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‘t Hooft Anomaly & Modular Bootstrap

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Based on work with Ying-Hsuan Lin (Caltech)
ArXiv: 1904.xxxxx

Two Major Non-Perturbative Tools for QFT

't Hooft Anomaly

- Obstruction to gauging G
- Invariant under RG and duality

[See Zohar's Lectures]

Conformal Bootstrap

- Consistency of CFT
- Constrain operator spectrum

[See Leonardo's Talk]

Wouldn't it be nice if we can combine the two techniques?

Questions We'd Like to Ask:

In CFT with Global Symmetry G and Anomaly α:

1. **Is there an upper bound on the lightest G charged operator?**
2. **How does the bound depend on the anomaly α?**

Weak Gravity Conjecture like questions [See Matt's Lectures]

Today: Bootstrap with Anomaly

Setup:

2D Bosonic CFT with Global Symmetry $G = \mathbb{Z}_2$

Question:

Is there an upper bound on the lightest $\mathbb{Z}_2$ odd operator?

2D CFT with Global Symmetry $\mathbb{Z}_2$

	Non-Anomalous	Anomalous
Bound on Charged Operator	NO 😭	YES 😍

Moral: It is harder to "hide" a symmetry if it is anomalous

Reminder on ‘t Hooft Anomalies

- A global symmetry with ‘t Hooft anomaly is still a *perfectly healthy* symmetry in a consistent QFT.

- You just cannot gauge it.

- Different from the ABJ anomaly, where the axial “symmetry” is *not* a true global symmetry.

Outline

- Symmetry and Anomaly in Two Dimensions
- Modular Bootstrap

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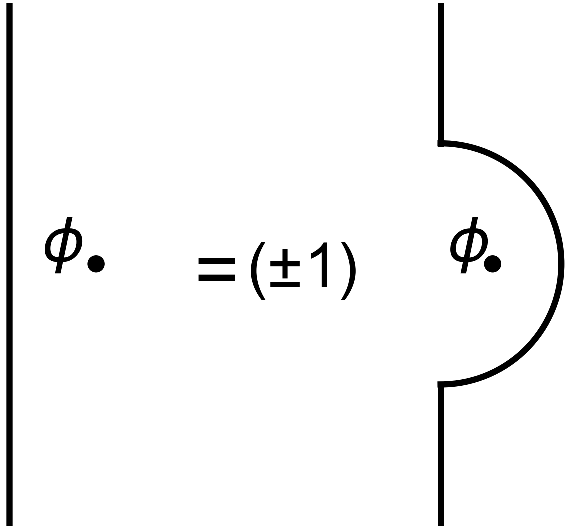
Symmetry and Topological Defect

- Continuous global symmetry → Noether charge
- More generally, a 0-form global symmetry $g \in G$ (continuous or discrete) is associated to a codimension-1 topological defect L_g.
- Topological defect acts on local operators by symmetry transformation.



0-form Codimension-1

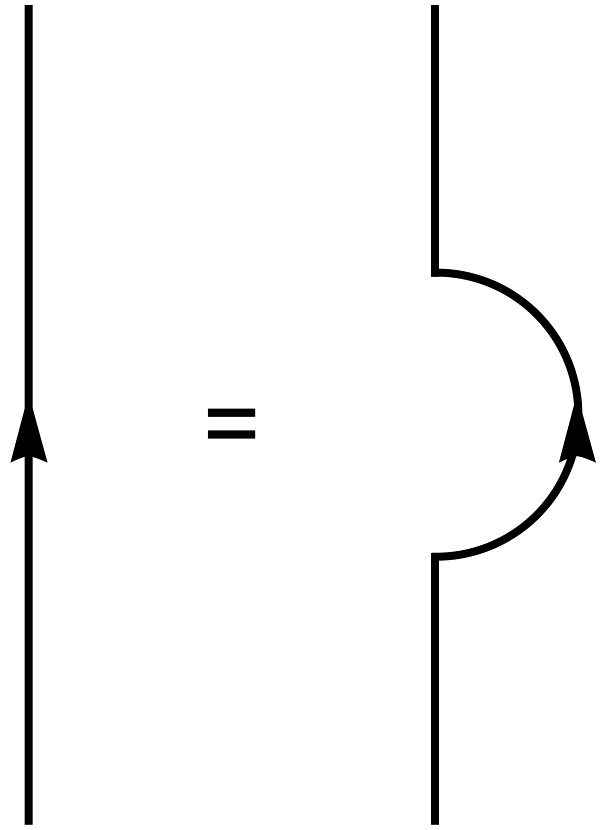
Global Symmetry $\longrightarrow$ Topological Defect

Basic Properties of Topological Lines

- They are topological

  - All physical observables are invariant under continuous deformation of topological lines.

  - They commute with **both** Virasoro algebras.

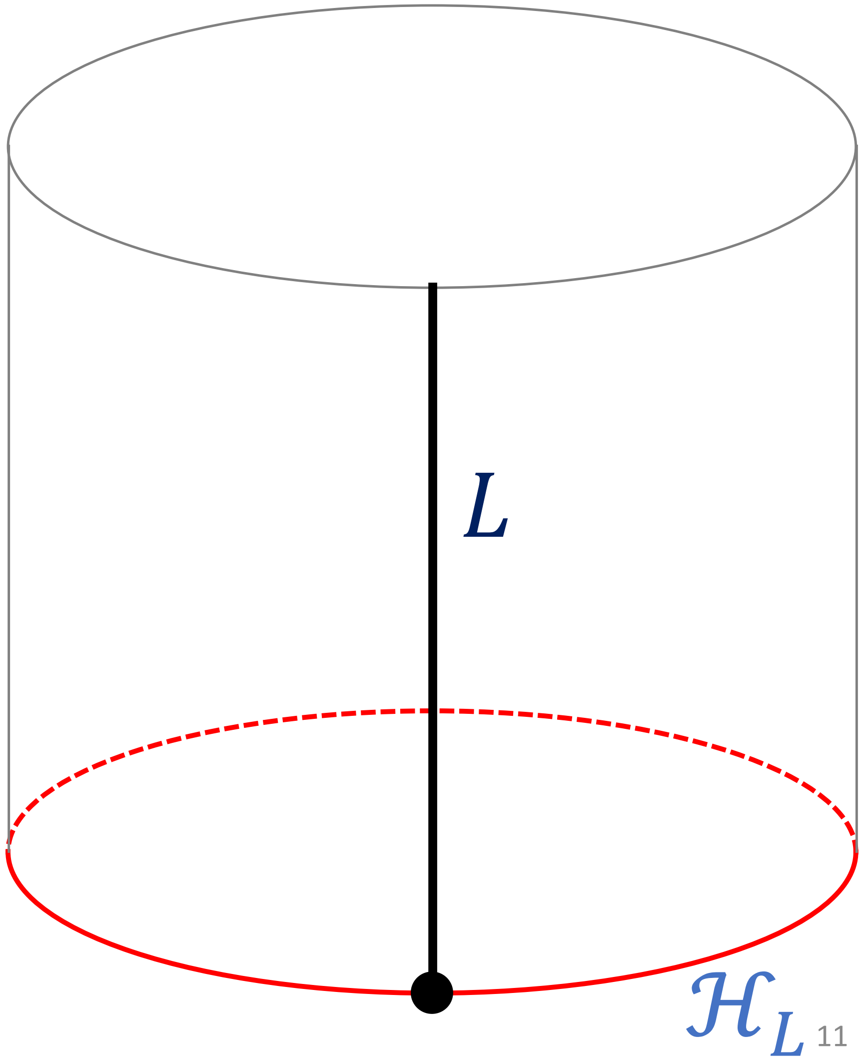


Defect Hilbert Space $\mathcal{H}_L$

- Followed from the topological property, states in $\mathcal{H}_L$ are in representations of **both** Virasoro algebras.

$$\mathcal{H}_L = \sum_{h,\bar{h}\in\mathcal{H}_L} (n_L)_{h,\bar{h}}\, Vir_h \otimes \overline{Vir}_{\bar{h}}$$

