M-theory S-Matrix from 3d SCFT

Silviu S. Pufu, Princeton University

Based on:

- arXiv:1711.07343 with N. Agmon and S. Chester
- arXiv:1804.00949 with S. Chester and X. Yin
- arXiv:1808.10554 with D. Binder and S. Chester

Also:

- arXiv:1406.4814, arXiv:1412.0334 with S. Chester, J. Lee, and R. Yacoby
- arXiv:1610.00740 with M. Dedushenko and R. Yacoby

Trieste, October 17, 2018

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- Learn about (reconstruct?) gravity / string theory / M-theory from CFT using AdS/CFT.
  - Work toward a constructive proof of AdS/CFT.
- Most well-established examples:
  - 4d $SU(N)$ $\mathcal{N} = 4$ SYM at large N and large 't Hooft coupling / type IIB strings on $AdS_5 \times S^5$
  - 3d $U(N)_k \times U(N)_{-k}$ ABJM theory at large N / M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$.
- Both have maximal SUSY (for ABJM only when $k = 1$ or 2).
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  - using supersymmetric localization;
  - using conformal bootstrap in CFTs.
- Example: using SUSic loc., the S^3 partition function of ABJM theory can be written as a $2N$-dim'l integral [Kapustin, Willett, Yaakov '09]

$$Z = \int d^N\lambda\, d^N\mu \frac{\prod_{i<j} 4\sinh^2(\lambda_i - \lambda_j)\sinh^2(\mu_i - \mu_j)}{\prod_{i,j}\cosh^2(\lambda_i - \mu_j)} e^{i\frac{k}{\pi}\sum_i(\lambda_i^2 - \mu_i^2)}\,.$$

- Expand at large N to find $F = -\log Z = \frac{\pi\sqrt{2}}{3}k^{1/2}N^{3/2} + O(N^{1/2})$.
  - $N^{3/2}$ scaling matches # of d.o.f.'s on N coincident M2-branes as computed using 11d SUGRA.
- Subleading corrections contain info beyond 11d SUGRA. What exactly can we learn from them??

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M-theory S-matrix

- **This talk:** Reconstruct M-theory S-matrix perturbatively at small momentum.
  - scatter gravitons and superpartners in 11d.
- Equivalently, reconstruct the derivative expansion of the M-theory effective action. Schematically,

$$S = \int d^{11}x \sqrt{g} \left[R + \text{Riem}^4 + \cdots + (\text{SUSic completion}) \right] .$$

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- Restrict momenta to be in 4 out of the 11 dimensions.

- In 11d, we can scatter: gravitons, gravitini, 3-form gauge particles.
- Momenta within 4d $\Longrightarrow$ can use 4d $\mathcal{N} = 8$ language. We scatter:
  - graviton (1);
  - gravitinos (8);
  - gravi-photons (28);
  - gravi-photinos (56);
  - scalars ($70 = 35 + 35$)
- At **leading** order in small momentum (i.e. p^2), scattering amplitudes are those in $\mathcal{N} = 8$ SUGRA at tree level. Examples:

$$\mathcal{A}_{\text{SUGRA, tree}}(h^- h^- h^+ h^+) = \frac{\langle 12 \rangle^4 [34]^4}{stu},$$

$$\mathcal{A}_{\text{SUGRA, tree}}(S_1 S_1 S_2 S_2) = \frac{tu}{s},$$

  where $s = (p_1 + p_2)^2$, $t = (p_1 + p_4)^2$, $u = (p_1 + p_3)^2$.
- Amplitudes depend on the particles being scattered, but they're all related by SUSY. (See Elvang & Huang's book.)

