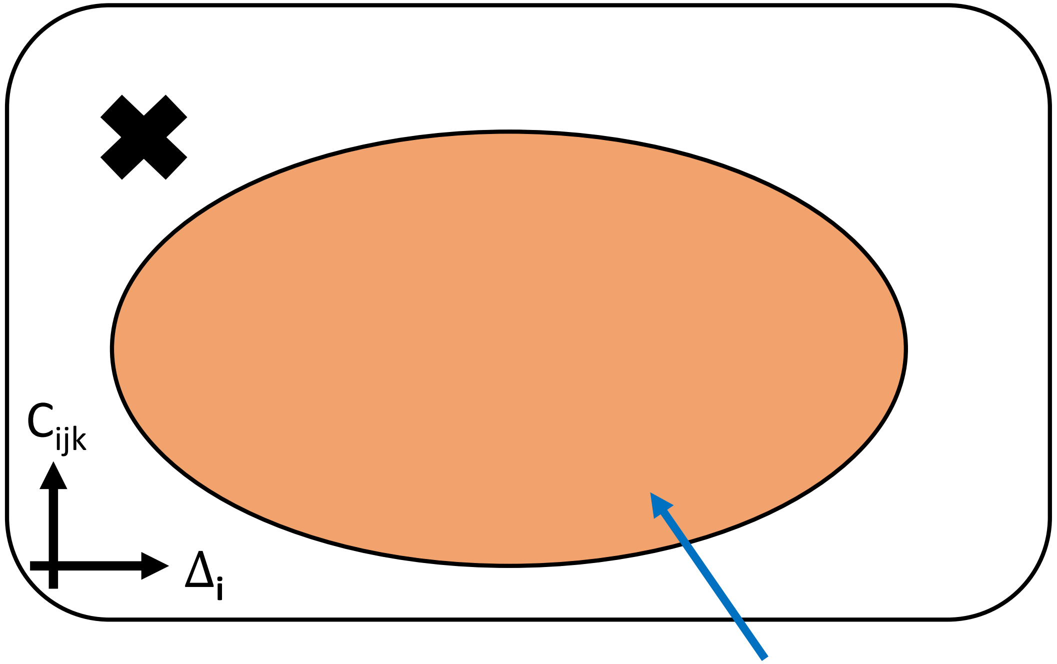
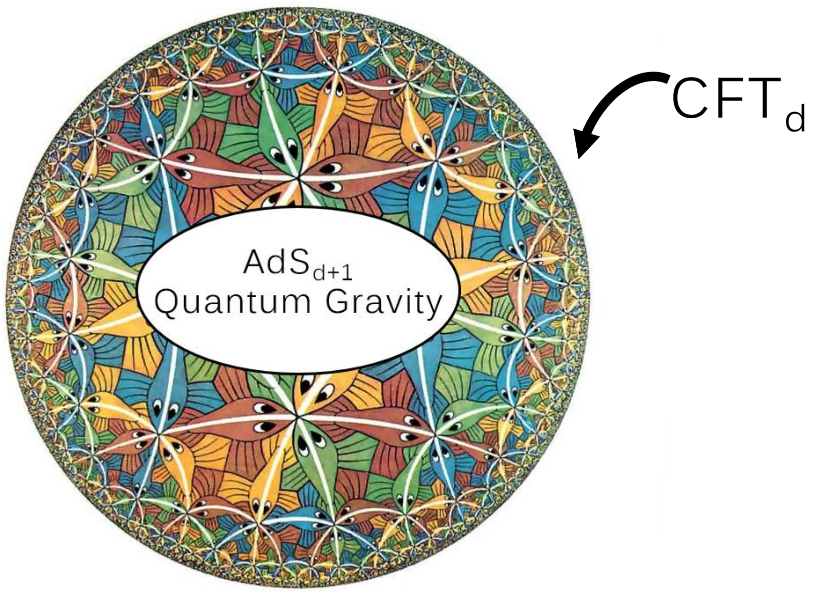
**Conformal bootstrap:** the program of classifying conformal field theories using symmetries and other abstract constraints.

- What is the range of possible quantum critical behaviors?
- What hidden structures govern CFTs?




Space of possible consistent CFTs

The bootstrap paradigm is especially powerful in the context of the **AdS/CFT Correspondence.**



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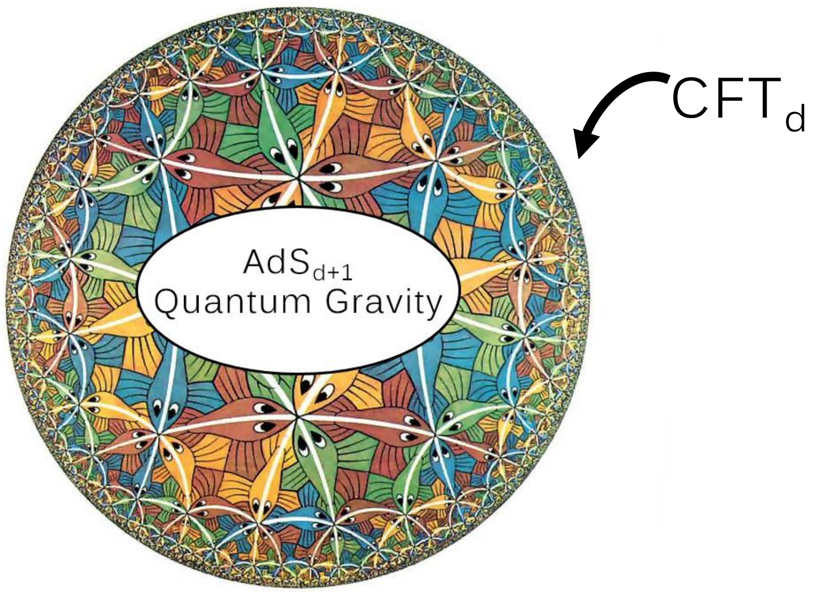


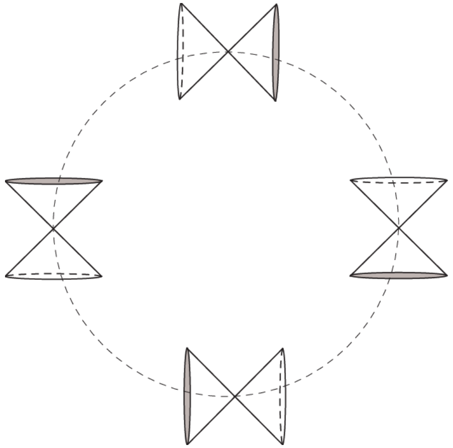

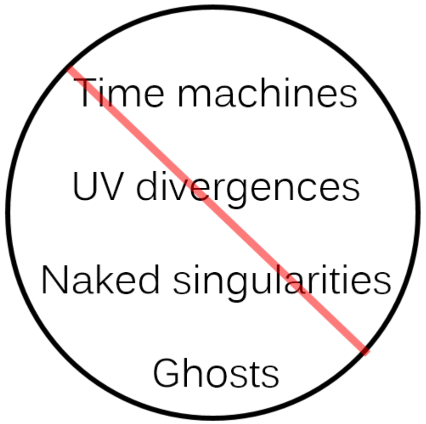


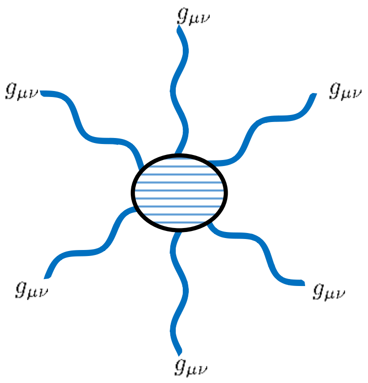


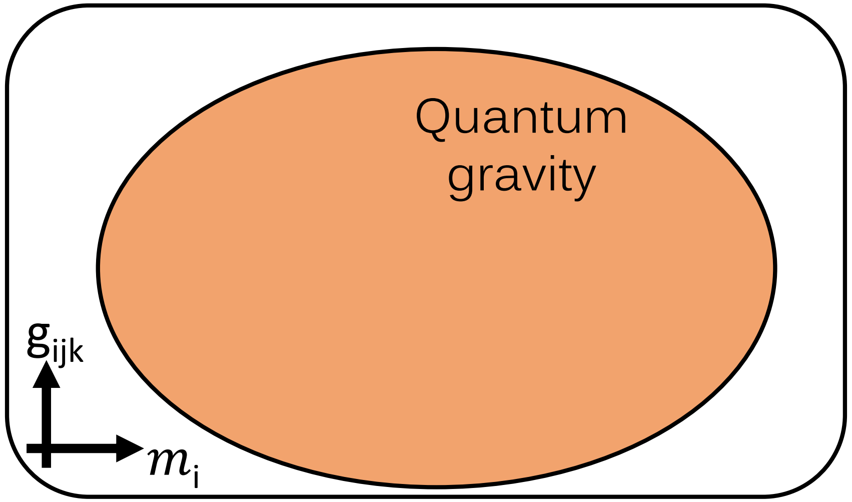

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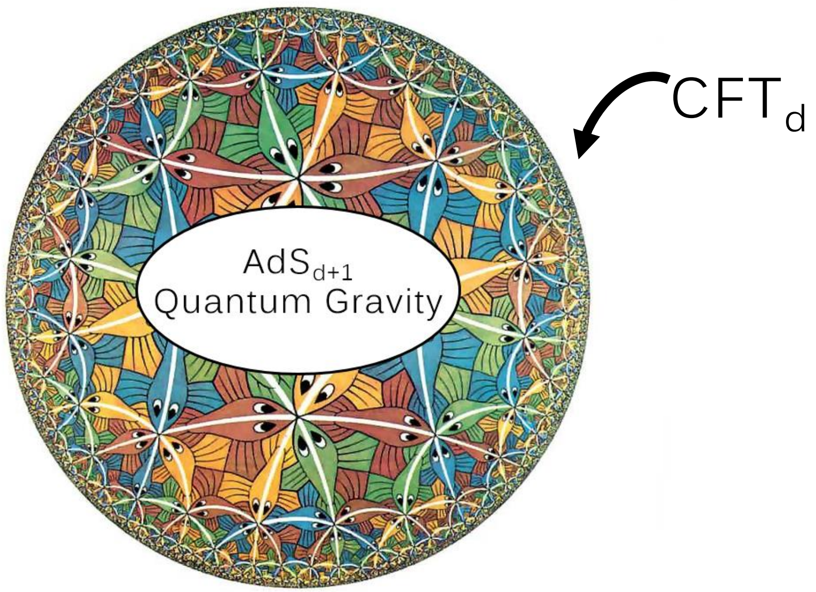


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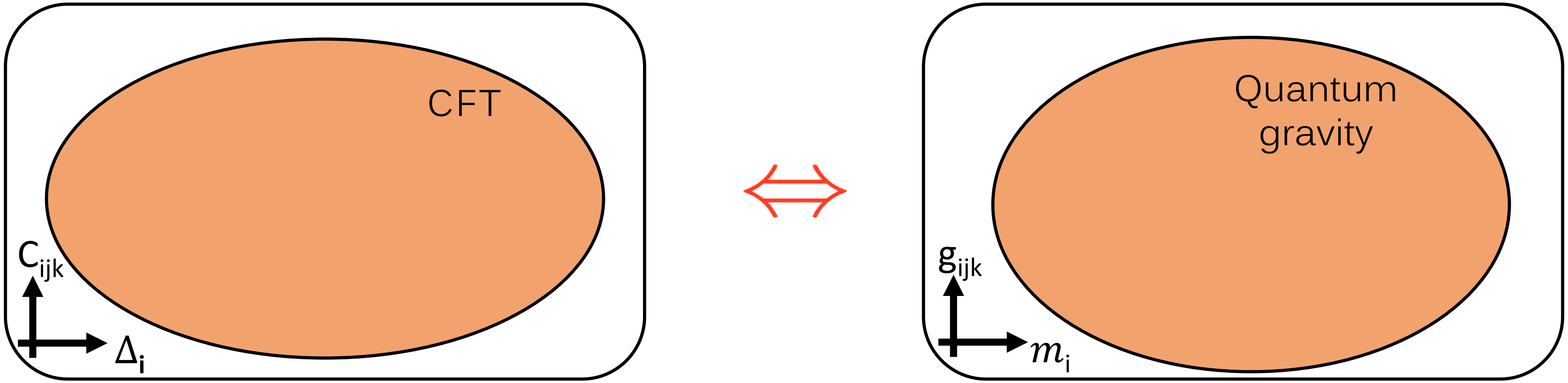

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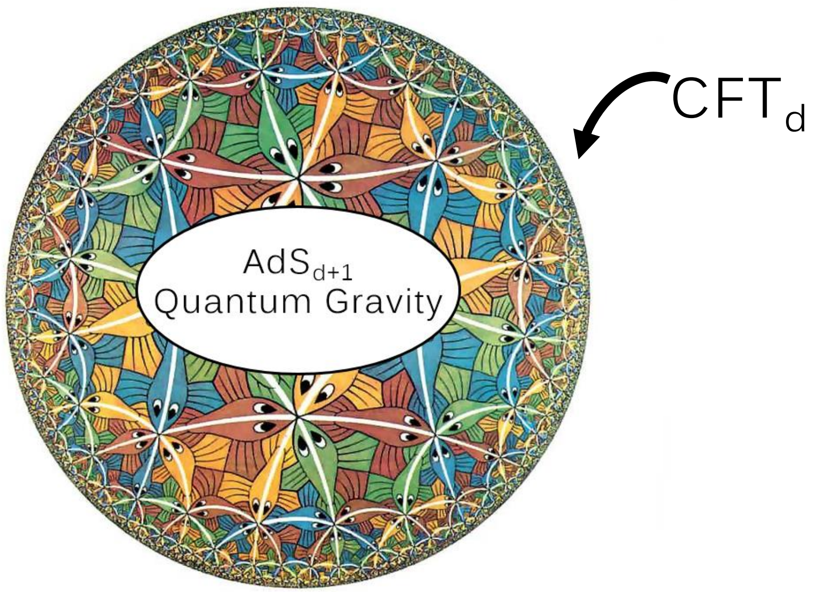