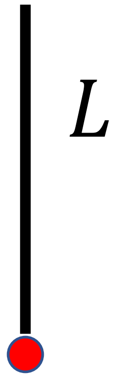
$$(n_L)_{h,\bar{h}} \in \mathbb{N}$$



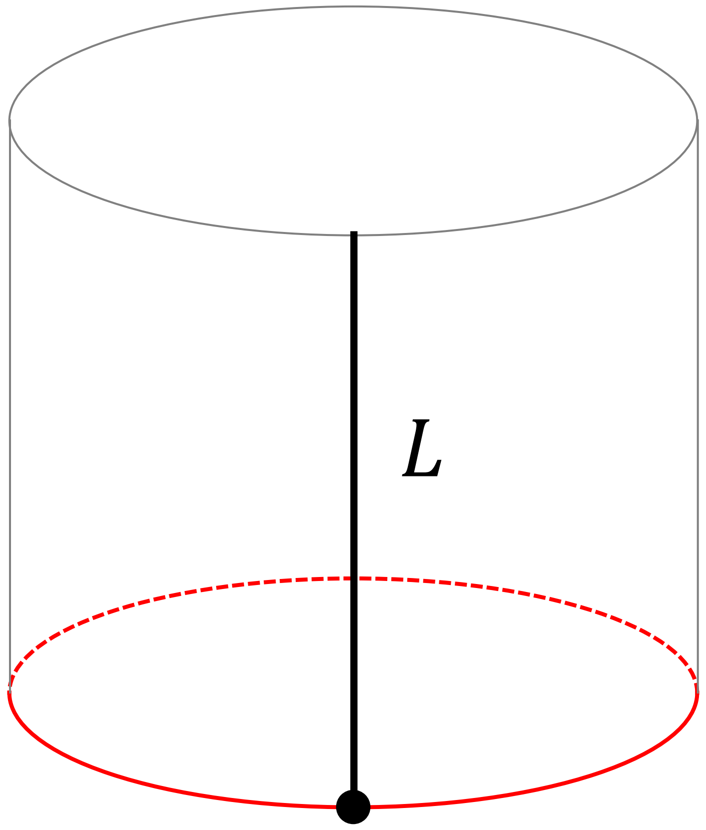
Operator-State Map for $\mathcal{H}_L$

Non-local operator living at the end of the defect line

E.g. Electron in QED

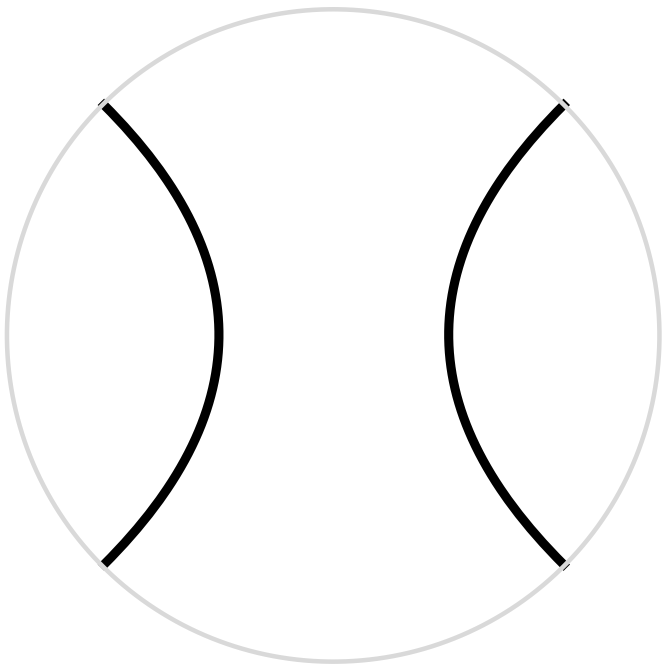


$\mu(x)$

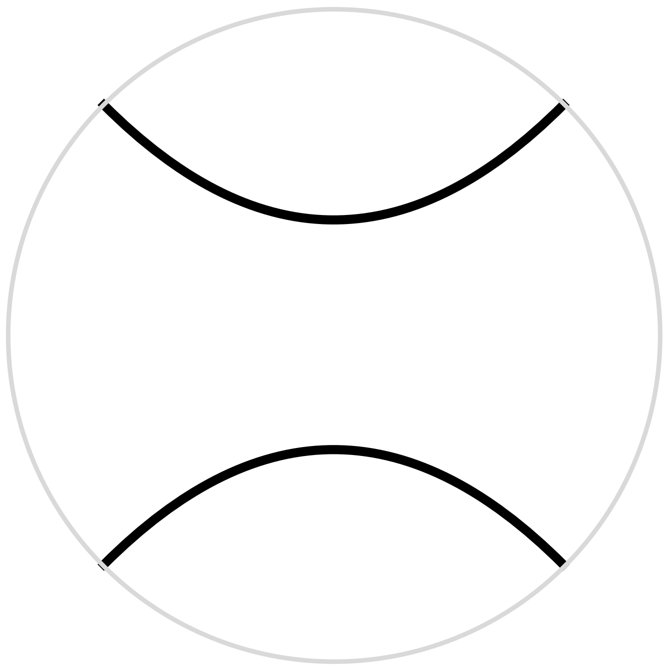


$|\mu\rangle \in \mathcal{H}_L$

Crossing Relations of the $\mathbb{Z}_2$ Line

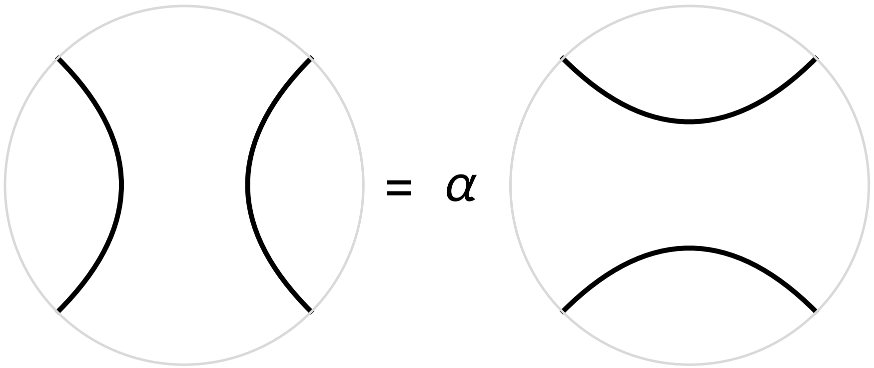


$= \alpha$



Do it twice: $\alpha^2 = 1$ (the cocycle condition)

Crossing and Anomaly

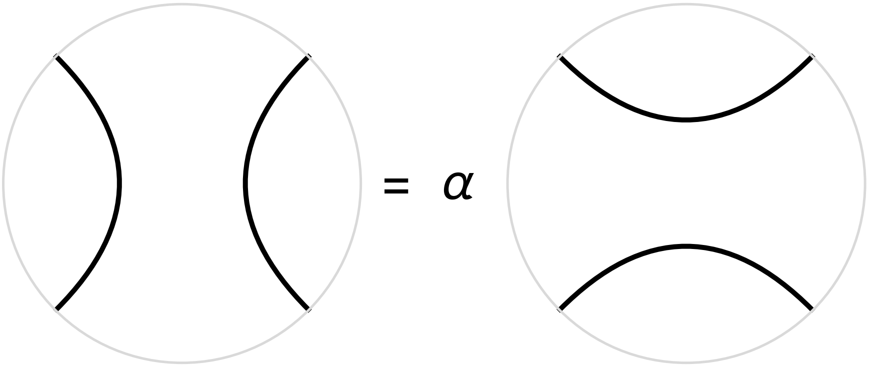


- $\alpha = +1$: Non-Anomalous (can be gauged)
- $\alpha = -1$: Anomalous (can *not* be gauged)