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Momentum expansion

- Momentum expansion takes a universal form (independent of the type of particle):

$$\mathcal{A} = \mathcal{A}_{\text{SUGRA, tree}} \Big(1 + \ell_p^6 f_{R^4}(s,t) + \ell_p^9 f_{\text{1-loop}}(s,t) + \ell_p^{12} f_{D^6 R^4}(s,t) + \ell_p^{14} f_{D^8 R^4}(s,t) + \cdots \Big) .$$

- $f_{D^{2n}R^4}$ = symmetric polyn in s, t, u of degree $n+3$
- Known from type II string theory + SUSY [Green, Tseytlin, Gutperle, Vanhove, Russo, Pioline, ...] :

$$f_{R^4}(s,t) = \frac{stu}{3 \cdot 2^7} , \qquad f_{D^6 R^4}(s,t) = \frac{(stu)^2}{15 \cdot 2^{15}} .$$

- $\ell_p^{10} f_{D^4 R^4}$ allowed by SUSY, but known to vanish.
- **This talk:** Reproduce f_{R^4} and $f_{D^4 R^4} = 0$ from ABJM theory.

Flat space limit of CFT correlators

- Idea: Flat space scattering amplitudes can be obtained as limit of CFT correlators [Polchinski '99; Susskind '99; Giddings '99; Penedones '10; Fitzpatrick, Kaplan '11] .
- For a CFT_3 operator $\phi(x)$ with $\Delta_\phi = 1$,

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_{\text{conn}} = \frac{1}{x_{12}^2 x_{34}^2} g(U, V)$$

with $U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$, go to Mellin space

$$g(U, V) = \int \frac{ds\, dt}{(4\pi i)^2} U^{t/2} V^{(u-2)/2} \Gamma^2\left(1 - \frac{s}{2}\right) \Gamma^2\left(1 - \frac{t}{2}\right) \Gamma^2\left(1 - \frac{u}{2}\right) M(s, t)$$

where $s + t + u = 4$.

- From the large s, t limit of $M(s, t)$ one can extract 4d scattering amplitude $\mathcal{A}(s, t)$ [Penedones '10; Fitzpatrick, Kaplan '11] .

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If L is the radius of AdS (and $L/2$ is the radius of S^7), then

$$\mathcal{A}(s,t) = \lim_{L\to\infty} \frac{\mathcal{N} L^7}{\ell_p^9} \int_{c-i\infty}^{c+i\infty} d\alpha \, e^\alpha \alpha^{-1/2} M\left(-\frac{L^2}{4\alpha}s, -\frac{L^2}{4\alpha}t\right)$$

- Expect $M(s,t)$ to have a series expansion in $\ell_p/L \propto N^{-1/6}$.
- At leach order in ℓ_p/L, it is only the large s, t behavior of $M(s,t)$ that contributes to $\mathcal{A}(s,t)$.
- In order for $\mathcal{A}(s,t)$ to have an expansion in ℓ_p times momentum, we need

$$M = \sum_{k=1}^{\infty} \left(\frac{\ell_p}{L}\right)^{7+2k} \text{(function that grows as } k\text{th power of } s, t, u)$$

- Instead of $1/N$ or ℓ_p/L I will use $1/c_T$, where $\langle T_{\mu\nu} T_{\rho\sigma} \rangle \propto c_T$.

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- To obtain scattering amplitude of graviton + superpartners in M-theory, look at **stress tensor multiplet** in $k = 1$ ABJM theory:
- Stress tensor multiplet of any $\mathcal{N} = 8$ SCFT (R-symm is $\mathfrak{so}(8)$):

	dimension	spin	$\mathfrak{so}(8)_R$	couples to
focus on this $\longrightarrow$	1	0	$\mathbf{35}_c$	scalars
	3/2	1/2	$\mathbf{56}_v$	gravi-photinos
focus on this $\longrightarrow$	2	0	$\mathbf{35}_s$	pseudo-scalars
	2	1	$\mathbf{28}$	gravi-photons
	5/2	3/2	$\mathbf{8}_v$	gravitinos
	3	2	$\mathbf{1}$	graviton

- Easier to look at scalars than at operators with spin.
- There are 35 $\Delta = 1$ scalars S_{IJ} (traceless symmetric) and 35 $\Delta = 2$ pseudo-scalars P_{AB} (traceless symmetric).
- Task: find the Mellin amplitudes M_{SSSS} (6 fns), M_{SSPP} (3 fns), M_{PPPP} (6 fns) in the $1/c_T$ expansion, and then take flat space limit.