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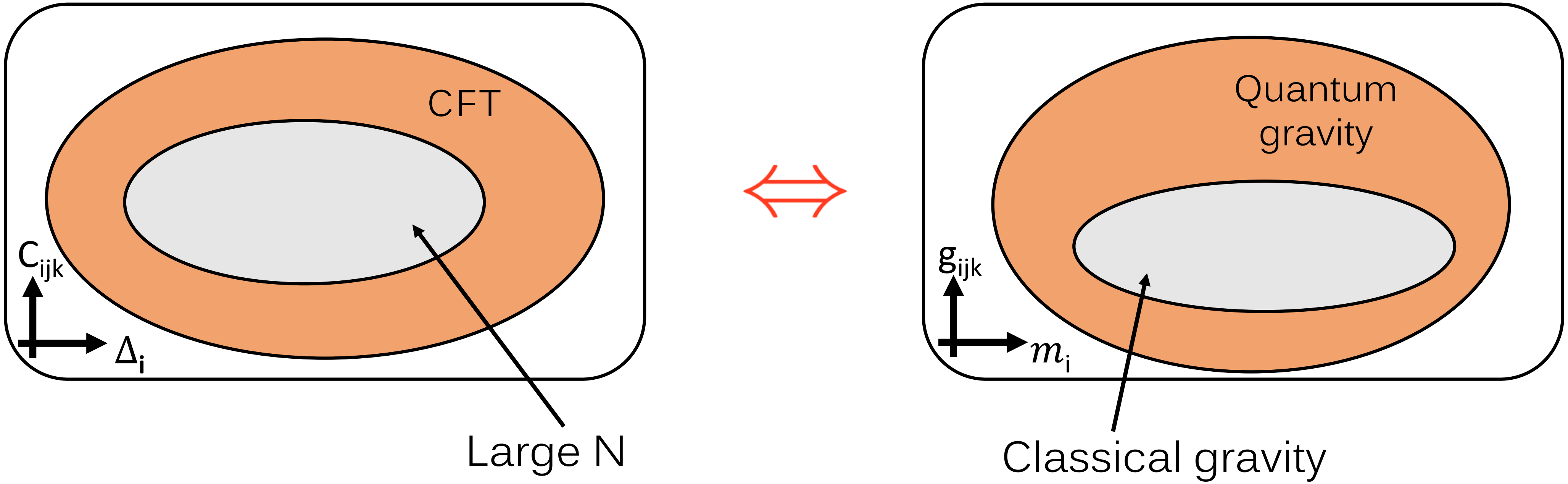

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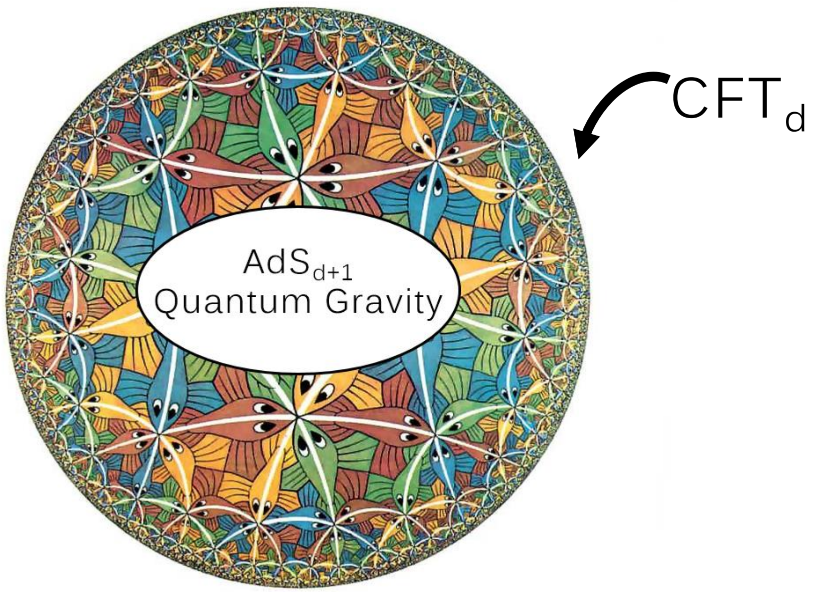


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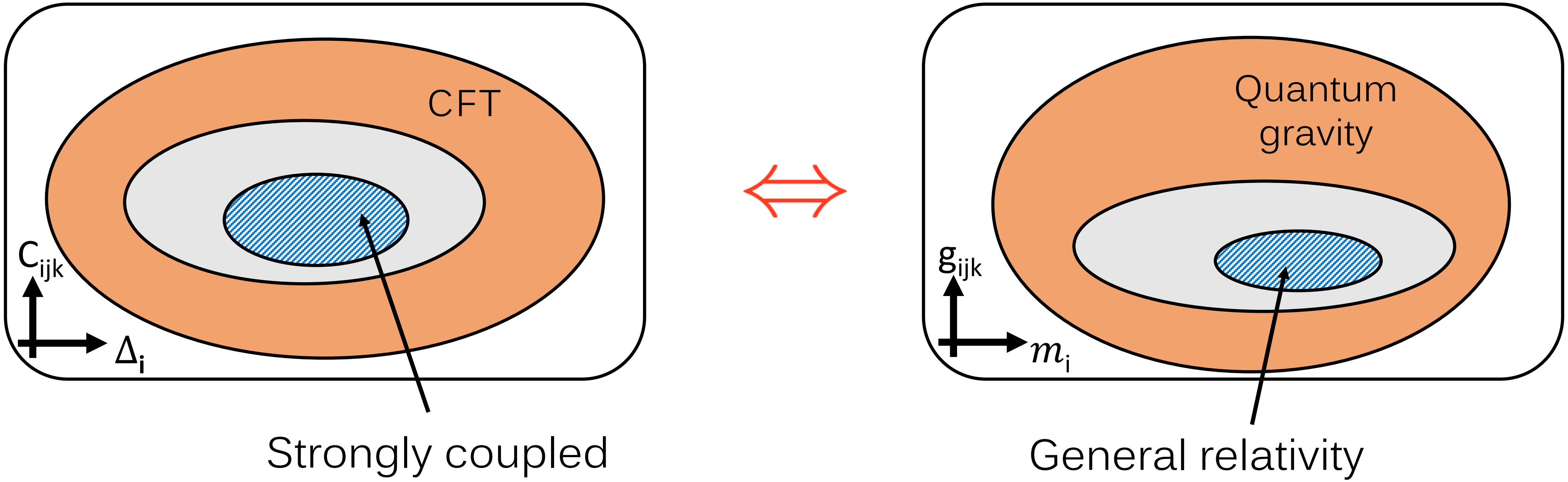

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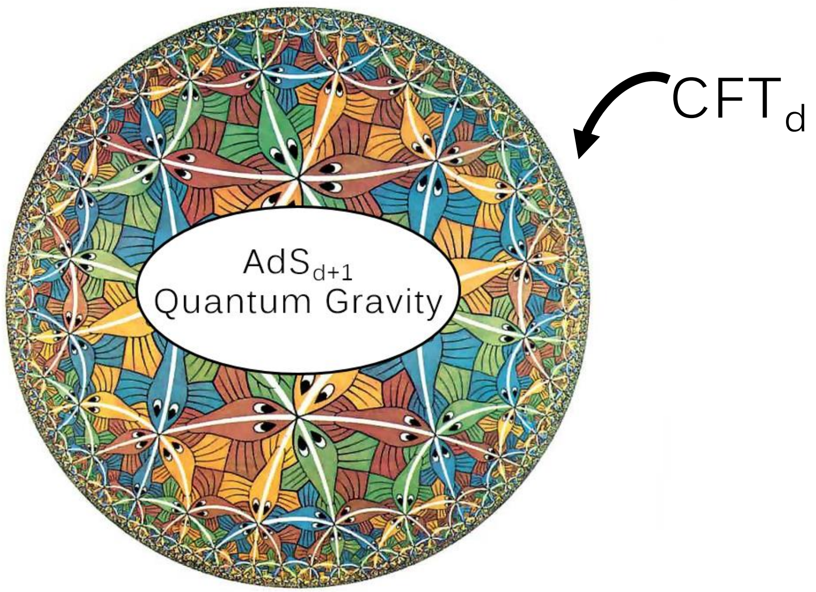


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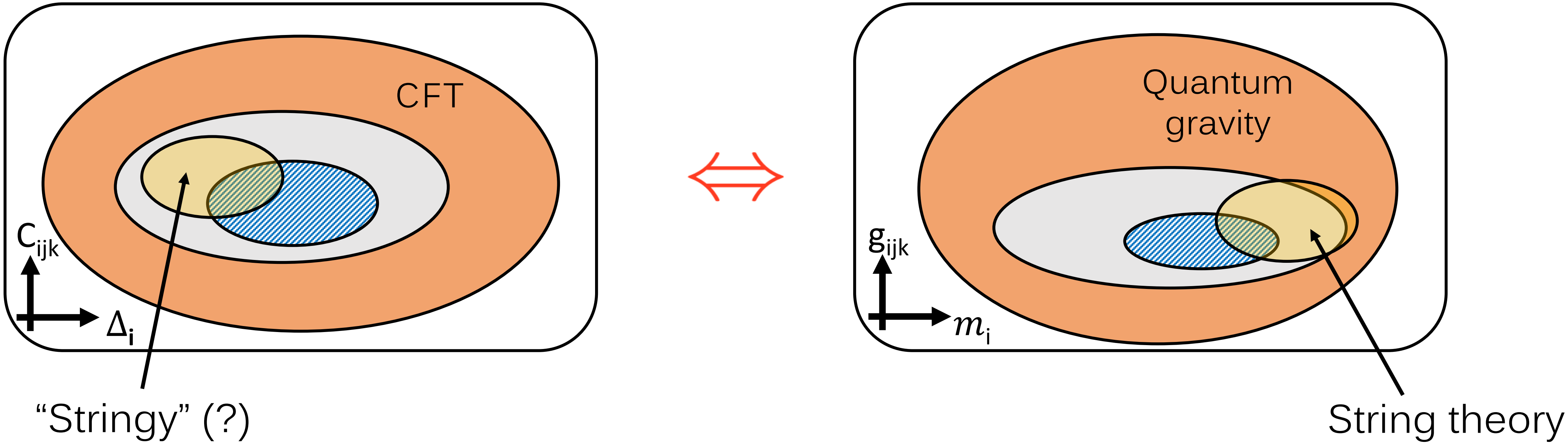


At first, AdS/CFT was mostly used as a tool for determining strongly coupled field theory dynamics from simple, semiclassical calculations in gravity.

$$\text{AdS} \longrightarrow \text{CFT}$$

More recently,

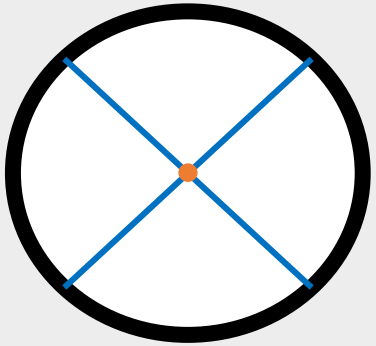
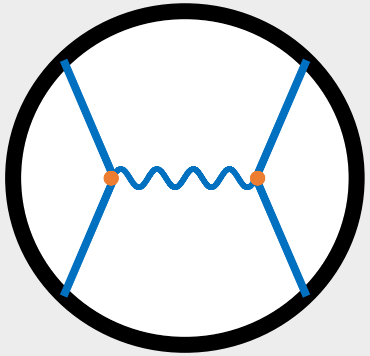
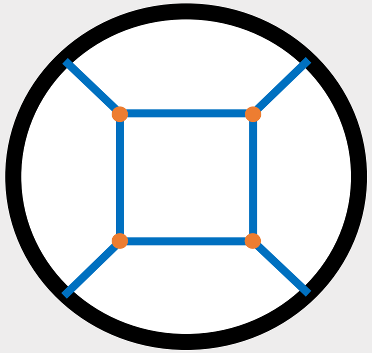
$$\text{AdS} \longleftarrow \text{CFT}$$

We are learning about quantum gravity from insights and precision computations in CFT.

The conformal bootstrap typically constrains CFT correlation functions.

AdS scattering amplitudes $\Longleftrightarrow$ CFT correlation functions

Loop expansion in AdS $\Longleftrightarrow$ 1/N expansion in CFT

$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle =$  $+$  $+$  $+ \ldots$

Planar ($1/N^2$)

Non-planar ($1/N^4$ + ...)

Today's talk will focus on **AdS loop amplitudes**: their computation, using bootstrap-inspired techniques, and their utility in answering questions about string theory.

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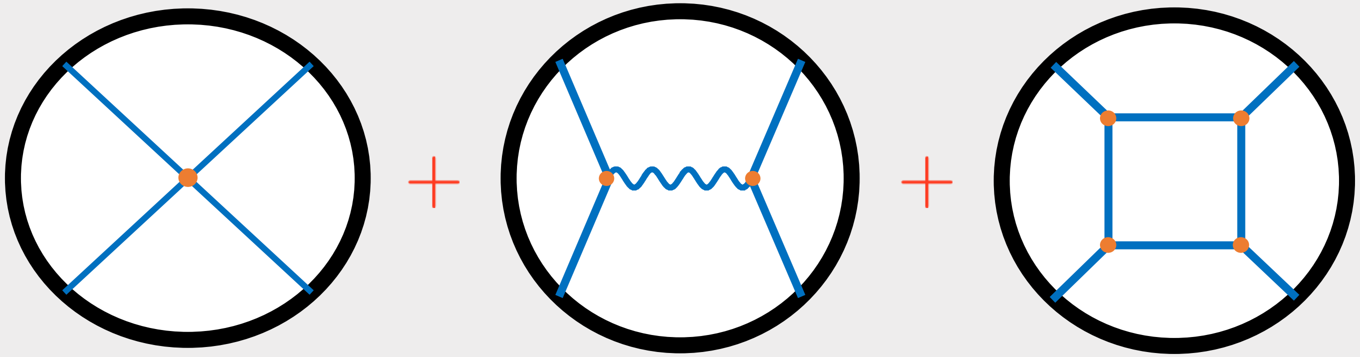

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The talk has 3 components.

# I. Loops in AdS

Why loops?

1. **Curved space** amplitude-ology
2. The only known approach to generic **non-planar CFT** data at strong coupling
3. **Fundamental** objects in AdS quantum gravity

# I. Loops in AdS

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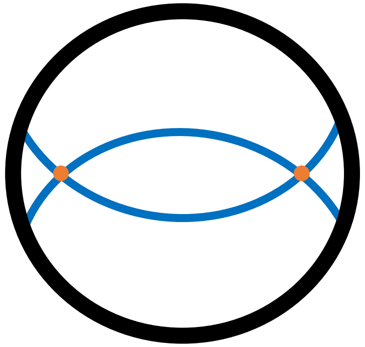
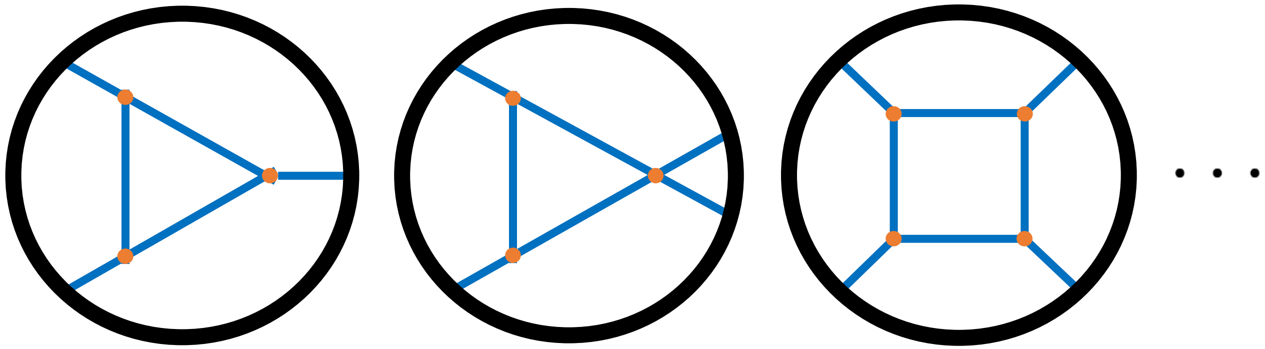
1. **Curved space** amplitude-ology
2. The only known approach to generic **non-planar CFT** data at strong coupling
3. **Fundamental** objects in AdS quantum gravity

Before 2016, what was known?