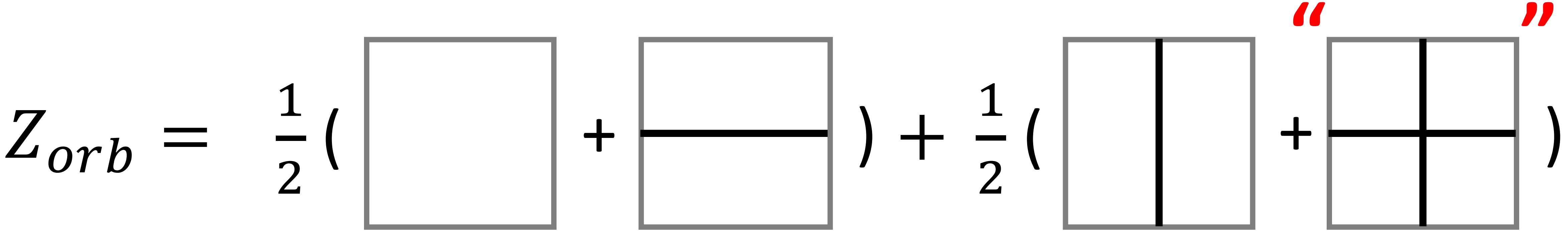
Indeed, the bosonic, unitary $\mathbb{Z}_2$ anomaly is classified by

$$\alpha \in H^3(\mathbb{Z}_2, \mathrm{U}(1)) = \mathbb{Z}_2$$

Crossing and Anomaly



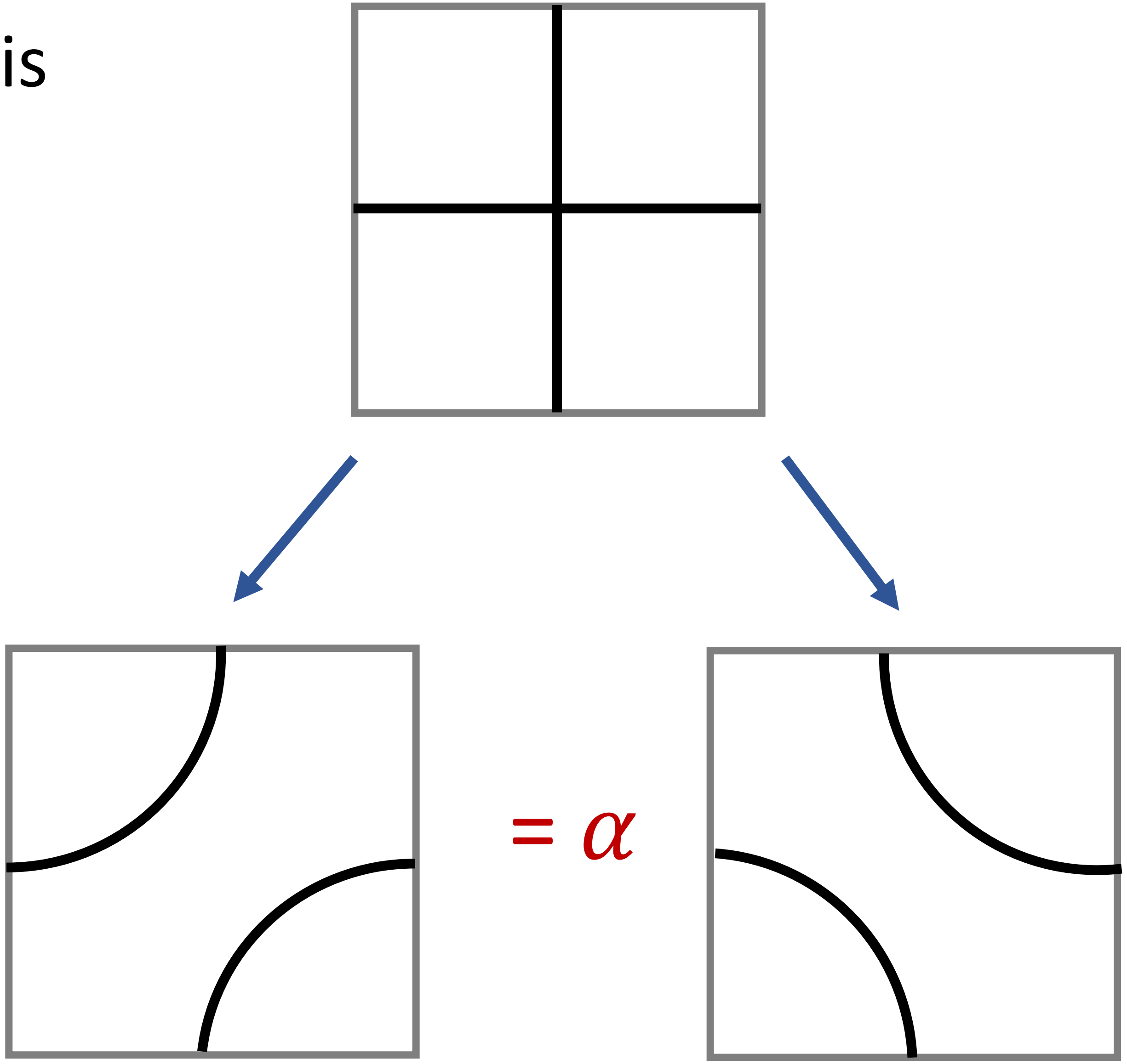
- Consider the torus partition function of the would-be $\mathbb{Z}_2$ orbifold theory:



Untwisted sector

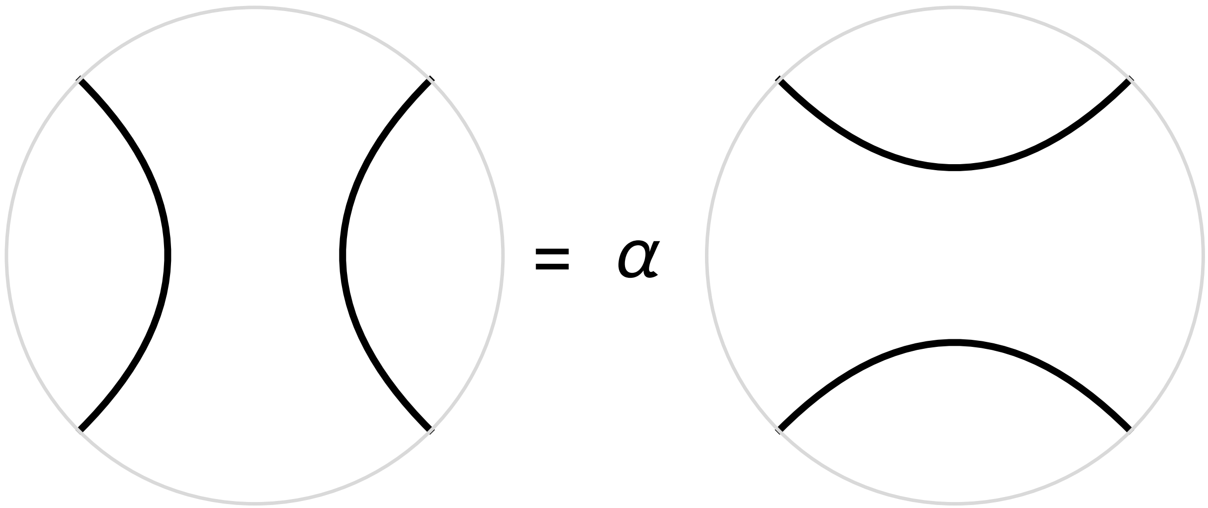
Twisted sector

When $\alpha = -1$, this is an ambiguity
$\Longrightarrow$ anomaly



Crossing and Anomaly (Recap)

- $\alpha = +1$: Non-Anomalous
- $\alpha = -1$: Anomalous



Outline

- Symmetry and Anomaly in Two Dimensions
- Modular Bootstrap

Modular Bootstrap

- **Positivity**: Expansion on Virasoro characters
- **Crossing**: Modular S Transformation