Ward identity

- $\langle SSSS \rangle = \frac{1}{x_{12}^2 x_{34}^2} \times$ 6 functions $\mathcal{S}_i(U,V)$, $i = 1, \ldots 6$
- $\langle SSPP \rangle = \frac{1}{x_{12}^2 x_{34}^4} \times$ 3 functions $\mathcal{R}_i(U,V)$, $i = 1, \ldots 3$
- $\langle PPPP \rangle = \frac{1}{x_{12}^4 x_{34}^4} \times$ 6 functions $\mathcal{P}_i(U,V)$, $i = 1, \ldots 6$

- The $\mathcal{S}_i(U,V)$ obey differential relations [Dolan, Gallot, Sokatchev '04] . Example:

$$\partial_U \mathcal{S}_4(U,V) = \frac{1}{U}\mathcal{S}_4(U,V) + \left(\frac{1}{U} - \partial_U - \partial_V\right)\mathcal{S}_2(U,V) + \left(\frac{1}{U} + (U-1)\partial_U + V\partial_V\right)\mathcal{S}_3(U,V)\,,$$

$$\partial_V \mathcal{S}_4(U,V) = -\frac{1}{2V}\mathcal{S}_4(U,V) - \frac{1}{V}\left(1 - U\partial_U + (U-1)\partial_V\right)\mathcal{S}_2(U,V) - \left(\partial_U + \partial_V\right)\mathcal{S}_3(U,V)\,.$$

- One can derive differential relations relating $\mathcal{R}_i$ and $\mathcal{P}_i$ to $\mathcal{S}_i$ [Binder, Chester, SSP '18] . (Pretty hard!) Example:

$$\mathcal{R}_1(U,V) = \frac{1}{4}\Big[4 + \left(U^2 - 4U\right)\partial_U + \left(4 + U - 2U^2 + 7UV - 4V^2\right)\partial_V$$
$$+ 2U(2V - U + 2)(U\partial_U^2 + (U + V - 1)\partial_U\partial_V + V\partial_V^2)\Big]\mathcal{S}_1(U,V)\,.$$

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CFT 4-pt correlators at large N

- In general, one cannot compute 4-pt functions using SUSic localization.
- However, the requirements
  - The Mellin amplitudes obey SUSY Ward identities;
  - The Mellin amplitudes are consistent with crossing symmetry;
  - At order $1/c_T^{\frac{7}{9}+\frac{2}{9}n}$, the Mellin amplitude grows at most as the n^{th} power of s, t, u;
  - The Mellin amplitudes have the analytic properties appropriate for tree-level Witten diagrams

determine M_{SSSS}, M_{SSPP}, M_{PPPP} up to a few constants whose number depends on n.

- Number of solutions:

degree in s, t, u	1	2	3	4	5	6	7	...
11D vertex	R			R^4		$D^4 R^4$	$D^6 R^4$	...
scaling	c_T^{-1}			$c_T^{-\frac{5}{3}}$		$(0\times)c_T^{-\frac{19}{9}}$	$c_T^{-\frac{7}{3}}$	
# of params	1			2		3	4	...

(degree 1 in [Zhou '18]); degree ≥ 2 in [Chester, SSP, Yin '18] .)

- The number of solutions matches number of solutions to the Ward identity for the flat space scattering amplitudes.

  So:

- To determine $M(s,t)$ to order $1/c_T$ we should compute **one** CFT quantity.
- To determine $M(s,t)$ to order $1/c_T^{5/3}$ we should compute **two** CFT quantities. Etc.
- Luckily, we can compute **three** CFT quantities (one being c_T) and relate them to $M(s,t)$.