Yes:



No:

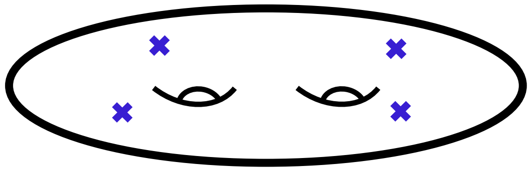


New idea: **AdS Unitarity Method**

# II. Application: String amplitudes from N=4 SYM

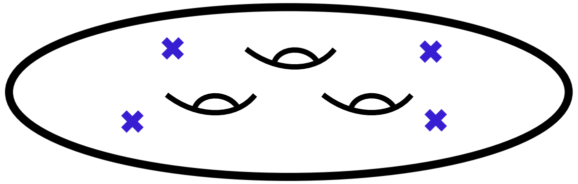
String perturbation theory is stuck in the genus expansion.

State-of-the-art for graviton 4-pt amplitude in Minkowski space:



✓ $\forall\, \alpha'$

[D'Hoker, Phong '05:
"Two-loop superstrings VI: Non-renormalization theorems and the 4-point function"]



$\sim \alpha'^6 f(p_i) + \ldots$

✓ X

[Gomez, Mafra '13]

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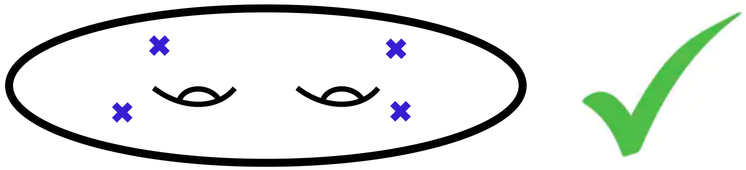

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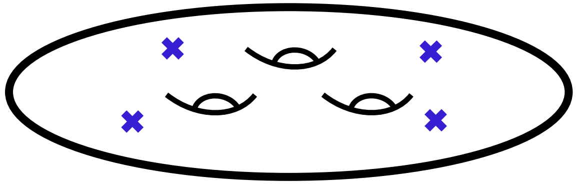

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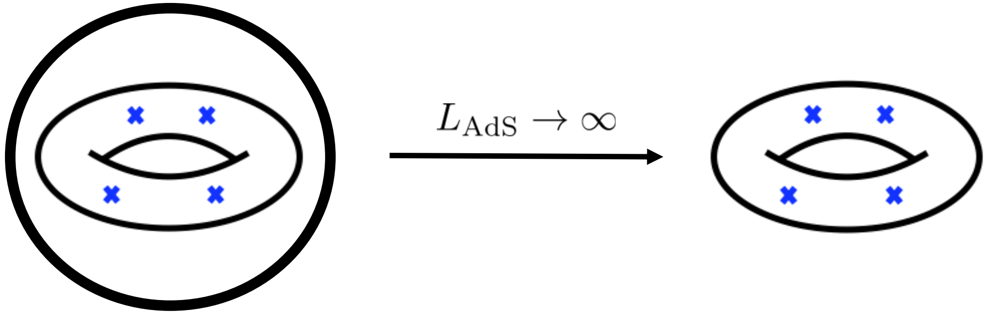
N=4 SYM has a type IIB string dual on $AdS_5$ x $S^5$.

Its non-planar correlators encode bulk string loop amplitudes…

→ Compute **string amplitudes** holographically.



# III. The String Landscape and Extra Dimensions in AdS/CFT

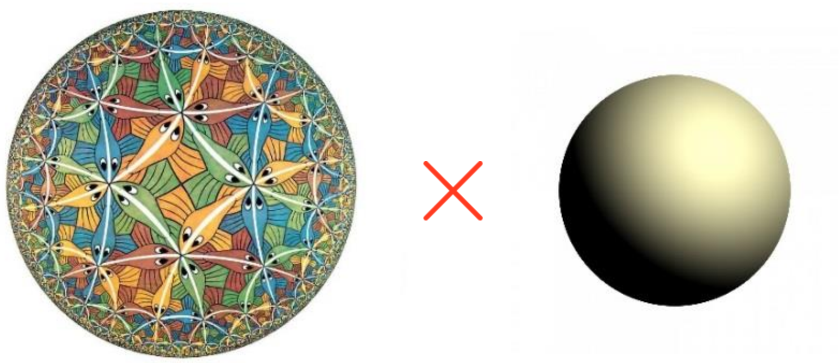
What is the landscape of AdS vacua in string/M-theory?

AdS M

One simpler (but still hard!) question is whether there exist fully rigorous AdS x M vacua with parametrically small extra dimensions (i.e. hierarchy/scale-separation).

Define D as the total number of large (AdS sized) bulk dimensions.
The question is whether D = d+1 is possible. (There are no fully controlled examples.)

# III. The String Landscape and Extra Dimensions in AdS/CFT



Consider the uniqueness question for N=4 SYM. Why $AdS_5$ x $S^5$ instead of "pure" $AdS_5$?

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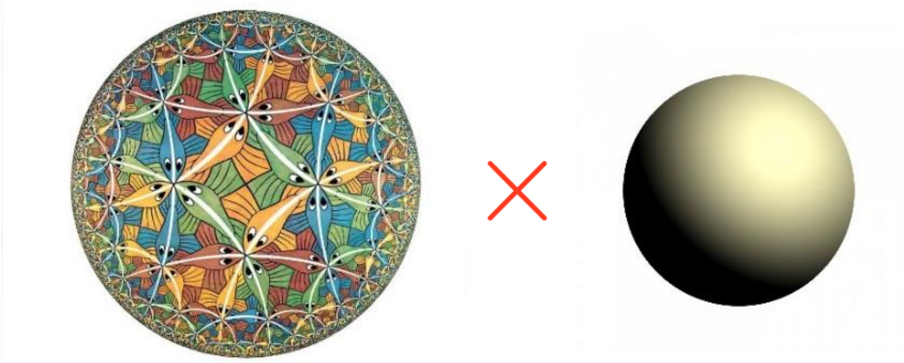

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These are toy models for deeper questions about our own universe:

- Why does our universe appear 3+1-dimensional?
- Could it have been otherwise? What symmetry principles govern this?

# III. The String Landscape and Extra Dimensions in AdS/CFT

Today we will address the following modest question about the AdS landscape:

Take D = number of "large" (= AdS-sized) bulk dimensions.

Given the planar OPE data of a large N, strongly coupled CFT, what is D?

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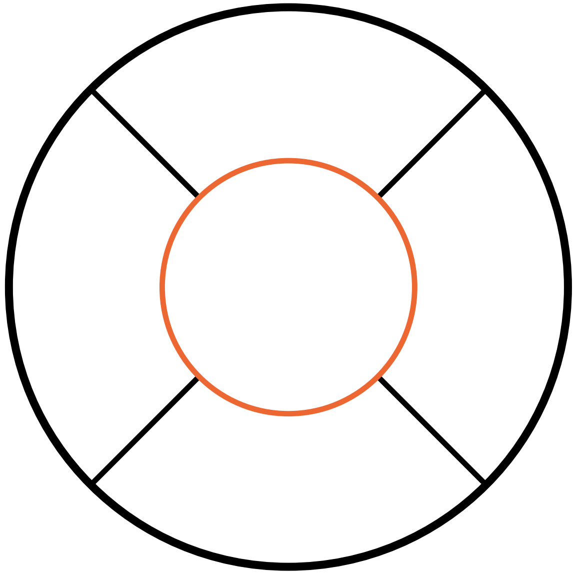


# Outline

1. Bootstrap basics and large N CFT

2. Loops in AdS

3. Application: String amplitudes from N=4 super-Yang-Mills

4. The String Landscape and Extra Dimensions in AdS/CFT

Based on:

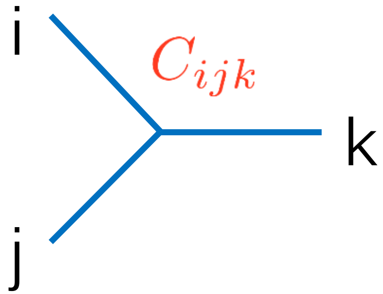
- 1612.03891, with O. Aharony, F. Alday, A. Bissi
- 1808.00612, with J. Liu, V. Rosenhaus, D. Simmons-Duffin
- 1809.10670, with F. Alday, A. Bissi
- 1906.01477, with F. Alday
- To appear, with D. Meltzer, A. Sivaramakrishnan

# What are Conformal Field Theories (made of)?

**I. Local operators**: $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \ldots$


These carry a conformal dimension (Δ), Lorentz spins, and maybe other charges.

**II. Their interactions:** $\mathcal{O}_i(x)\mathcal{O}_j(0) \sim \sum_k C_{ijk}\mathcal{O}_k(0)x^{\Delta_k - \Delta_i - \Delta_j}$




This is the operator product expansion (OPE).

"OPE data" **{$\Delta_i$, $C_{ijk}$}** completely determine local operator dynamics of a CFT.

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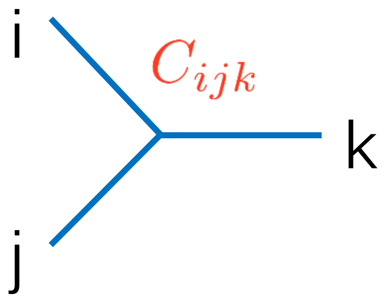


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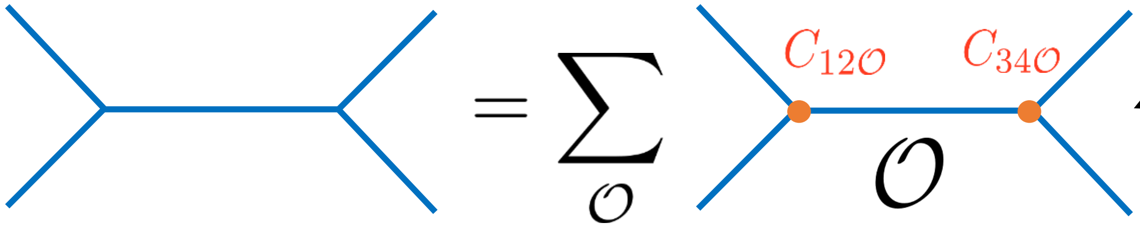
"OPE data" $\{\Delta_i, C_{ijk}\}$ completely determine local operator dynamics of a CFT.

Charting theory space = Constraining the sets $\{\Delta_i, C_{ijk}\}$

Note: No reference to Lagrangians!

# What are Conformal Field Theories (made of)?

We can glue these vertices to make higher-point correlation functions.




Conformal partial wave (CPW)

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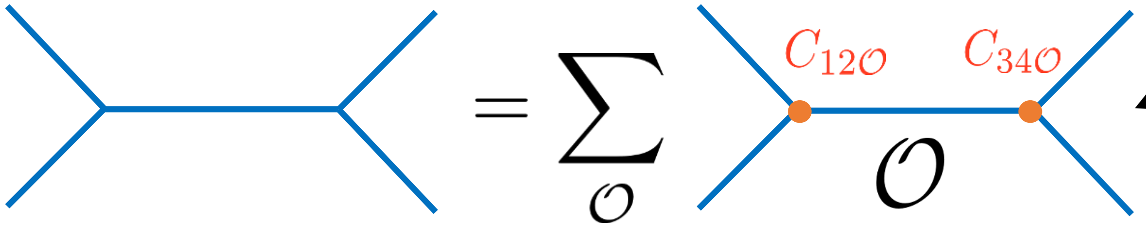


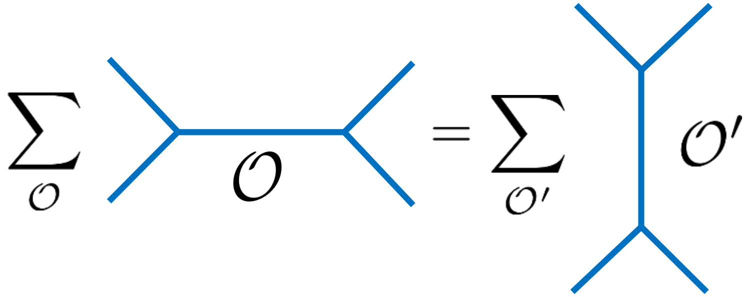

Conformal partial wave (CPW)

These obey dynamical laws which constrain the underlying data **{Δ$_i$, C$_{ijk}$}** .

- **Unitarity**: $\Delta_i \geq \Delta_* \geq 0$ and $C_{ijk}^2 \geq 0$

- **Associativity**: $$\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3 = \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3$$

The latter implies crossing symmetry of four-point functions.



The conformal bootstrap program has three main threads:

1. The **space** of CFTs

2. The **properties** of *all* CFTs

3. The **properties** of *specific* (universality classes of) CFTs

Originally, these investigations were numerical. Now, **analytics** are exploding.

*How* the bootstrap works – i.e. what symmetries and abstract constraints are used – is time-dependent, as we discover new facts about field theory.

# Some classic bootstrap questions:

Is there an upper bound on the dimension of the lightest operator in any CFT? In a given OPE?

Are there bounds on OPE coefficients – for example, central charges or anomaly coefficients?

Assuming certain features, is there a CFT at all? If so, can we determine the precise value of its critical exponents, etc?

How special are the CFTs we already know about?

In a given CFT, what hidden structures relate apparently independent OPE data?

# Bootstrap 2.0: Analytics

Some landmark results:

- Every CFT has an infinite number of primaries.
- Every 2d CFT has a lightest primary below a universal upper bound.
- CFTs with higher spin currents are free.
- Central charges – measures of anomalies and/or degrees of freedom – are bounded.
- Many classes of superconformal theories have soluble subsectors that are completely determined by 2d chiral algebras.

[Komargodski, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Hellerman; Maldacena, Zhiboedov; Hofman, Maldacena; Beem, Rastelli, van Rees; Afkhami-Jeddi, Hartman, Kundu, Jain; Caron-Huot]

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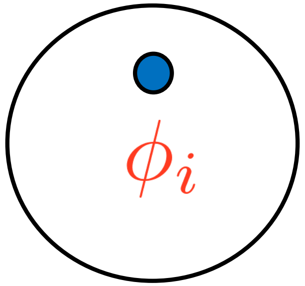
Some of these proven using new approaches, not just crossing symmetry!