Torus Partition Function

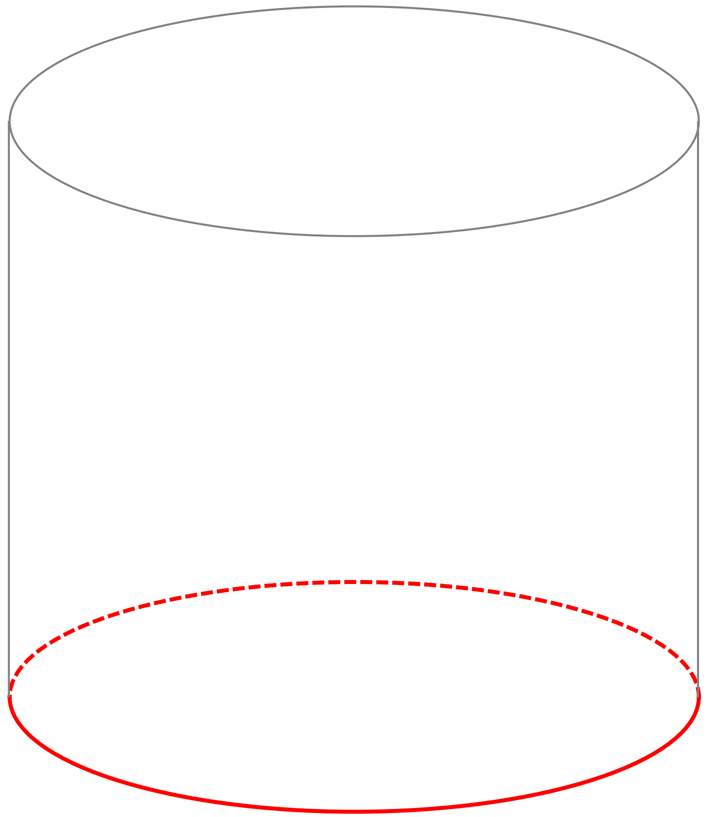
$$\chi_h(\tau) = \frac{q^{h-\frac{c-1}{24}}}{\eta(\tau)}$$

- The torus partition function can be expanded on the Virasoro characters with positive coefficients

$$Z(\tau,\bar{\tau}) = Tr_{\mathcal{H}}\left[q^{h-c/24}\bar{q}^{\bar{h}-c/24}\right]$$

$$= \sum_{h,\bar{h}\in\mathcal{H}} n_{h,\bar{h}}\ \chi_h(\tau)\ \chi_{\bar{h}}(\bar{\tau})$$

$$n_{h,\bar{h}} \in \mathbb{N}$$



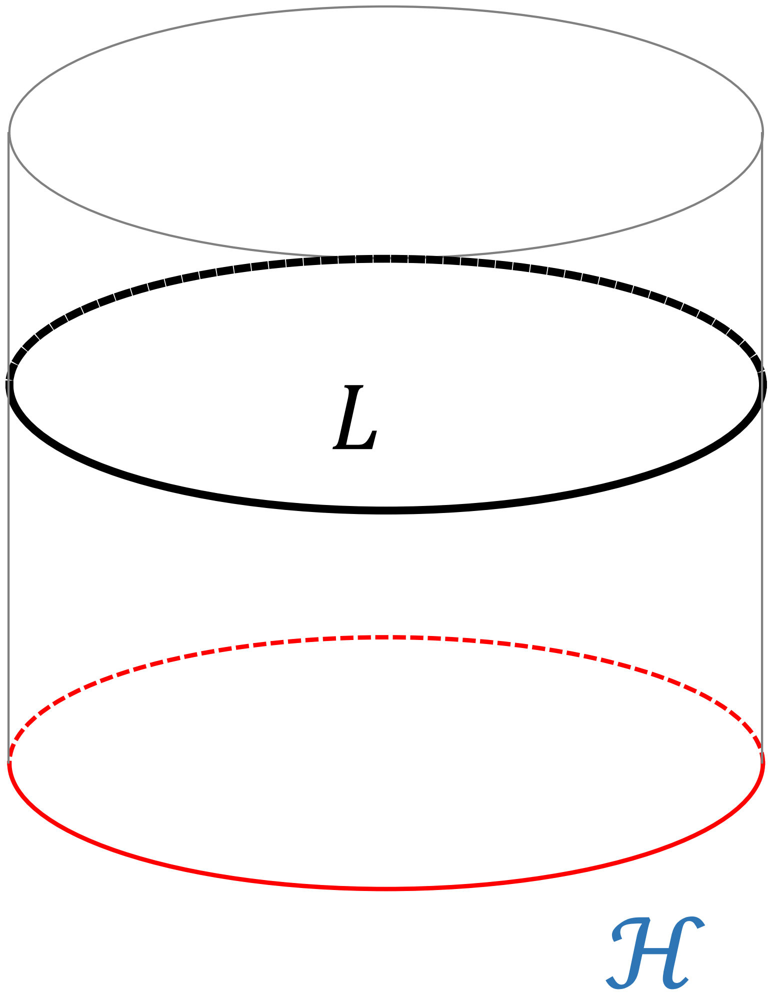
$\mathcal{H}$

Torus Partition Function with $\mathbb{Z}_2$ Line

$$Z^L(\tau,\bar\tau) = Tr_{\mathcal{H}}\,[\hat{L}\; q^{h-c/24}\bar{q}^{\bar{h}-c/24}]$$

$$= \sum_{h,\bar{h}\in\mathcal{H}} (n^{+}_{h,\bar{h}} - n^{-}_{h,\bar{h}})\;\chi_h(\tau)\;\chi_{\bar{h}}(\bar\tau)$$

$$n^{\pm}_{h,\bar{h}} \in \mathbb{N}$$

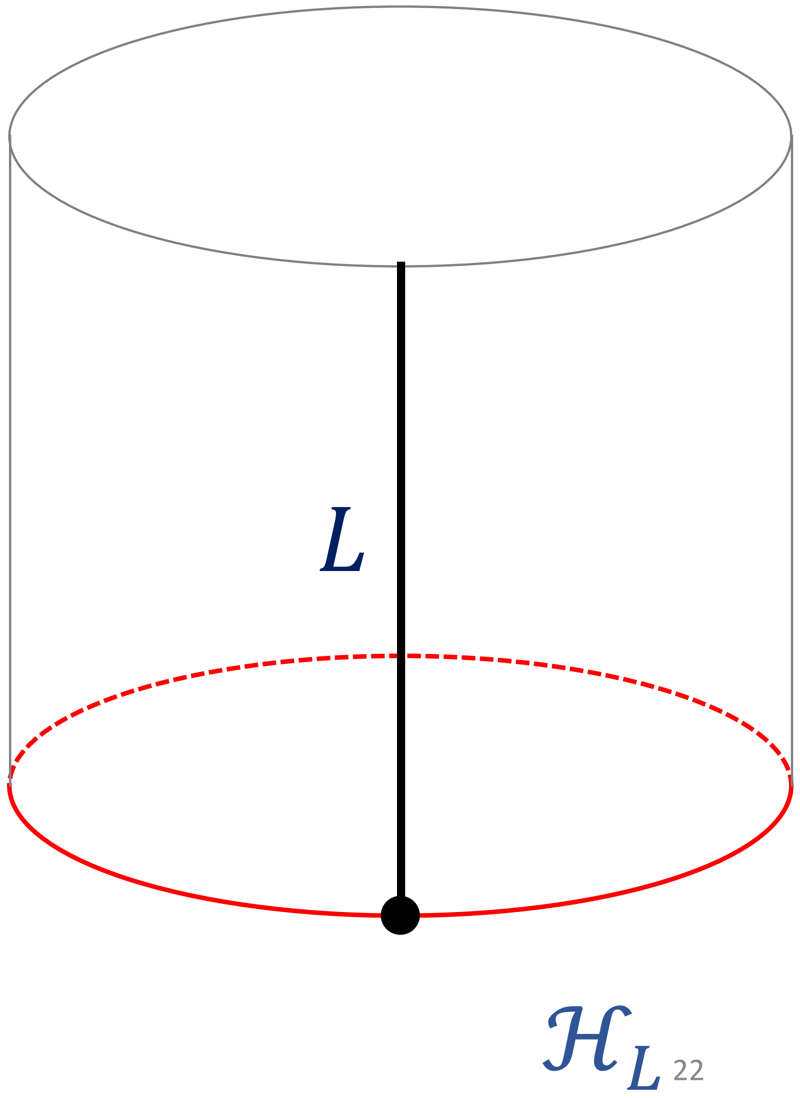


Defect Hilbert Space $\mathcal{H}_L$

$$Z_L(\tau,\bar{\tau}) = Tr_{\mathcal{H}_L}\,[q^{h-\frac{c}{24}}\bar{q}^{\bar{h}-\frac{c}{24}}]$$