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What to compute

- What to compute: mass deformed S^3 partition function $Z(m_1, m_2) = e^{-F(m_1,m_2)}$ as a function of two masses m_1, m_2.
- How to compute: use SUSic localization to write $Z(m_1, m_2)$ as an integral [Kapustin, Willett, Yaakov '09] ; then use statistical physics techniques to extract $F(m_1, m_2)$ to all orders in the $1/N$ expansion! [Marino, Putrov '11; Nosaka '15]
- The following quantities

$$\left.\frac{\partial^2 F}{\partial m_1^2}\right|_{m_1=m_2=0}, \qquad \left.\frac{\partial^4 F}{\partial m_1^4}\right|_{m_1=m_2=0}, \qquad \left.\frac{\partial^4 F}{\partial m_1^2 \partial m_2^2}\right|_{m_1=m_2=0}$$

can be related to $\langle SSSS\rangle$, $\langle SSPP\rangle$, and $\langle PPPP\rangle$ and thus can be used to determine $M(s, t)$ up to order $1/c_T^{19/9}$.

Mass-deformed S^3 partition function

- ABJM theory at $k = 1$ (or 2) has $\mathfrak{so}(8)_R$ R-symmetry.
- As an $\mathcal{N} = 2$ SCFT, it has $\mathfrak{so}(2)_R$ R-symmetry as well as $\mathfrak{su}(4)$ flavor symmetry.
- So there exists an $\mathfrak{su}(4)$ flavor current multiplet (conserved current j_μ, scalar J with $\Delta = 1$, pseudoscalar K with $\Delta = 2$, fermions).
- Can couple it to an $\mathfrak{su}(4)$ background vector multiplet (A_μ, D, σ, fermions):

$$\int d^3x \ \mathrm{tr}\Big(A_\mu j^\mu + DJ + K\sigma + \text{fermions}\Big) \,.$$

- On S^3, the following background preserves SUSY:

$$A_\mu(x) = 0 \,, \qquad D(x) = \frac{im}{r} \,, \qquad \sigma(x) = m \,, \qquad \text{fermions} = 0 \,.$$

The deformation $\int d^3x \ \mathrm{tr}\, m\Big(\frac{i}{r}J + K\Big)$ is a real mass deformation.

Two mass parameters

- The Cartan of $\mathfrak{su}(4)$ is 3-dimensional, so there are 3 independent mass parameters.
- However, expanded to order m^4, the S^3 free energy takes the form

$$F(m) = f_0 + f_2 \operatorname{tr} m^2 + f_3 \operatorname{tr} m^3 + f_{4,1} (\operatorname{tr} m^2)^2 + f_{4,2} \operatorname{tr} m^4 + O(m^5)\,.$$

- Up to order m^4, we don't lose any info by considering only 2 mass parameters (instead of 3):

$$m = \operatorname{diag}\{m_1, -m_1, m_2, -m_2\}\,.$$

Localization result

- Using supersymmetric localization [Kapustin, Willett, Yaakov '09] :

$$Z_{S^3}(m_1, m_2) = \int d^N\lambda\, d^N\mu\, \frac{e^{ik\sum_i(\lambda_i^2-\mu_i^2)}\prod_{i<j}\sinh^2(\lambda_i-\lambda_j)\sinh^2(\mu_i-\mu_j)}{\prod_{i,j}\cosh(\lambda_i-\mu_j+\frac{m_1}{2})\cosh(\lambda_i-\mu_j+\frac{m_2}{2})}$$

- Small N: can evaluate integral exactly.
- Large N: rewrite $Z_{S^3}(m)$ as the partition function of non-interacting Fermi gas of N particles with [Marino, Putrov '11; Nosaka '15]

$$U(x) = \log(2\cosh x) - m_1 x\,, \qquad T(p) = \log(2\cosh p) - m_2 p\,.$$

Resummed perturbative expansion [Nosaka '15] :

$$Z_{S^3}(m) \sim \mathrm{Ai}\left(f_1(m_1, m_2)N - f_2(m_1, m_2)\right)$$

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Integrated correlators

Relate mass derivatives to $\langle SSSS\rangle$, $\langle SSPP\rangle$, $\langle PPPP\rangle$.