- Causality and analyticity
- Regge physics/quantum chaos
- Energy conditions (e.g. ANEC)
- In 2d, modular invariance

[Komargodski, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Hellerman; Maldacena, Zhiboedov; Hofman, Maldacena; Beem, Rastelli, van Rees; Afkhami-Jeddi, Hartman, Kundu, Jain; Caron-Huot]

# Large N Conformal Field Theory

| CFT | AdS |
|---|---|
| "Single-trace" operators $\mathcal{O}_i$ | Elementary fields $\phi_i$ |



# Large N Conformal Field Theory

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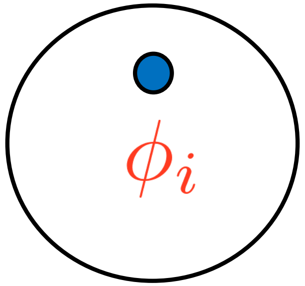
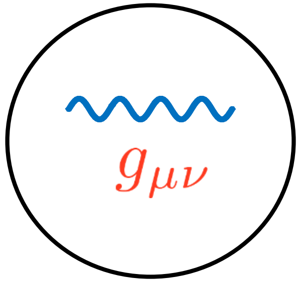
| Stress tensor | $T_{\mu\nu}$ | Graviton | $g_{\mu\nu}$ |



# Large N Conformal Field Theory

| CFT | | AdS | |
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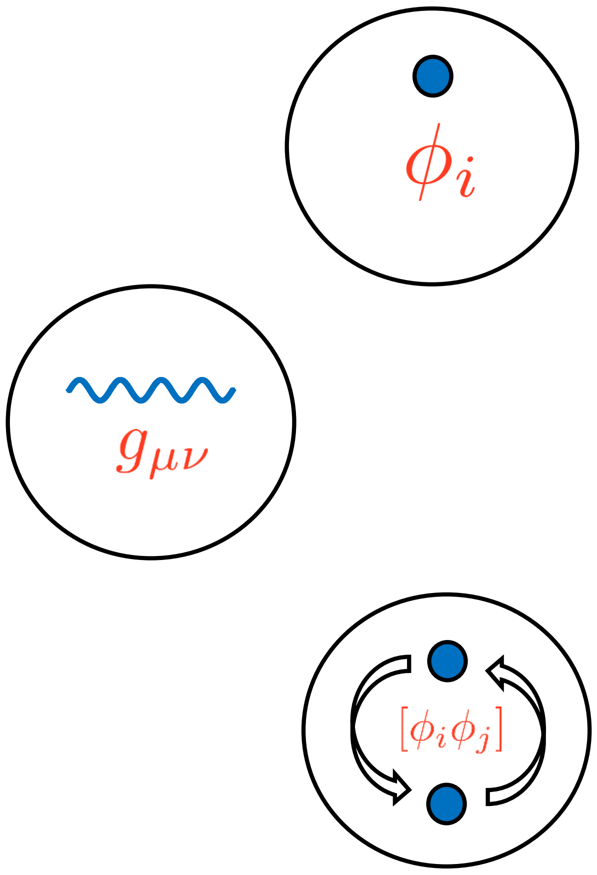

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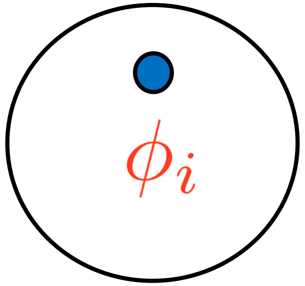


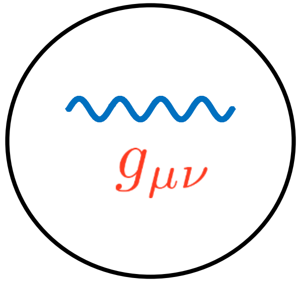


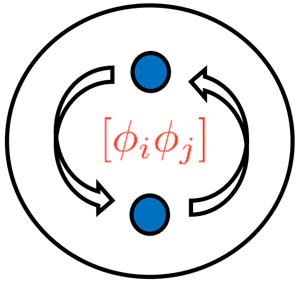

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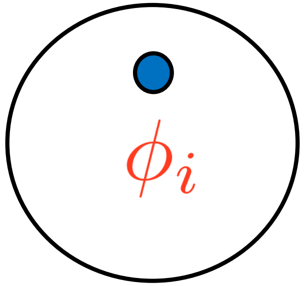


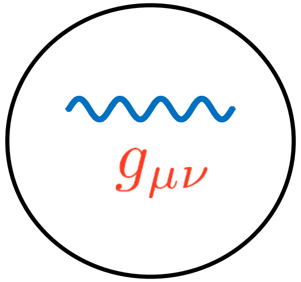


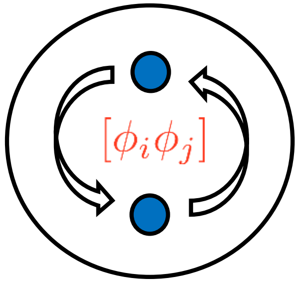

# Large N Conformal Field Theory

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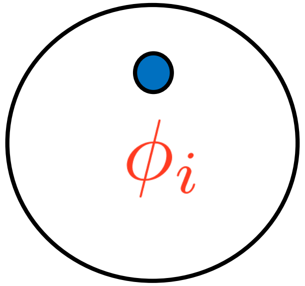
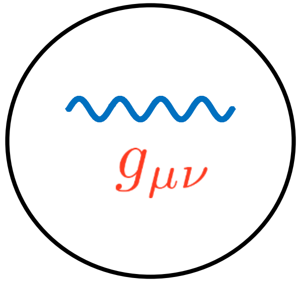
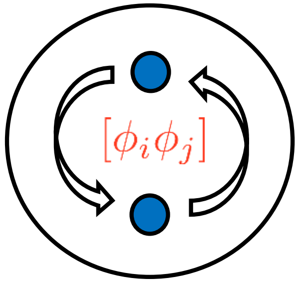
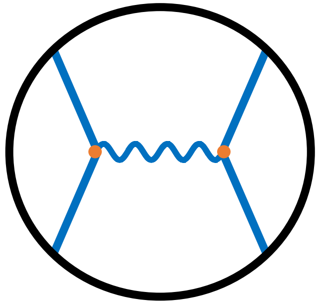


Strongly-coupled quark-gluon plasma

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Area law entanglement

$$S_{EE} = \frac{A_{RT}}{4G_N}$$

# $\text{AdS} \to \text{CFT}$

Huge landscape of non-Lagrangian CFTs

Strongly coupled anomalous dimensions

$$\Delta \sim M_{\text{string}} \sim \lambda^{\# > 0}$$

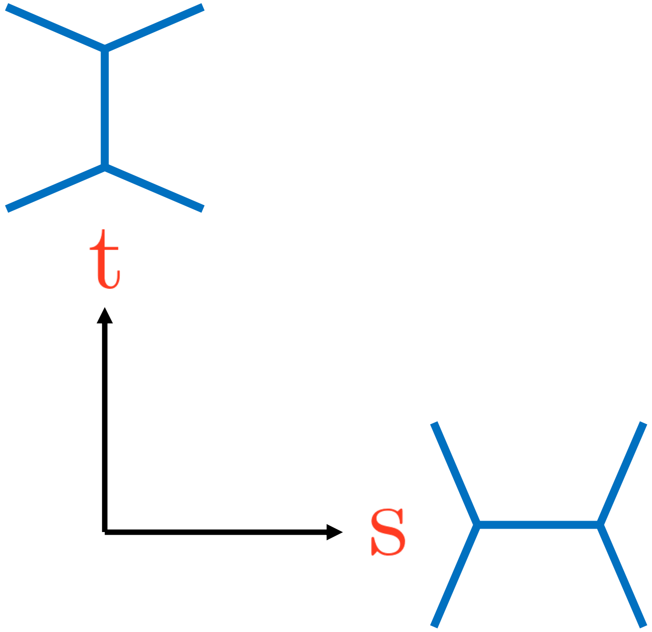
AdS $\longleftarrow$ CFT

# Outline

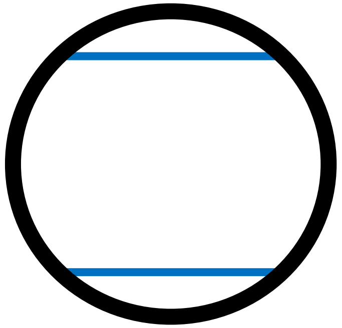
1. Bootstrap basics and large N CFT
2. **Loops in AdS**
3. Application: String amplitudes from N=4 super-Yang-Mills
4. The String Landscape and Extra Dimensions in AdS/CFT

(A quick word on notation:)

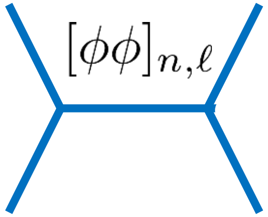
$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_\phi}x_{34}^{2\Delta_\phi}}\mathcal{A}(z,\bar{z})$$



CFT decomposition of bulk amplitude <$\phi\phi\phi\phi$>.



$$= \sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty}$$




Double-trace composites:

$$[\phi\phi]_{n,\ell} \simeq \phi \Box^n \partial_{\mu_1} \dots \partial_{\mu_\ell} \phi$$

$$\Delta_{n,\ell} = 2\Delta_\phi + 2n + \ell + \gamma_{n,\ell}$$

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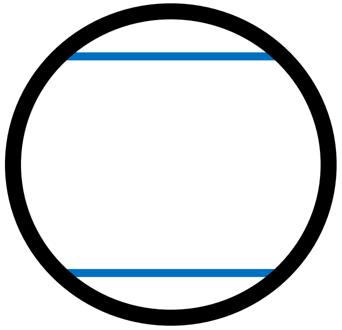

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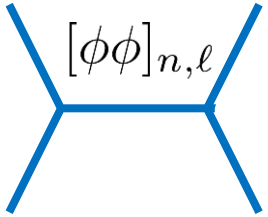


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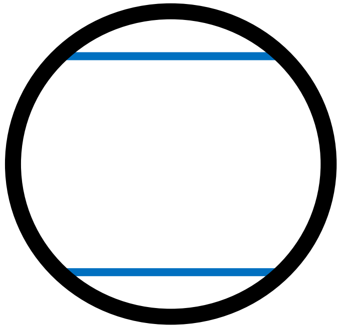
$$\Delta_{n,\ell} = 2\Delta_\phi + 2n + \ell + \cancel{\gamma_{n,\ell}}$$

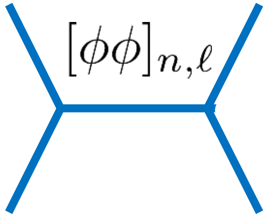
=0 in MFT

CFT decomposition of bulk amplitude <$\phi\phi\phi\phi$>.

Squared OPE coefficients of MFT



$$= \sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}$$




$$= \sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty} a^{(0)}_{n,\ell}\, G^{(s)}_{n,\ell}$$

S-channel conformal blocks

Double-trace composites:

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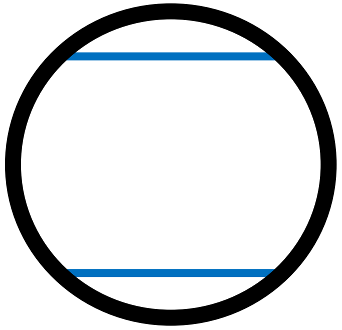

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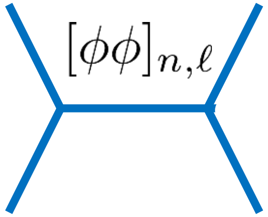


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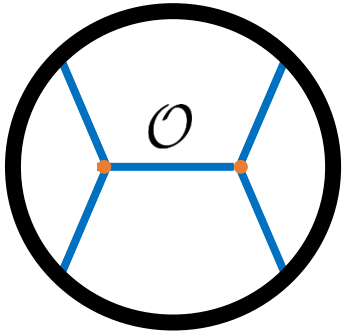
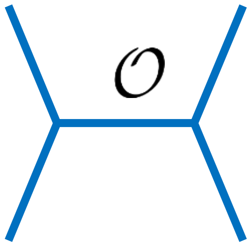
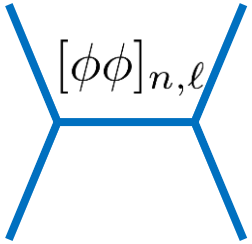



$$=$$




$$+\sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}$$




Single-trace

Double-trace

Double-trace composites:

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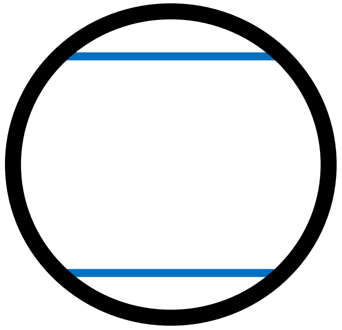
[$\phi\phi$] anomalous dimension:

$$\gamma_{n,\ell} = \frac{\gamma^{(1)}_{n,\ell}}{c} + \frac{\gamma^{(2)}_{n,\ell}}{c^2} + \ldots$$

**Tree-level**
Fixed by single-trace data

CFT decomposition of bulk amplitude <$\phi\phi\phi\phi$>.



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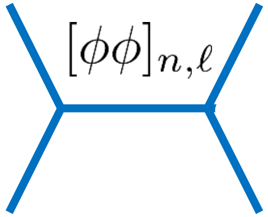


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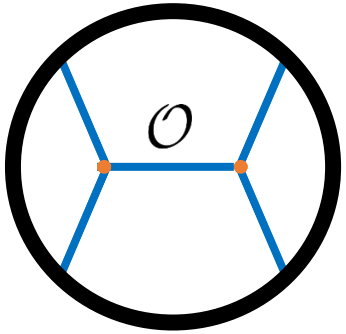
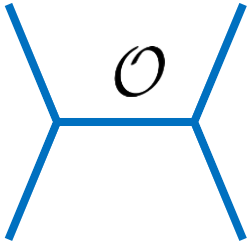



$$=$$

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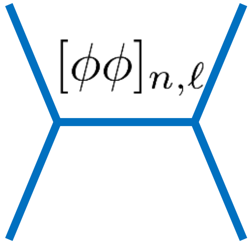


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**Tree-level**
Fixed by single-trace data

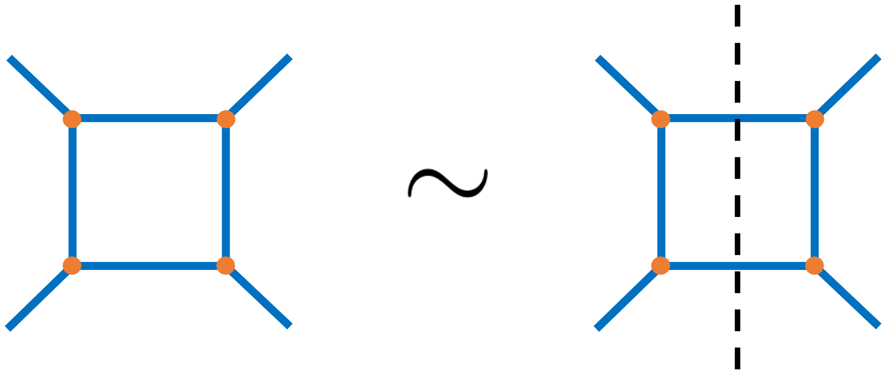
**1-loop**
Fixed by **tree-level** data... how?