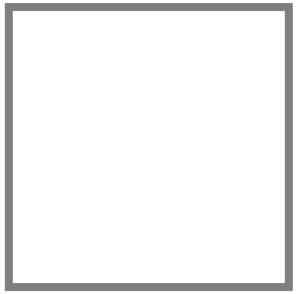
$$= \sum_{h,\bar{h}\in\mathcal{H}_L} (n_L)_{h,\bar{h}}\,\chi_h(\tau)\,\chi_{\bar{h}}(\bar{\tau})$$

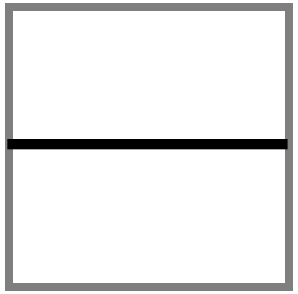
$$(n_L)_{h,\bar{h}} \in \mathbb{N}$$



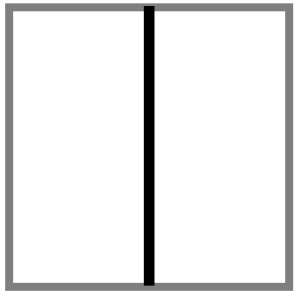
Positivity

$$Z^{\pm}(\tau,\bar{\tau}) = \sum_{(h,\bar{h})\in\mathcal{H}^{\pm}} \underset{\substack{\vee \\ \mathbf{0}}}{n^{\pm}_{h,\bar{h}}}\, \chi_h(\tau)\chi_{\bar{h}}(\bar{\tau}) = \frac{1}{2}\Big(\quad \pm \quad\Big)$$

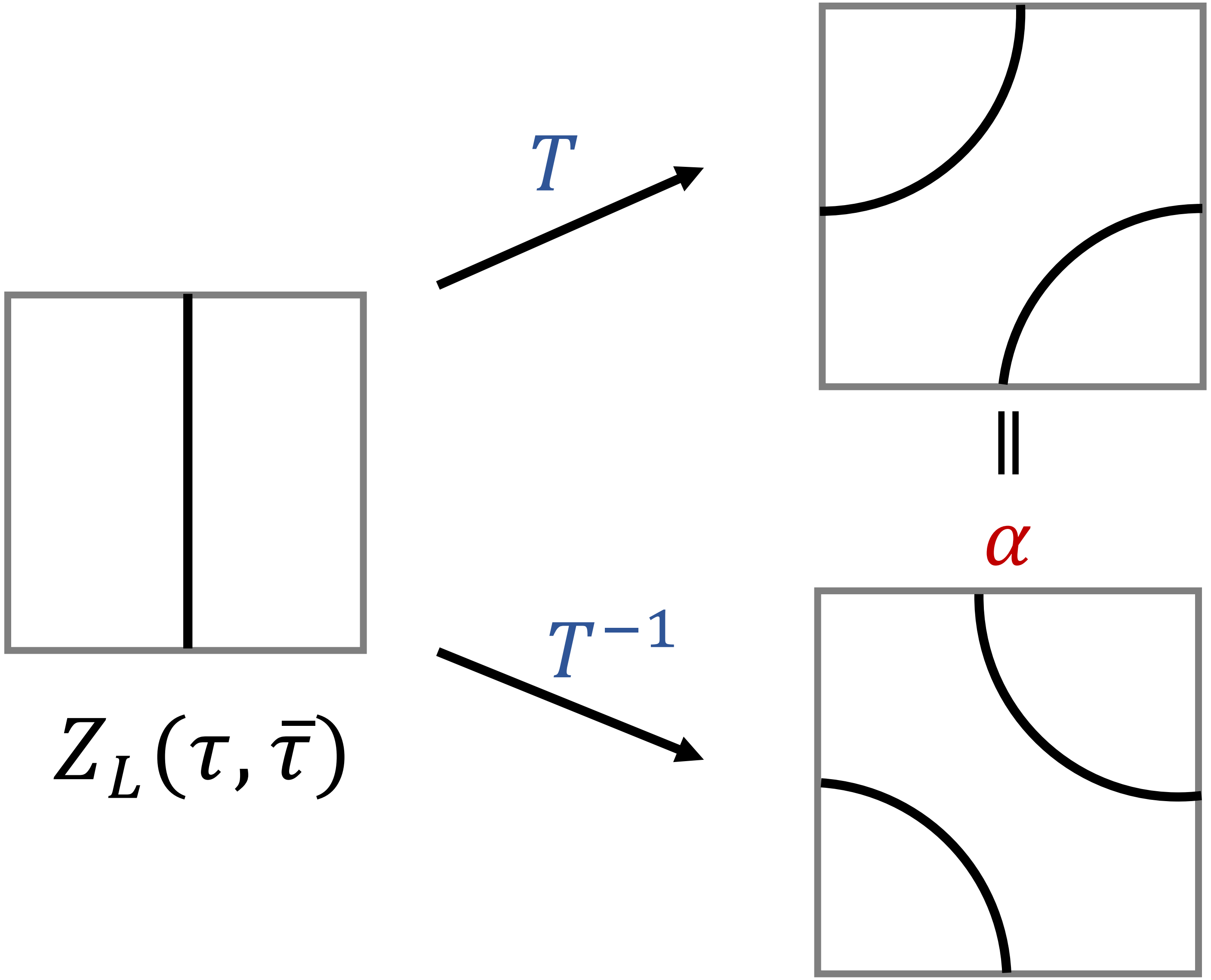




$$Z_L(\tau,\bar{\tau}) = \sum_{h,\bar{h}\in\mathcal{H}_L} \underset{\substack{\vee \\ \mathbf{0}}}{(n_L)_{h,\bar{h}}}\, \chi_h(\tau)\, \chi_{\bar{h}}(\bar{\tau}) \;=$$



Anomaly and Modular Transformation



$Z_L(\tau, \bar{\tau})$ is invariant under

$$\begin{cases} T^2 & \text{If } \alpha = +1 \\ T^4 & \text{If } \alpha = -1 \end{cases}$$

Spin Selection Rule in $\mathcal{H}_L$

- The spin $s = h - \bar{h}$ of a state in the defect Hilbert space $\mathcal{H}_L$ is constrained by the anomaly α [Chang-Lin-SHS-Wang-Yin]:

$$s \in \begin{cases} \dfrac{\mathbb{Z}}{2} & \text{If } \alpha = +1 \text{ (Non-Anomalous)} \\ \dfrac{1}{4} + \dfrac{\mathbb{Z}}{2} & \text{If } \alpha = -1 \text{ (Anomalous)} \end{cases}$$

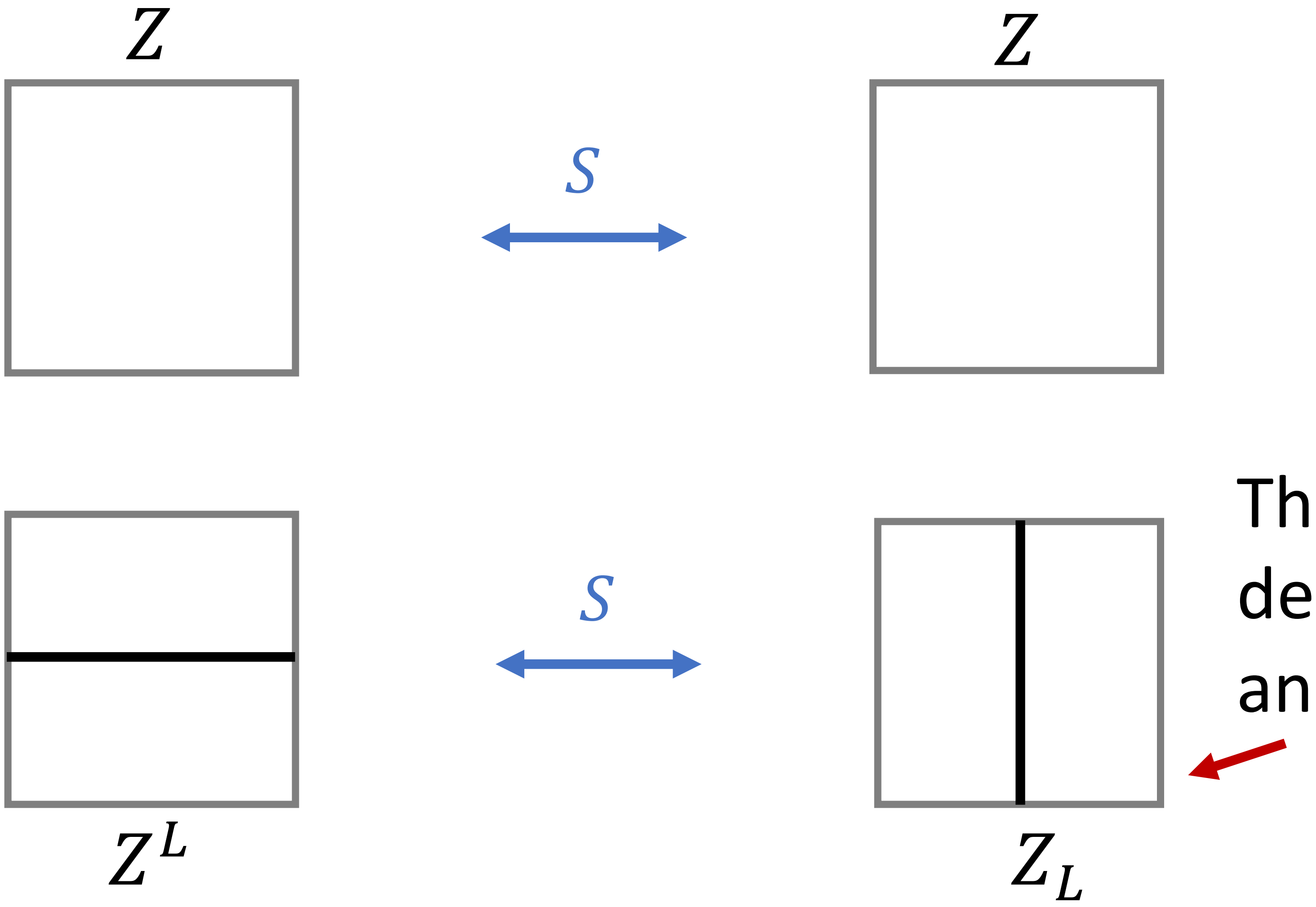
Spin Selection Rule in $\mathcal{H}_L$

- Now we see that the anomaly controls the spin of **non-local operators living at the end of the line**.

- How do we convert this information to constraints on **local operators**?