In the world of amplitudes, the dominant paradigm is that of “**unitarity methods**”.

Recall the optical theorem for an S-matrix:

$$S = \mathbf{1} + iT \quad \underset{\text{Unitarity of S}}{\Longrightarrow} \quad \mathrm{Disc}(T) = T^\dagger T$$

This buys you one order in perturbation theory. e.g. at 1-loop,

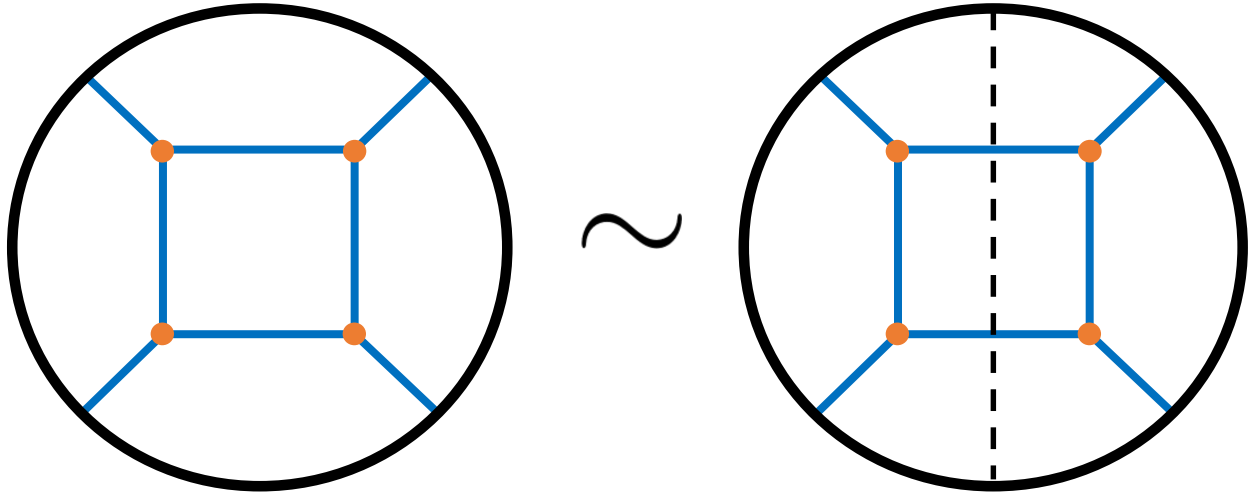


**Unitarity methods** = constructing loop-level amplitudes from low-order ones by cutting.

(Basic idea: A Lagrangian defines the set of tree-level amplitudes, so from these, one must be able to construct the S-matrix to all orders in perturbation theory.)

In AdS, no asymptotic states: instead, dual CFT operators...

"**AdS Unitarity Method**": a prescription for constructing loop-level AdS amplitudes from OPE data of lower-order ones.



Like ordinary unitarity methods, but reconstructed from operations in the CFT.

[Aharony, Alday, Bissi, EP]

# AdS Unitarity Method

Nicely phrased using CFT dispersion relation(Lorentzian inversion). [Caron-Huot]

Schematically:
$$\mathcal{A}(z,\bar{z}) \approx \int K(z,\bar{z};z',\bar{z}')\,\mathrm{dDisc}(\mathcal{A}(z',\bar{z}'))$$
("dDisc constructibility")

where
$$\mathrm{dDisc}_t(\mathcal{A}(z,\bar{z})) \equiv \frac{1}{2}\mathrm{Disc}^{\circlearrowleft}_{\bar{z}=1}(\mathcal{A}(z,\bar{z})) + \frac{1}{2}\mathrm{Disc}^{\circlearrowright}_{\bar{z}=1}(\mathcal{A}(z,\bar{z}))$$

For identical external scalar operators, dDisc acts on conformal blocks G as follows:
$$\mathrm{dDisc}_t(G^{(t)}_{\Delta,\ell}) = 2\sin^2\left(\frac{\pi}{2}\underline{\underline{(\Delta-\ell-2\Delta_\phi)}}\right) G^{(t)}_{\Delta,\ell}$$

→ Annihilates double-trace operators with $\gamma = 0$.

→ In the 1/c expansion,
$$\mathrm{dDisc}_t(\mathcal{A}^{1-\mathrm{loop}}) \supset \frac{\pi^2}{2}\sum_{n,\ell} a^{(0)}_{n,\ell}\underline{\underline{(\gamma^{(1)}_{n,\ell})^2}}G^{(t)}_{n,\ell}$$
[Aharony, Alday, Bissi, EP]

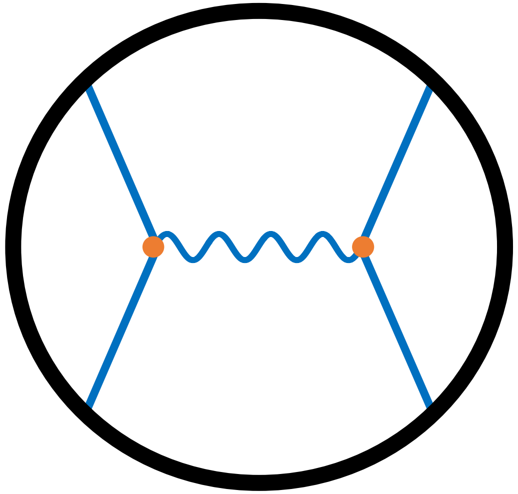
1-loop anomalous dimension does not appear = Fixed by tree-level!

# AdS Unitarity Method

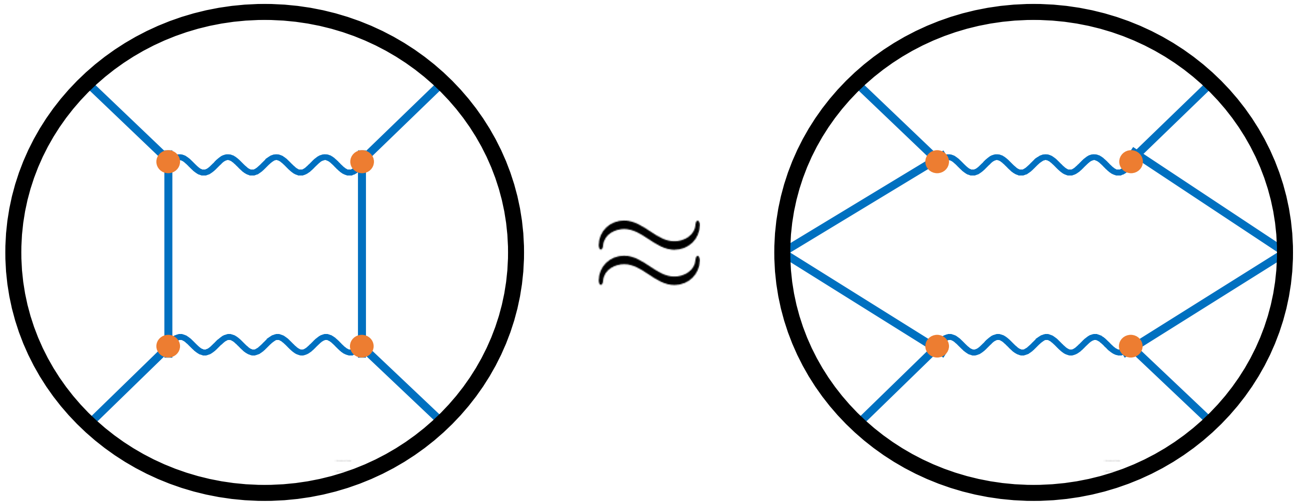
"Glue" CFT data at leading order in 1/N (AdS tree) to compute higher orders (AdS loops).

**Diagrammatic suggestion:**

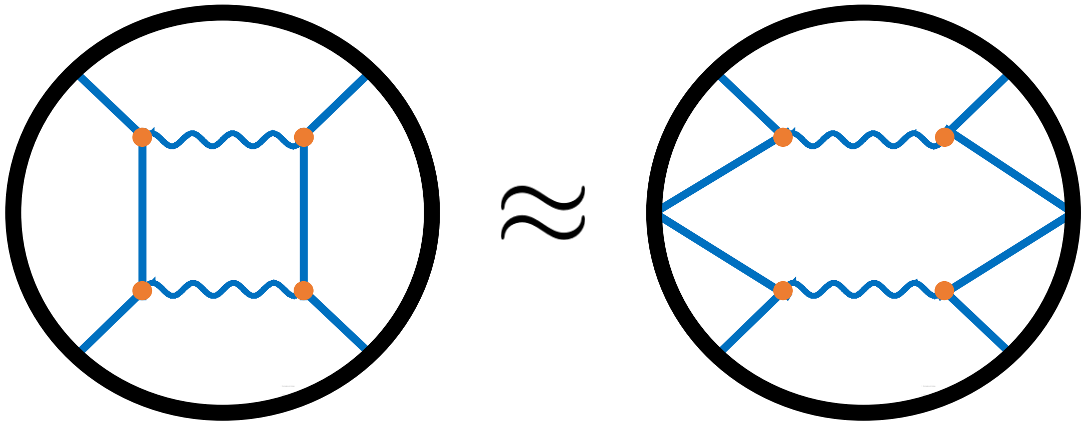
**Tree**



**1-loop**



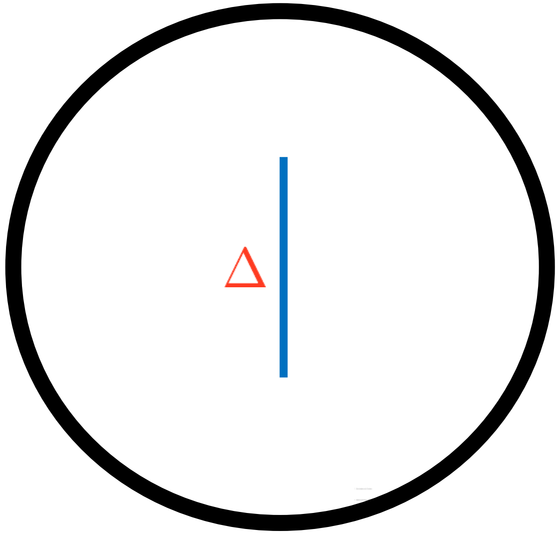
# AdS Unitarity Method



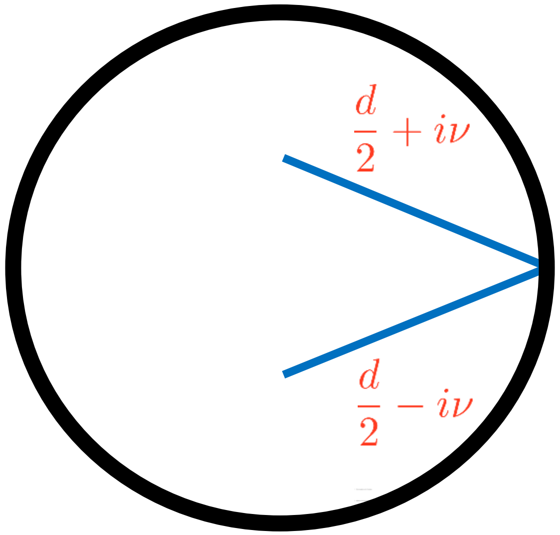
This picture can be made precise.

- We can glue two tree amplitudes together (CFT shadow techniques).
- Because the internal propagator is off-shell, the correct procedure requires an infinite sum of such pairs of glued trees, with specific dimensions.

(Reverse: “split representation” of bulk-bulk propagator)




$$= \int_{-i\infty}^{i\infty} d\nu \frac{\nu^2}{\nu^2 + (\Delta - \frac{d}{2})^2}$$



# AdS Unitarity Method

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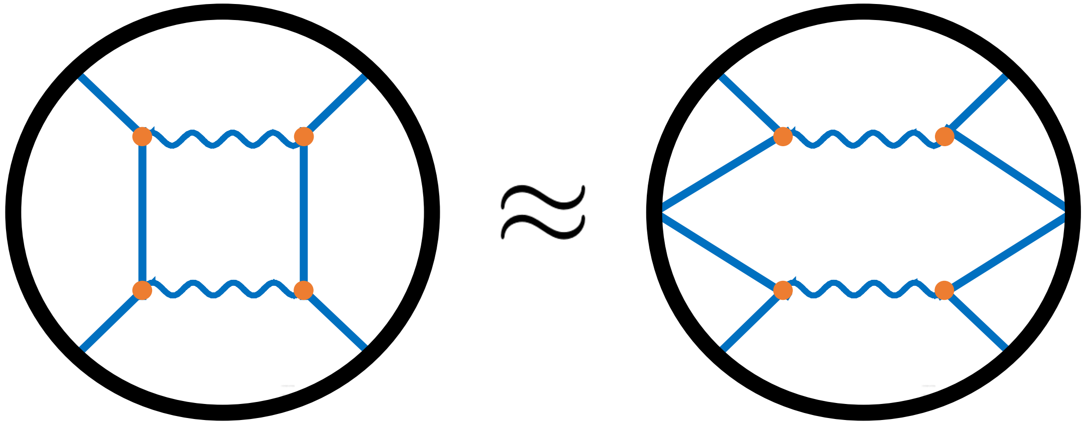
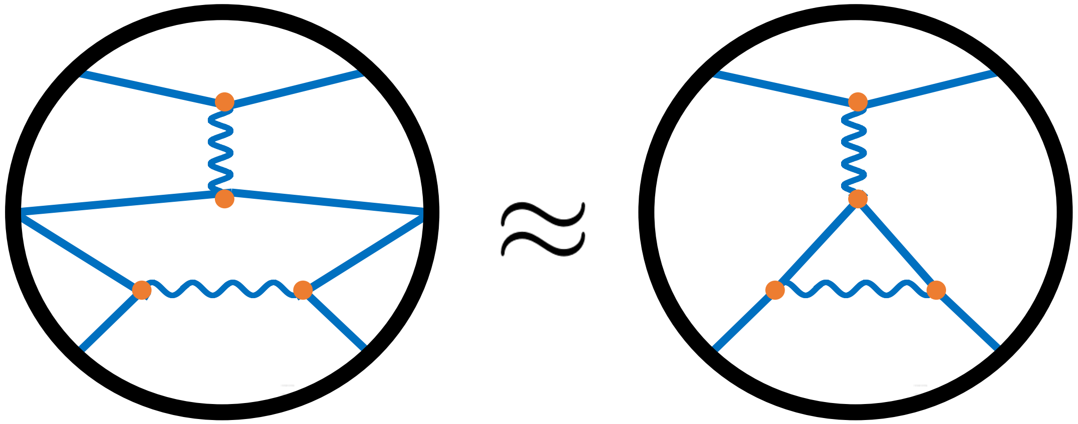


- We can glue two tree amplitudes together (CFT shadow techniques).
- Because the internal propagator is off-shell, the correct procedure requires an infinite sum of such pairs of glued trees, with specific dimensions.