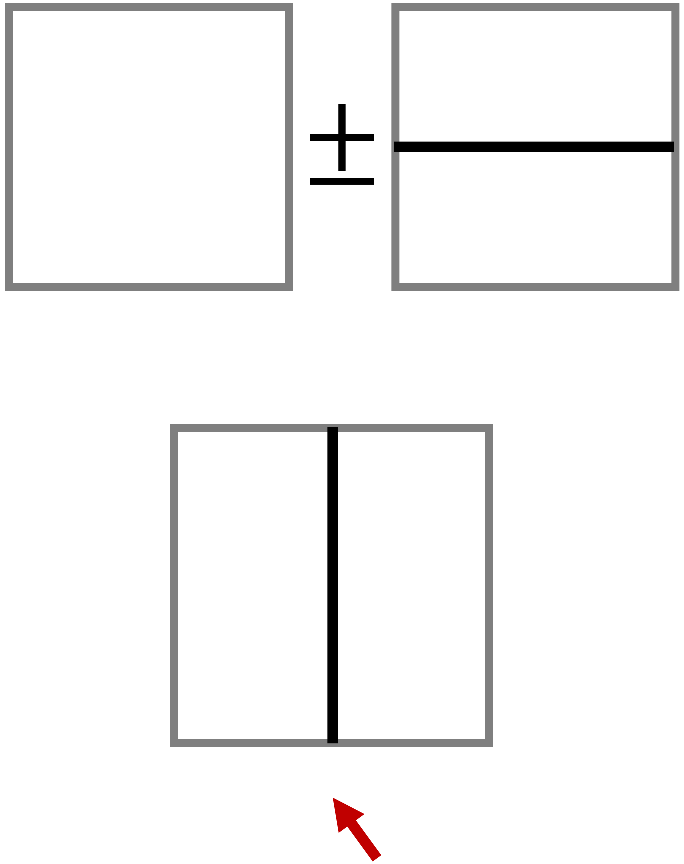
- Modular Transformation

Crossing

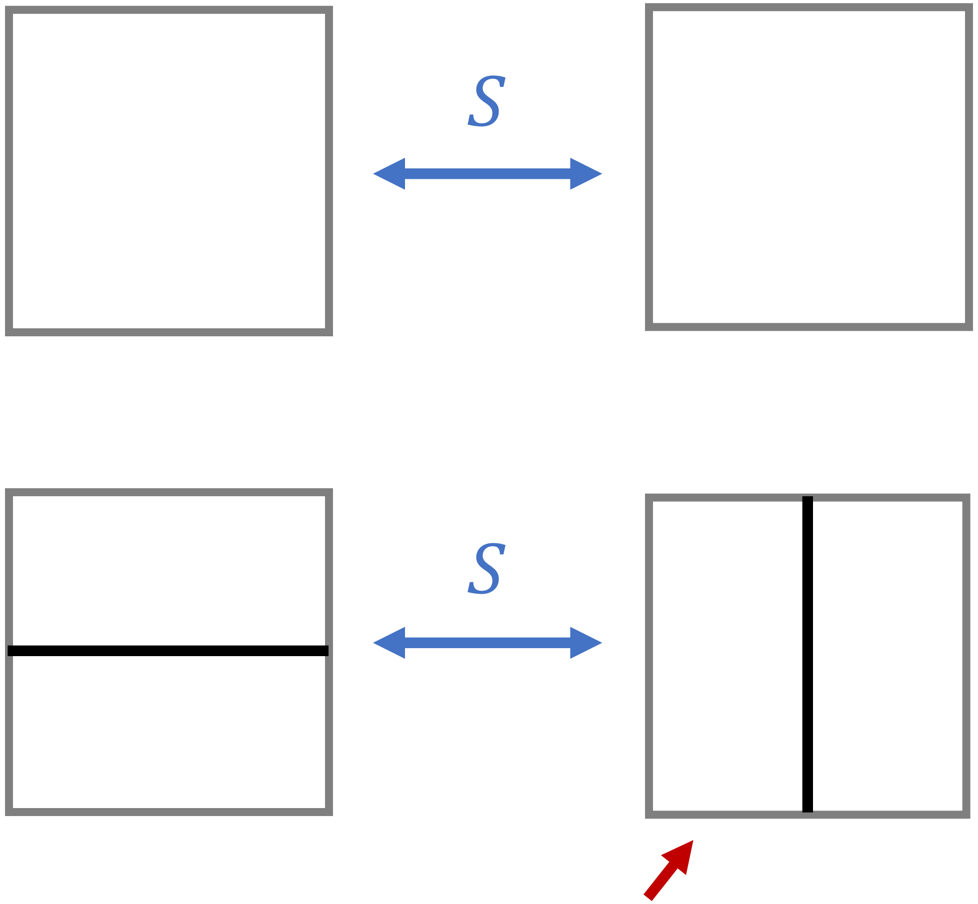


The spins here depend on the anomaly

Positivity



Crossing



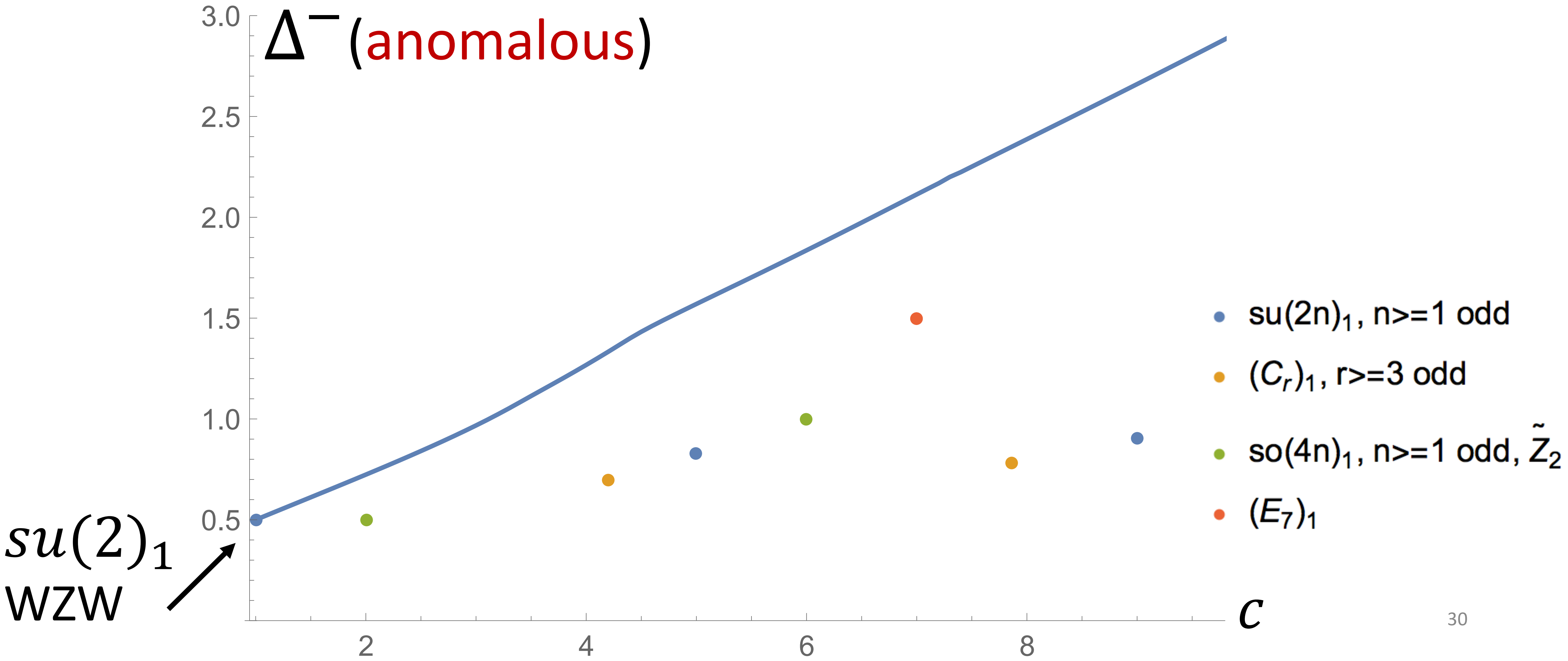
The spins here and there depend on the anomaly

2D CFT with Global Symmetry $\mathbb{Z}_2$

	Non-Anomalous	Anomalous
Bound on Charged Operator	NO 😭	YES 😍

Moral: It is harder to "hide" a symmetry if it is anomalous

Upper Bounds on $\mathbb{Z}_2$ Odd Operators Δ^-



Why is this result interesting? (at least to me…)

- As a Bootstrapper…
- As an Anomaler…

For a Bootstrapper…

- Global symmetry helps us target the CFT we want to bootstrap.

  E.g. 3d O(2) bootstrap [See Zohar's and Leonardo's Talks]

- 't Hooft anomaly is a more refined information for the global symmetry.

- Even the very existence of a bound might depend on the anomaly!

For an Anomaler…

- In a **gapped** phase, discrete anomalies imply that:

    - The symmetry is either spontaneously broken, or
    - There is a TQFT matching the anomaly.

- Rather surprisingly, discrete anomalies also constrain the spectrum of <u>local operators</u> in a **gapless** CFT phase.

Generalization

- 2d CFT with $U(1)$ Global Symmetry
- Higher Dimensions

Generalization

- 2d CFT with $U(1)$ Global Symmetry

- Higher Dimensions

2D CFT with Global Symmetry $U(1)$

- A U(1) Noether current has a holomorphic component $\boldsymbol{J(z)}$ and an antiholomorphic component $\boldsymbol{\bar{J}(\bar{z})}$.

- The topological line generating the U(1) is

$$U_\theta = exp\left[i\theta \oint dz\, J(z) - i\theta \oint d\bar{z}\, \bar{J}(\bar{z})\right]$$

- While $\boldsymbol{J(z)}$ and $\boldsymbol{\bar{J}(\bar{z})}$ are separately conserved, $\partial \bar{J}(\bar{z}) = \bar{\partial} J(z) = 0$, each of them generically generates an $\mathbb{R}$ symmetry, instead of $U(1)$.

2D CFT with Global Symmetry $U(1)$

	Non-Anomalous	Anomalous
Bound on U(1) Charged Operator	NO 👿	YES 😇

Previous Work

- The authors of [Benjamin-Dyer-Fitzpatrick-Kachru] and [Montero-Shiu-Soler] derived a bound on the lightest U(1) charged operator for a **holomorphic** U(1).
- A holomorphic U(1) is always anomalous (chiral anomaly), which is consistent with our observation.
- However, there is ***no*** bound for a non-anomalous U(1) (free boson example).

Example: 2d Compact Boson

[See Leonardo's Discussion Session]

- At any radius R, there are two U(1)'s: the winding $\boldsymbol{U(1)_w}$ and the momentum $\boldsymbol{U(1)_n}$.

- Both U(1)'s are non-anomalous (and non-holomorphic), but there is a mixed anomaly between the two.

- Hence $\boldsymbol{U(1)_{diag}} = diag(\boldsymbol{U(1)_w}$ x $\boldsymbol{U(1)_n})$ is anomalous.

2d c=1 Compact Boson at Radius R

	$U(1)_w$	$U(1)_n$	$U(1)_{diag}$
Lightest Charged Op.	Winding mode	KK mode	Winding or KK mode
Δ^-	$\frac{R^2}{2}$	$\frac{1}{2R^2}$	$\text{Min}[\frac{R^2}{2}, \frac{1}{2R^2}]$
Bound?	NO😭	NO😭	YES😍
Anomaly?	NO	NO	YES

Generalization

- 2d CFT with $U(1)$ Global Symmetry
- Higher Dimensions

Higher Dimensions

- Is there such an anomaly-dependent bound on local operators in higher than **2** dimensions?

- NO! It was shown in [Wang-Wen-Witten 2016] that given a discrete, unitary, bosonic symmetry G and its anomaly α in d dimensions, there is a d-dimensional TQFT carrying this symmetry and anomaly.

Higher Dimensions

- In $d > 2$, these TQFTs have a unique vacuum:

  Trivial local operators, but non-trivial anomalies.

- Hence, discrete, unitary, bosonic anomalies do not constrain local operators in $d > 2$.

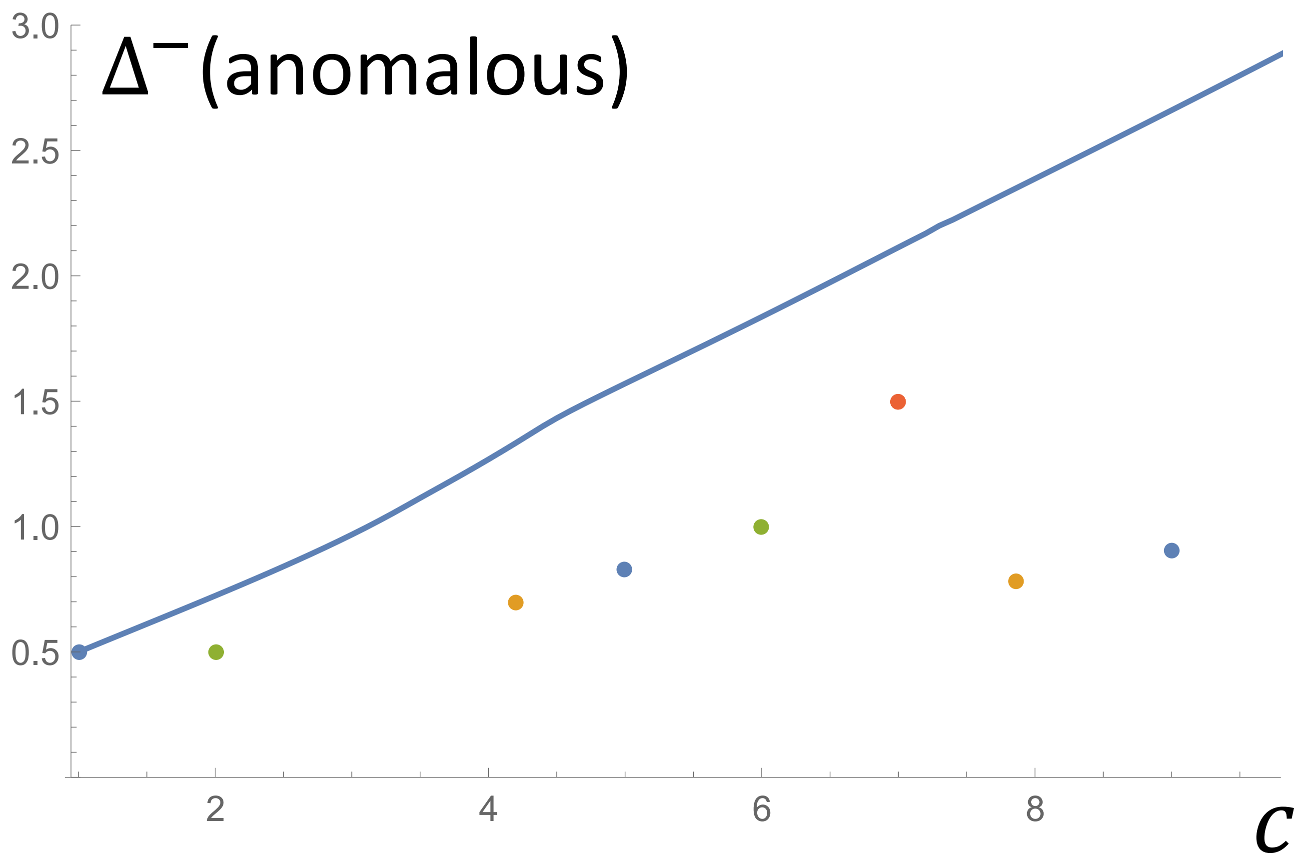
- In $d = 2$, those TQFTs have degenerate vacua (spontaneous symmetry breaking):

  Non-trivial local operators, non-trivial anomalies.

Turning to the non-anomalous case…

- So we have derived a bound for an anomalous $\mathbb{Z}_2$.

- What can we say about a non-anomalous $\mathbb{Z}_2$?



Order-Disorder Bound

- For a non-anomalous $\mathbb{Z}_2$, even though there is no bound on the $\mathbb{Z}_2$ odd operator, there is a bound on the lightest operator in

$$\mathcal{H}^- \oplus \mathcal{H}_L$$

$\mathbb{Z}_2$ odd (order)	Defect Hilbert space (disorder)

- The "order" and "disorder" operators cannot both be too heavy (analogous to [Levin 2019]).

2d Ising CFT [See Leonardo's Talk]

$\sigma(x)$

order

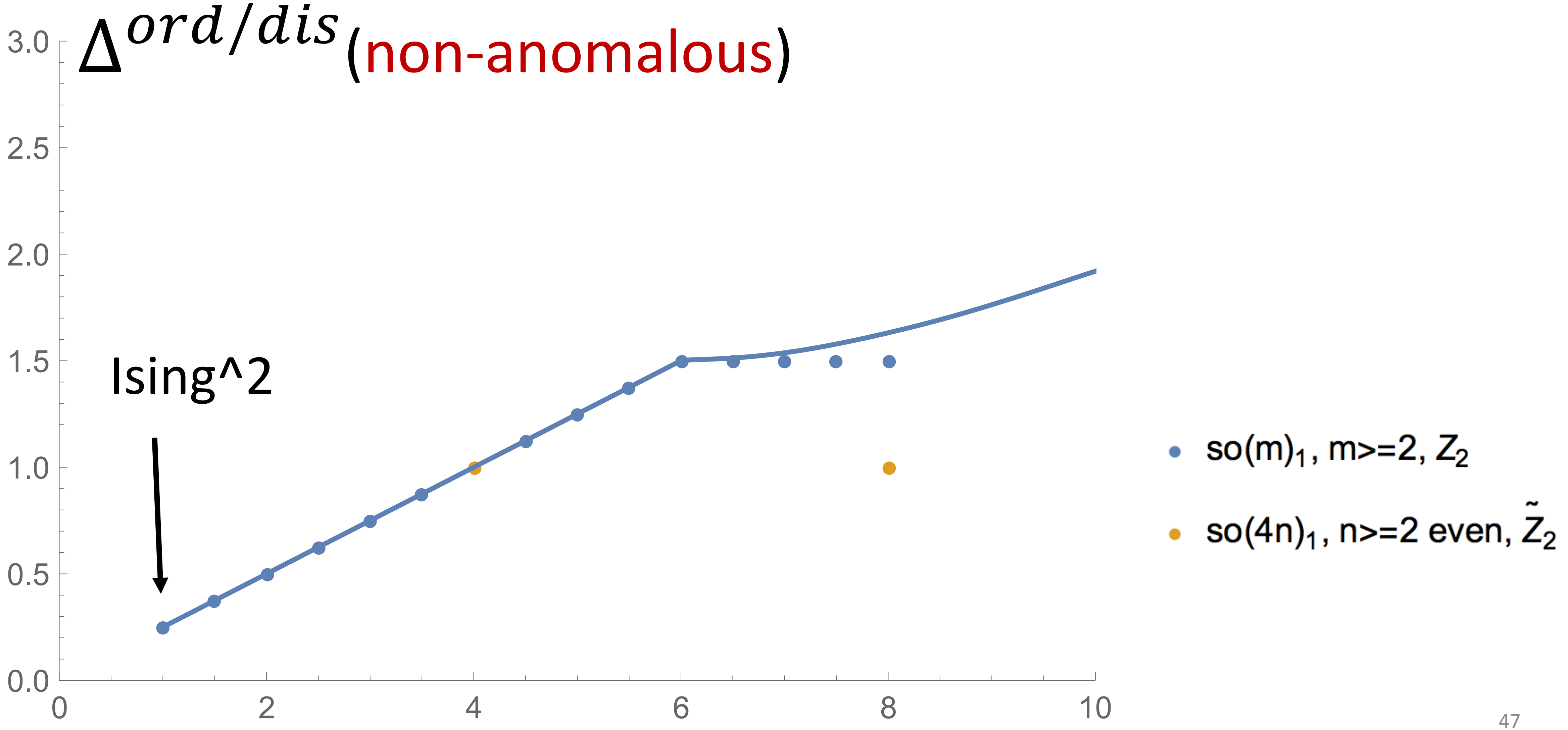
$$h = \bar{h} = \frac{1}{16}$$

$\mu(x)$

disorder

$$h = \bar{h} = \frac{1}{16}$$

Order-Disorder Bound



Conclusion

- Weak Gravity Conjecture Question in CFT: Is there a bound on the lightest charged operator?

- In 2d CFT, there is a bound if the symmetry is anomalous, but not otherwise.

- Discrete 't Hooft anomalies constrain local operators in the gapless phase.

Outlook

- Generalize to anomalies involving spacetime action, e.g. time-reversal anomaly.

- For anomalies that cannot be carried by TQFT, do they constrain bound on charged operators? E.g. Continuous global symmetry in 4d.

- Implications on the Weak Gravity Conjecture in AdS.

  Chern-Simons terms and Confinement.

Thank You!