  (Reverse: "split representation" of bulk-bulk propagator)

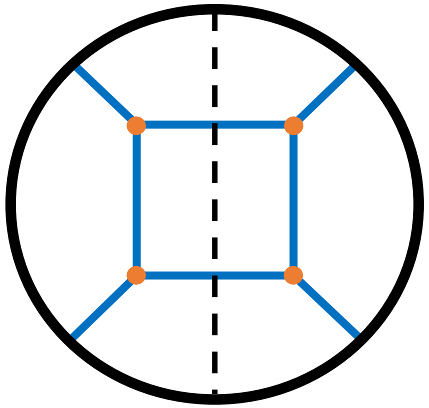
- Choosing to glue s- or t-channel trees gives the different 1-loop diagrams. e.g.



**Q**: What corresponds to a cut?

**A in bulk**: Taking the internal legs on-shell.

**A in CFT**: Isolating part of the conformal block expansion from double-trace operators whose constituents are dual to the internal lines.



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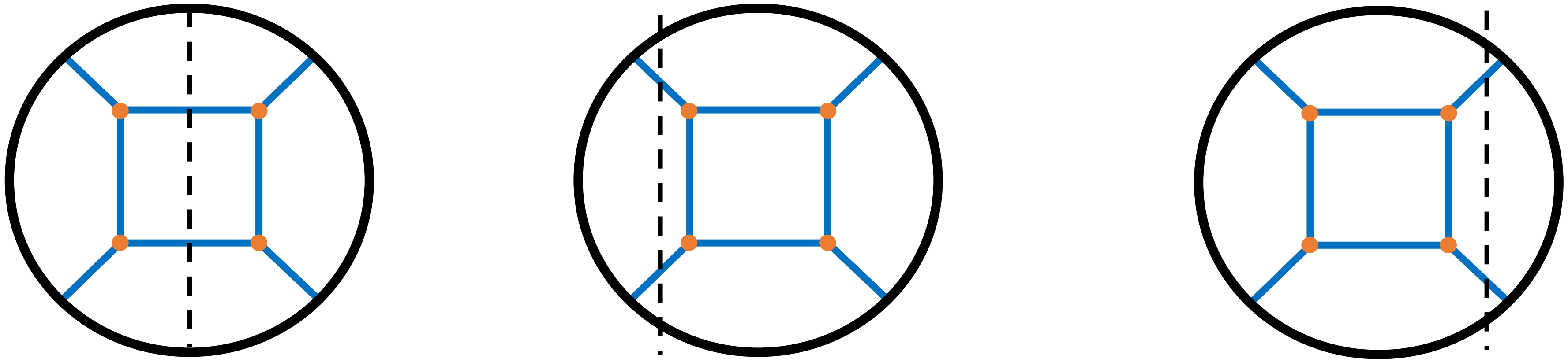

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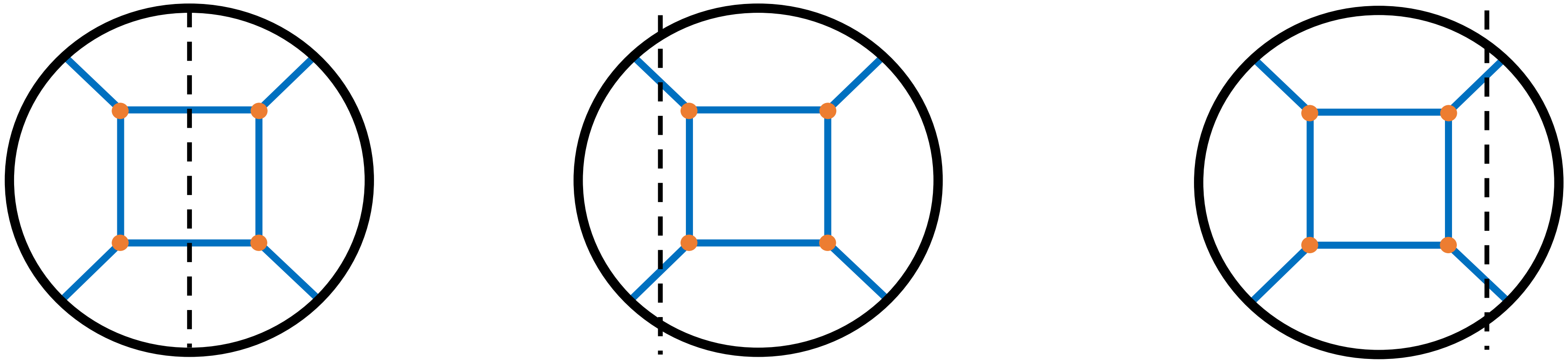


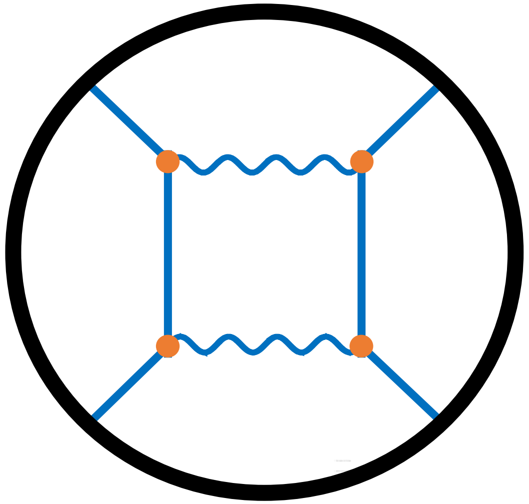
$\|$

$\mathrm{dDisc}_t(\mathrm{box})$

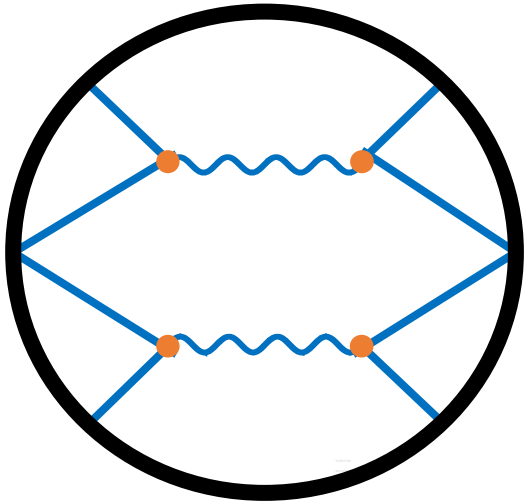
dDisc is the cut operator!

# Aside: A nice mathematical connection

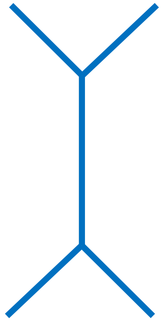
When gluing diagrams, *6j symbols* for the conformal group appear.



$$\approx$$



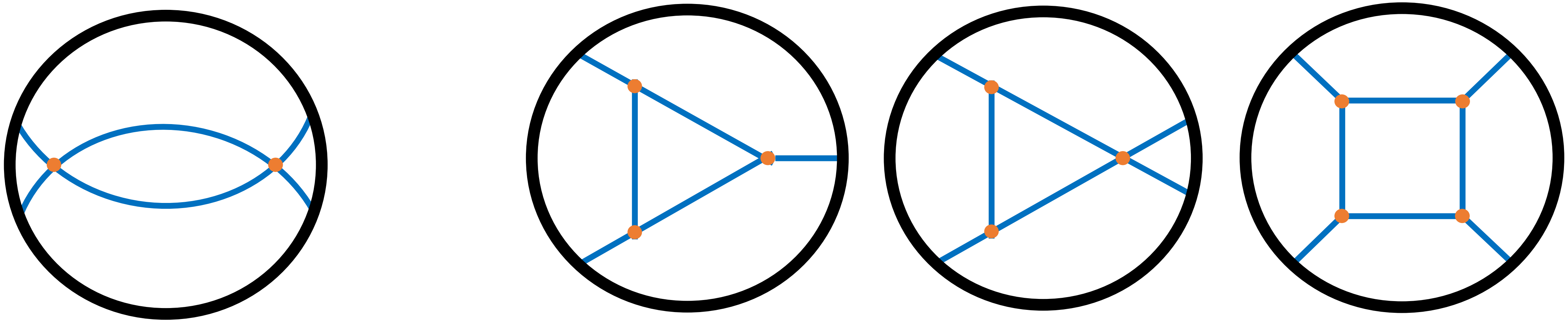
$$\approx \sum \int (6j)^2$$



6j symbol ~ AdS ladder kernel

[Liu, EP, Rosenhaus, Simmons-Duffin]

We are now able to compute all of these amplitudes (and various others) using AdS unitarity:



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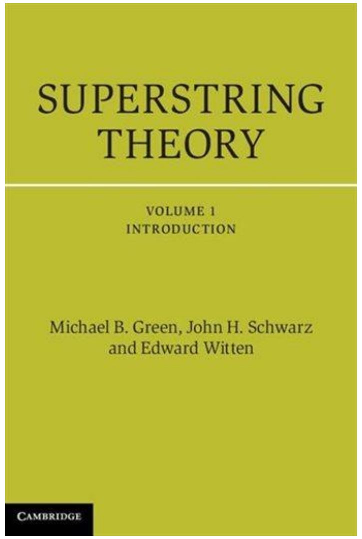
One of the most beautiful – and elemental – aspects of string/M-theory is that they predict specific corrections to general relativity.




What are they?

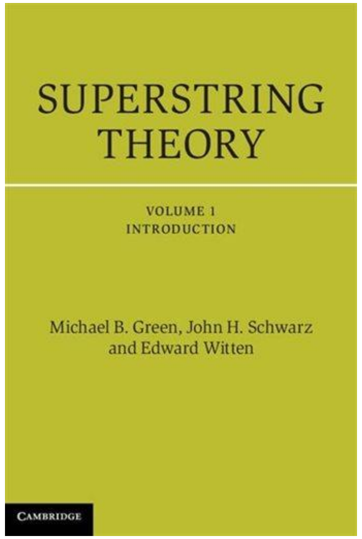
One of the most beautiful – and elemental – aspects of string/M-theory is that they predict specific corrections to general relativity.

What are they?

In M-theory,

$$S_{\text{M-theory}} \sim \int d^{11}x\sqrt{g}(R + \overbrace{R^4 + D^6R^4}^{\text{Known (fixed by SUSY)}} + \ldots)$$

Unknown



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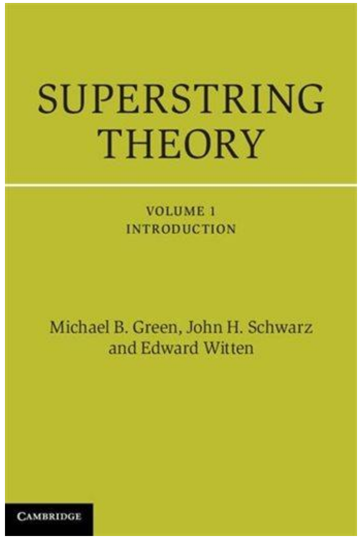



In M-theory,

Unknown

$$S_{\text{M-theory}} \sim \int d^{11}x\sqrt{g}(\overbrace{R + R^4 + D^6R^4}^{\text{Known (fixed by SUSY)}} + \ldots)$$

In type II string theory,

Unknown

$$S_{\text{type-II strings}} \sim \int d^{10}x\sqrt{g}(\overbrace{R + \mathfrak{f}(g_s)R^4 + \mathfrak{g}(g_s)D^4R^4 + \mathfrak{h}(g_s)D^6R^4}^{\text{Known (fixed by SUSY)}} + \ldots)$$

(Non-holomorphic SL(2,Z) modular forms)

Missing terms reflect a paucity of results in scattering amplitudes (as noted earlier).

# Taking stock of string perturbation theory:

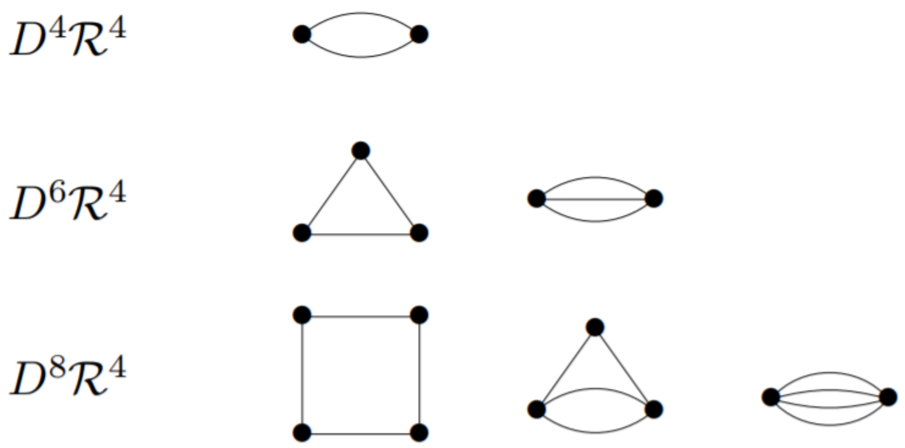
Most recent work on string perturbation theory has focused on issues at low-genus.

i) Shoring up issues of principle: unitarity, renormalization

[De Lacroix, Erbin, Pius, Rudra, Sen, Witten]

ii) Mathematical structure of moduli space integrands = “Modular graph functions”

[Basu, D’Hoker, Duke, Green, Gurdogan, Kaidi, Miller, Pioline, Vanhove]




iii) Transcendentality and double-copy

[D’Hoker, Green, Mafra, Schlotterer]

# Taking stock of string perturbation theory:

A couple of big questions:

1. **$D^8R^4$ coefficient is unknown.** Has been conjectured to obey **perturbative non-renormalization** beyond **four loops**.

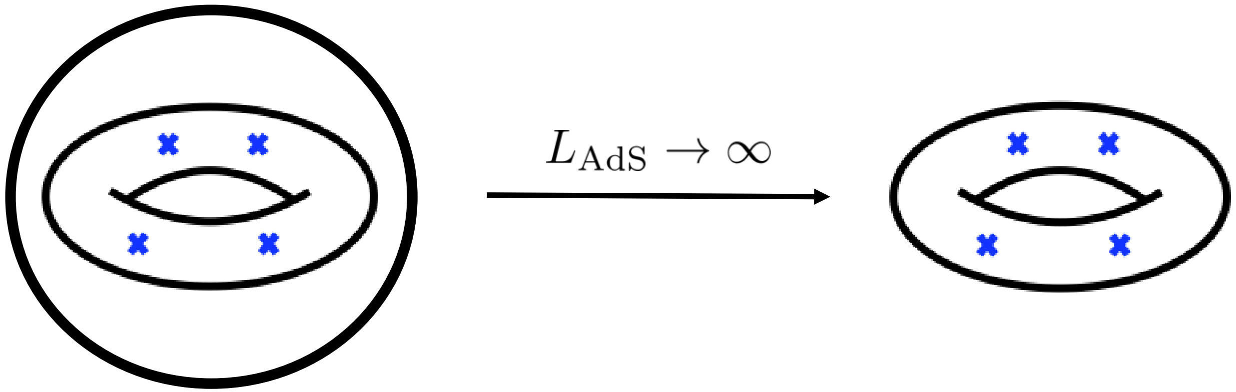
$$S_{\text{type-II strings}} \sim \int d^{10}x\sqrt{g}\left(R + \mathfrak{f}(g_s)R^4 + \mathfrak{g}(g_s)D^4R^4 + \mathfrak{h}(g_s)D^6R^4 + \ldots\right)$$

Perturbative truncation at g=1,2,3, respectively...

2. The coefficients appearing in the low-energy expansion appear to obey a form of **maximal transcendentality.** No one knows why.

# A new approach to string perturbation theory

1) Holographically compute the one-loop amplitude for strings in AdS, as a nonplanar correlator in a dual CFT.
2) Take a "flat space limit"




[Alday, Caron-Huot;
Alday, Bissi, EP]

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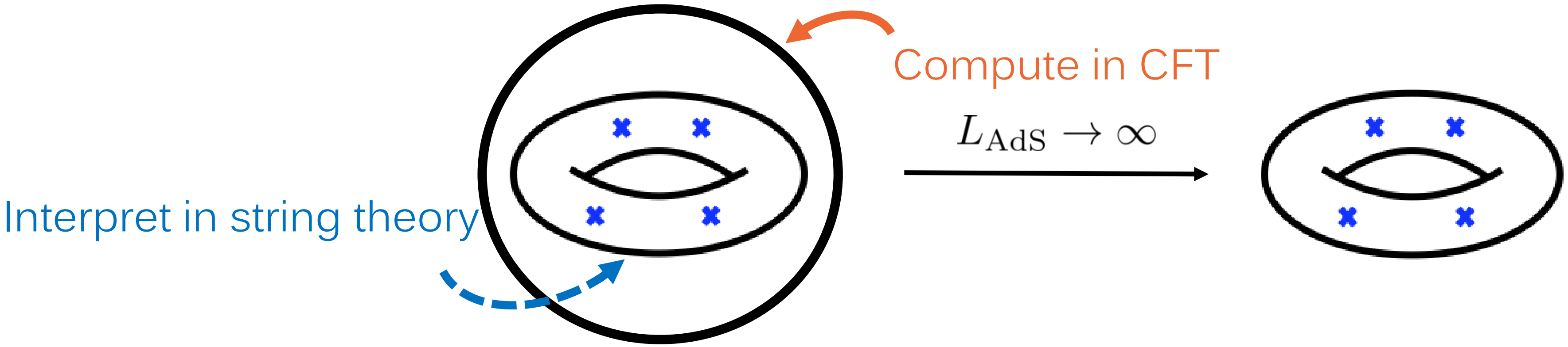


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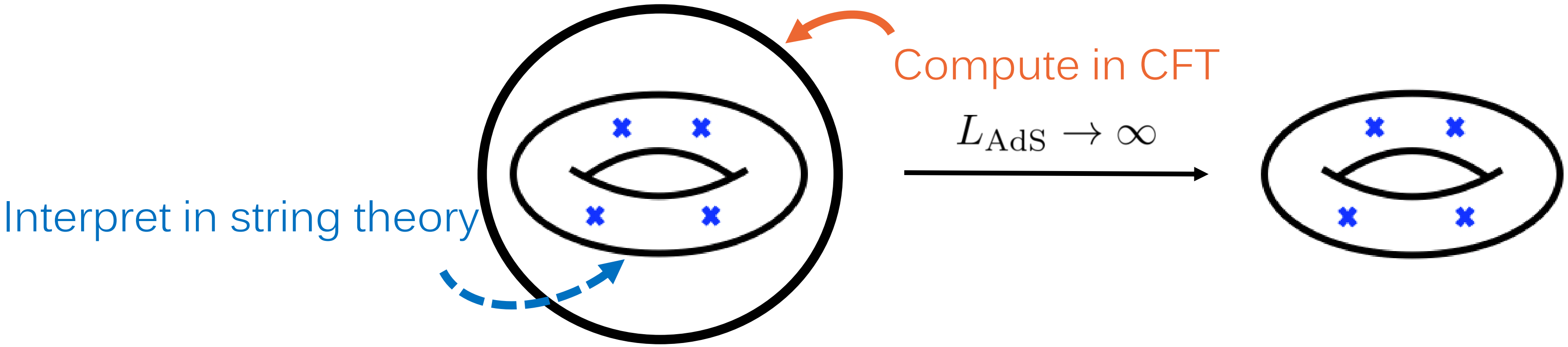


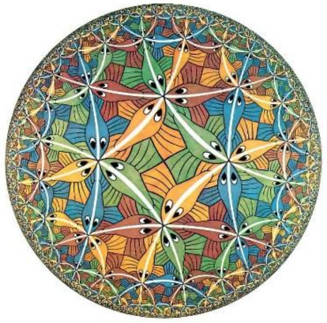
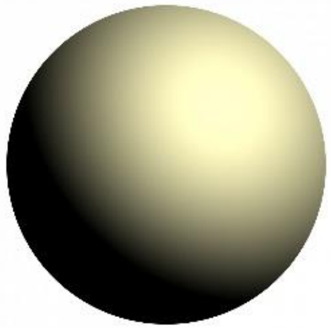

No strings attached.

String scattering amplitudes from 1/N expansion of *local* operator data in CFT.

[Alday, Caron-Huot; Alday, Bissi, EP]

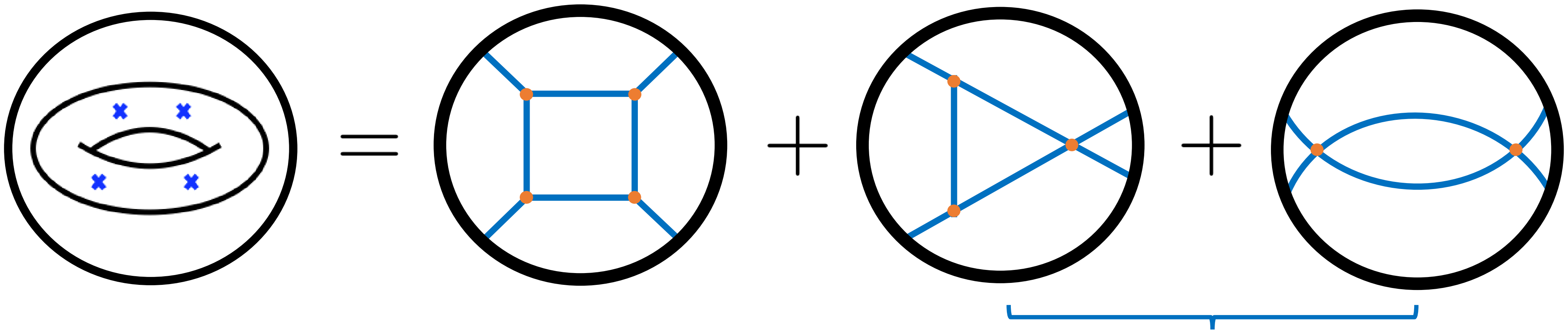
# A new approach to string perturbation theory

The prototypical CFT with a string dual is 4d maximally supersymmetric Yang-Mills:

$AdS_5$  $\times$  $S^5$ where $L_{\mathrm{AdS}_5} = L_{S^5}$

We compute the leading non-planar correction to a four-point function.

In practice, we take a **low-energy limit = CFT strong coupling expansion**.



Pure (super)gravity

Stringy corrections ~ $(\ell_{\text{string}})^{\#}$

# Genus-one strings in $AdS_5$ x $S^5$

Strong coupling ($\lambda \to \infty$) single-trace spectrum:

Only ½-BPS multiplets.

$$\mathcal{O}_p \in [0p0] \text{ of } SU(4)_R$$
$$\Delta_p = p \;, \;\; \text{where } p = 2, 3, \ldots$$

Study <2222>, where 2 = Stress-tensor multiplet.

$$\text{dDisc}_t(\mathcal{A}^{1-\text{loop}}_{2222}) = \frac{\pi^2}{2} \sum_{n,\ell} \langle \gamma^{(1)}_{n,\ell} \rangle^2 a^{(0)}_{n,\ell} G^{(t)}_{n,\ell}$$

Indicates mixing:

$$[\mathcal{O}_2\mathcal{O}_2]_{n,\ell} \;, \;\; [\mathcal{O}_3\mathcal{O}_3]_{n-1,\ell} \;, \;\; \ldots \;, \;\; [\mathcal{O}_{n+2}\mathcal{O}_{n+2}]_{0,\ell}$$

Evaluate in strong coupling ($1/\lambda$) expansion:

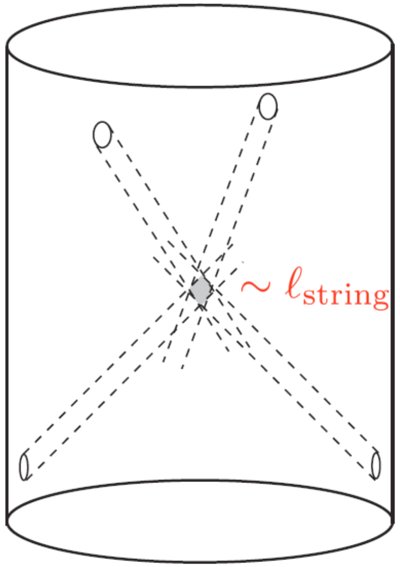
$$\gamma^{(1)} = \gamma^{(1)|\text{sugra}} + \frac{1}{\lambda^{3/2}} \gamma^{(1)|\mathcal{R}^4} + \frac{1}{\lambda^{5/2}} \gamma^{(1)|D^4\mathcal{R}^4} + \ldots$$

→ Non-planar <2222> in $1/\lambda$ expansion $\Longleftrightarrow$ $A^{1-loop}_{2222}$ in $AdS_5$ x $S^5$ in $\alpha'$ expansion.

# Genus-one strings in $R^{10}$

We can then take **flat space limit.**

$$L_{\mathrm{AdS}} \to \infty$$




[Okuda,Penedones; Penedones; Maldacena, Simmons-Duffin, Zhiboedov]

AdS amplitude → Graviton amplitude in $R^{10}$, with momenta in a five-plane.

At each order in $1/c$ and $1/\lambda$ expansion, leading power of s must match string result.

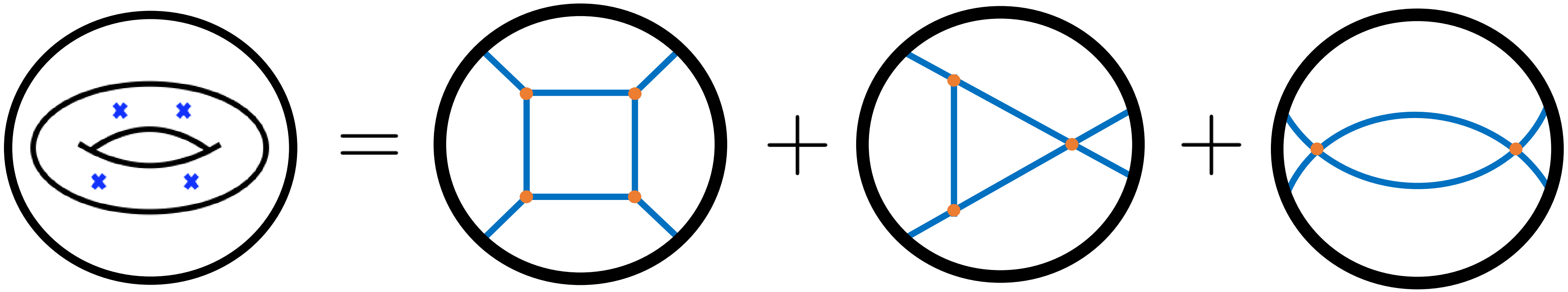
First few orders in $1/\lambda$ : ✓

[Green, Schwarz]

e.g.

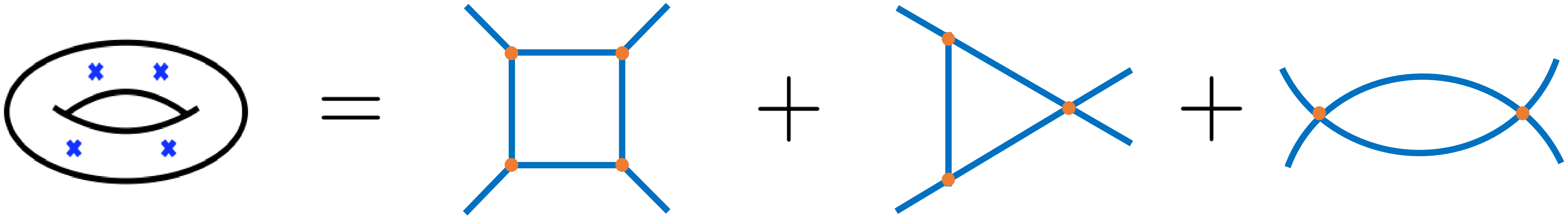
$$\left.\frac{\mathrm{dDisc}(\mathcal{A}_{2222}^{1-\mathrm{loop}})^{\mathcal{R}^4|\mathcal{R}^4}}{\mathrm{dDisc}(\mathcal{A}_{2222}^{1-\mathrm{loop}})^{\mathrm{sugra}|\mathcal{R}^4}}\right|_{\mathrm{flat}} = \frac{\mathrm{Disc}(A_{IIB}^{g=1})^{\mathcal{R}^4|\mathcal{R}^4}}{\mathrm{Disc}(A_{IIB}^{g=1})^{\mathrm{sugra}|\mathcal{R}^4}} = \frac{3\zeta(3)}{14}\left(\frac{\alpha' s}{4}\right)^3$$

# Summary



Compute these diagrams via the strong-coupling expansion of the CFT.

Flat space limit → Low-energy expansion of the genus-one string amplitude in **10d** flat space.



This matches the first several terms in genus-one string perturbation theory.

# Outline

1. Bootstrap basics and large N CFT
2. Loops in AdS
3. Application: String amplitudes from N=4 super-Yang-Mills
4. The String Landscape and Extra Dimensions in AdS/CFT

# III. The String Landscape and Extra Dimensions in AdS/CFT

We know necessary CFT conditions for bulk locality…

**Large N** + **higher-spin gap** (s>2) → **Local AdS bulk** [Heemskerk, Penedones, Polchinski, Sully; …]

…but in what dimension is it local? (CFT: how sparse is the low-spin spectrum?)

All fully-controlled examples of the AdS/CFT Correspondence involve bulk solutions which contain manifolds of parametrically large positive curvature: D > d+1

$AdS_{d+1} \times M_{D-d-1}$

$AdS_5 \times S^5/T^{1,1}/Y^{p,q}/L^{p,q,r}$, $AdS_{4/7} \times S^{7/4}$, $AdS_3 \times S^3 \times T^4$, $AdS_{3/2} \times S^{2/3} \times CY_3$, …

Large transverse manifolds means light KK towers, dual to CFT local operators.

No pure gravity, or even close!?

# III. The String Landscape and Extra Dimensions in AdS/CFT

There *are* attempts at constructing AdS x Small solutions in string/M-theory. e.g.:

1. Large Volume Scenario (non-SUSY $AdS_4$, IIB) [Balasubramanian, Berglund, Conlon, Quevedo]
2. KKLT (SUSY $AdS_4$, IIB) [Kachru, Kallosh, Linde, Trivedi]
3. DGKT (SUSY $AdS_4$ IIA) [DeWolfe, Giryavets, Kachru, Taylor]
4. Polchinski-Silverstein (SUSY $AdS_4$, $AdS_5$ from F-theory)

These all involve assumptions or arguments based on effective field theory, perturbative/non-perturbative effects in $\alpha$’ and/or $g_s$, and backreaction of sources.

What we want is to make fully rigorous, quantitative statements from the bootstrap.

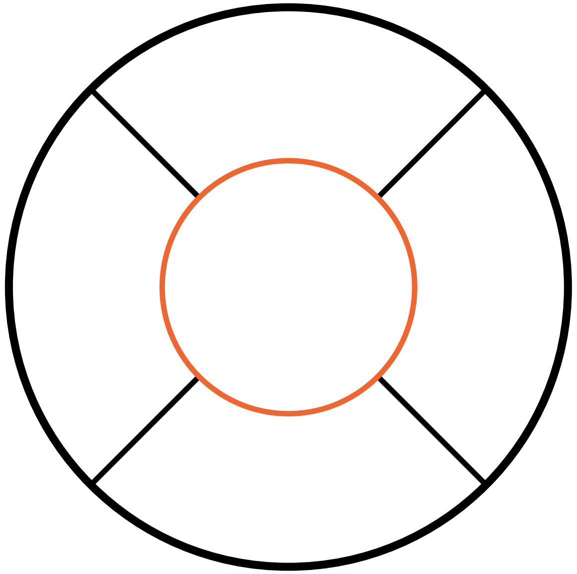
Today: set up a dictionary.

# III. The String Landscape and Extra Dimensions in AdS/CFT

Today we will address the following modest question about the AdS landscape:

Take D = number of "large" (= AdS-sized) bulk dimensions.

Given the planar OPE data of a large N, strongly coupled CFT, what is D?



# Segue

Q: In the N=4 calculation, why did we get a **D=10** string amplitude?

A: The bulk dual is **$AdS_5$ x $S^5$** where $\boldsymbol{L_{S^5} = L_{AdS_5}}$…

How exactly does the CFT correlator "know" about the extra five dimensions?

# Segue

To match to flat space, either:

1. Match amplitudes
2. Match partial wave coefficients

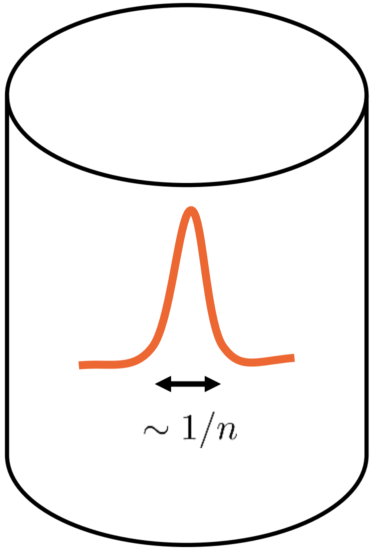
$$\mathrm{dDisc}_t(\mathcal{A}^{1-\mathrm{loop}}_{2222}) = \frac{\pi^2}{2}\sum_{n,\ell}\langle\gamma^{(1)}_{n,\ell}\rangle^2 \underline{\underline{a^{(0)}_{n,\ell}G^{(t)}_{n,\ell}}}$$

$$\Big\downarrow \; L \to \infty$$

Partial waves

The dictionary between OPE data and flat space momentum:

$$L\sqrt{s} \sim n$$



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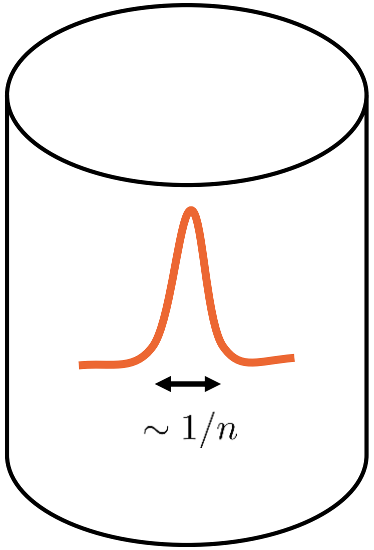


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In N=4 SYM at $\lambda = \infty$, $\gamma^{(1)}_{n\gg 1,\ell} \sim n^3$ but $\langle\gamma^{(1)}_{n\gg 1,\ell}\rangle^2 \sim n^{11} = n^{6+5}$

The 5 of $S^5$?

[Alday, Caron-Huot]

# A 1-loop sum rule for D

Consider a D-dimensional two-derivative theory of gravity + spin ≤ 2 matter.

$$A_D(s,t) = G_N A_D^{\text{tree}}(s,t) + G_N^2 A_D^{1-\text{loop}}(s,t) + \mathcal{O}(G_N^3)$$

Suppose there exists an $\text{AdS}_{d+1} \times \mathcal{M}_{D-d-1}$ vacuum.

EFT:
$L_{\mathcal{M}} \sim L$

Define

$$A_{d+1}(s,t) \equiv \frac{A_D(s,t)}{\text{Vol}(\mathcal{M})} = \frac{L^{d-1}}{c} A_{d+1}^{\text{tree}}(s,t) + \frac{L^{D+d-3}}{c^2} A_{d+1}^{1-\text{loop}}(s,t) + \mathcal{O}(c^{-3})$$

where the CFT central charge $c \sim 1/G_N$.

At high-energy $s,t \gg 1$ and fixed-angle $\cos\theta = 1 + \frac{2t}{s}$,

$$A_{d+1}(s \gg 1, \theta) = \frac{(L\sqrt{s})^{d-1}}{c} f_{d+1}^{\text{tree}}(\theta) + \frac{(L\sqrt{s})^{D+d-3}}{c^2} f_{d+1}^{1-\text{loop}}(\theta) + \mathcal{O}(c^{-3})$$

**Order-by-order in 1/c, flat space limit of a CFT correlator must reproduce this.**

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**Order-by-order in 1/c, flat space limit of a CFT correlator must reproduce this.**

# A 1-loop sum rule for D

An arbitrary 1-loop correlator has a (t-channel) dDisc of the following form:

$$\mathrm{dDisc}_t(\mathcal{A}^{1-\mathrm{loop}}(z,\bar{z})) = \sum_{n,\ell} \beta^{1-\mathrm{loop}}_{n,\ell}\, a^{(0)}_{n,\ell}\, G^{(t)}_{n,\ell}(z,\bar{z})$$

In flat space limit, **matching** yields a **1-loop sum rule** for D:

$$L\sqrt{s} \sim n$$

$$\boxed{\beta^{1-\mathrm{loop}}_{n\gg 1,\ell} \sim n^{D+d-3} f^{1-\mathrm{loop}}(\ell)}$$

where

$$\beta^{1-\mathrm{loop}}_{n,\ell} \equiv 2\sum_{\mathcal{O}} \rho_{\mathrm{ST}}(\Delta_{\mathcal{O}}) \left( \frac{\pi^2}{4} \langle \gamma^{(1)}_{n,\ell}(\mathcal{O}) \rangle^2 + \sin^2(\pi(\tau_{\mathcal{O}} - \Delta_\phi)) \| C^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} \| \right)$$

Single-trace density of states

Degenerate operators

$$\Delta_{\mathcal{O}} - \Delta_\phi \in \mathbb{Z}$$

Non-degenerate operators

$$\Delta_{\mathcal{O}} - \Delta_\phi \notin \mathbb{Z}$$

# A 1-loop sum rule for D

$$\boxed{\beta^{1-\text{loop}}_{n\gg 1,\ell} \sim n^{D+d-3} f^{1-\text{loop}}(\ell)}$$

Comments:

1. **Positive-definite**, term-by-term → Lower bound D

$$\beta^{1-\text{loop}}_{n,\ell} \equiv 2\sum_{\mathcal{O}} \rho_{\text{ST}}(\Delta_{\mathcal{O}}) \left( \frac{\pi^2}{4} \langle \gamma^{(1)}_{n,\ell}(\mathcal{O}) \rangle^2 + \sin^2(\pi(\tau_{\mathcal{O}} - \Delta_\phi)) \| C^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} \| \right)$$

N.B. dDisc crucial!

2. **Trees are insensitive to D**, as they must be: consistent truncations exist.

$$\beta^{\text{tree}}_{n\gg 1,\ell} \sim n^{d-1} \quad : \quad \text{Einstein scaling}$$

[Cornalba, Costa, Penedones]

3. D+d-3 follows from two-derivative approx. (= large HS gap in CFT)

Let us explore some consequences of this sum rule for extra dimensions.

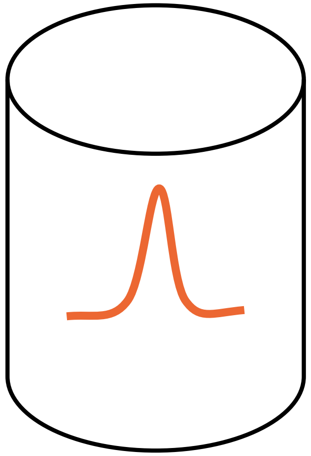
# A 1-loop sum rule for D

1. Suppose we have a power law density of non-degenerate single-trace operators:

$$\rho_{\text{ST}}(\Delta_{\mathcal{O}} \gg 1) \sim \Delta_{\mathcal{O}}^{\mathtt{x}-1}$$

Sum dominated by large double-trace dimensions, $1 \ll n \sim \Delta_{\mathcal{O}} \ll \Delta_{\text{gap}}$

$$\beta^{1-\text{loop}}_{n\gg 1,\ell} \sim n^{2d+\mathtt{x}-1} \quad \Rightarrow \quad D = d+1+\mathtt{x}$$



→ x large extra dimensions.

Converse of a holographic fact: Weyl's law growth of eigenvalues $\lambda$ on compact manifold $\mathcal{M}$ with smooth boundary.

Parameterizing $\lambda \sim \Delta^2$,

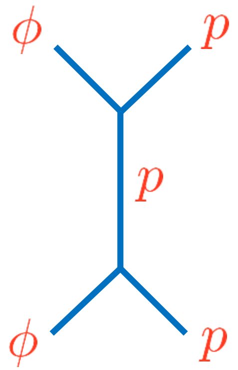
$$\int^{\Delta_* \gg 1} d\Delta\, \rho_{\mathcal{M}}(\Delta) \sim \frac{\text{vol}(\mathcal{M})}{(4\pi)^{\frac{\dim(\mathcal{M})}{2}}\Gamma\left(\frac{\dim(\mathcal{M})}{2}+1\right)} \Delta_*^{\dim(\mathcal{M})}$$

# A 1-loop sum rule for D

2. Suppose there is a tower of degenerate operators

$$\Delta_p = \Delta_\phi + p - 2 \ , \ \text{ where } p = 2, 3, \ldots$$

Assuming a cubic coupling $\phi p p$.




$$\xrightarrow[\text{(Crossing)}]{} \quad \langle \gamma^{(1)}_{[\phi\phi]_{n,\ell}}(p) \rangle$$

Result:

$$\langle \gamma^{(1)}_{[\phi\phi]_{n,\ell}}(p) \rangle \Big|_{1 \ll n \sim p} \sim n^{d-3} \cdot C^2_{\phi pp} \big|_{p \gg 1}$$

Depends on asymptotic of C! If $C_{\phi pp}\big|_{p \gg 1} \sim \dfrac{p^{1+\frac{\alpha}{4}}}{\sqrt{c}}$ then $D = d + 2 + \alpha$

# Stringy OPE universality

In familiar cases like $\phi = T_{\mu\nu}, \mathcal{L}$ , this OPE coefficient is linear ($\alpha = 0$).

**Conjecture (OPE universality):** for any light operator $\phi$ and heavy operator $\phi_p$ with

$$\Delta_\phi \ll \Delta_{\text{gap}} \quad \text{and} \quad \Delta_\phi \ll \Delta_p \ll c^{\#>0}$$

the normalized planar OPE coefficient $\phi pp$ has linear asymptotics, $C_{\phi pp}\Big|_{\Delta_p \gg 1} \sim \frac{p}{\sqrt{c}}$

p can be KK mode or massive string mode.

Copious evidence from literature (N=4 SYM semiclassical and KK correlators, ABJM, D1-D5)

(N.B. This is *NOT* the same “heavy-heavy-light” as in ETH, 2d CFT, or large charge.)

# Bounding holographic spectra

Now turn logic around.

Assume string/M-theory dual with D ≤ 10 or 11.

What does this imply about single-trace spectrum of planar CFT?

1. Density of states: $\rho_{\mathrm{ST}}(\Delta \gg 1) \lesssim \Delta^{8-d}$ (string)

$\rho_{\mathrm{ST}}(\Delta \gg 1) \lesssim \Delta^{9-d}$ (M)

2. If $\phi_p$ furnish sequence of irreps $R_p$ of global symmetry, with asymptotics

$$\dim(R_{p\gg 1}) \sim p^{\mathbf{r}_p}$$

then since D = d + 2 + $r_p$, the above inequalities **bound $\boldsymbol{r_p}$.**

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Why, from CFT, are these things true?

# A final speculation

So, then: what is the landscape of AdS vacua?

A possible Holographic Hierarchy Conjecture:

**Large Higher-Spin Gap + No Global Symmetries → Local AdS dual with D = d+1**

This generalizes arguments of [Polchinski, Silverstein]

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Let the bootstrapping begin.

# Summary

New techniques for AdS loop amplitudes using ideas from the bootstrap

A novel holographic approach to string perturbation theory

A dictionary for finding large extra bulk dimensions from CFT data

# Future directions

## 1. AdS loops:

- Complete analysis. e.g. a 1-loop basis?
- What is the L-loop function space/transcendentality properties?

## 2. Holographic string amplitudes

- Higher-genus, non-SUSY data

## 3. Extra dimensions

- We have a dictionary. Can we bootstrap the landscape?