- Recall that the mass deformations are

$$\sum_{i=1}^{2} m_i \int d^3x \left(\frac{i}{r} J_i + K_i\right)$$

- The $\Delta = 1$ operators J_i are linear combinations of the $\mathbf{35}_c$ scalars S_{IJ}.
- The $\Delta = 2$ operators K_i are linear combinations of the $\mathbf{35}_s$ pseudoscalars P_{IJ}.
- $\frac{\partial^2 F}{\partial m_i^2} = -\left\langle \left(\int \frac{i}{r} J_i + \int K_i\right)^2 \right\rangle$ = integrated 2-pt fn.
- $\frac{\partial^4 F}{\partial m_i^2 \partial m_j^2} = -\left\langle \left(\int \frac{i}{r} J_i + \int K_i\right)^2 \left(\int \frac{i}{r} J_j + \int K_j\right)^2 \right\rangle$ = integrated 4-pt fn.

Integrated 2-pt function

- Because S and P belong to the same multiplet as the stress tensor, $\langle T_{\mu\nu} T_{\rho\sigma} \rangle \propto c_T$ implies $\langle SS \rangle \propto c_T$ and $\langle PP \rangle \propto c_T$.
- Then $\langle JJ \rangle \propto c_T$ and $\langle KK \rangle \propto c_T$, and

$$\frac{\partial^2 F}{\partial m_i^2} = \frac{\pi^2}{32} c_T \,.$$

But what does this have to do with the 4-pt function?

- (Super)Conformal block decomposition

$$\langle SSSS \rangle = \frac{1}{x_{12}^2 x_{34}^2} \sum_{\text{multiplets } \mathcal{M}} \lambda_{\mathcal{M}}^2 g_{\mathcal{M}}(U, V)$$

- Then, in a normalization where $\lambda_{\text{id}}^2 = 1$,

$$\lambda_{\text{stress}}^2 = \frac{\langle SST_{\mu\nu} \rangle \langle SST_{\rho\sigma} \rangle}{\langle T_{\mu\nu} T_{\rho\sigma} \rangle} = \frac{256}{c_T} \,.$$

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Four mass derivatives

Expressing the four mass derivatives in terms of c_T:

$$-\frac{1}{c_T^2}\frac{\partial^4 F}{\partial m_1^4} = \frac{3\pi^2}{64}\frac{1}{c_T} + \frac{(3\pi)^{4/3}}{2^{10/3}}\frac{1}{c_T^{5/3}} + \frac{(\cdots)}{c_T^2} - (18\pi^2)^{1/3}\frac{1}{c_T^{7/3}} + \cdots$$

$$-\frac{1}{c_T^2}\frac{\partial^4 F}{\partial m_1^2 \partial m_2^2} = -\frac{\pi^2}{64}\frac{1}{c_T} + \frac{5\pi^{4/3}}{2^{10/3}3^{2/3}}\frac{1}{c_T^{5/3}} + \frac{(\cdots)}{c_T^2} - \frac{4(2\pi^2)^{1/3}}{3^{10/3}}\frac{1}{c_T^{7/3}} + \cdots$$

Impose these constraints on the 4-pt functions $\langle SSSS \rangle$, $\langle SSPP \rangle$, $\langle PPPP \rangle$ using

$$\frac{\partial^4 F}{\partial m_i^2 \partial m_j^2} = -\left\langle \left(\int \frac{i}{r} J_i + \int K_i\right)^2 \left(\int \frac{i}{r} J_j + \int K_j\right)^2 \right\rangle$$

Integrated 4-pt function

- If $\langle AABB \rangle = \frac{1}{x_{12}^{2\Delta_A} x_{34}^{2\Delta_B}} G(U, V)$ in flat space, then the integrated correlator on S^3 (with metric $ds^2 = \Omega^{-2}(\vec{x}) d\vec{x}^2$) is

$$\left\langle \left(\int A \right)^2 \left(\int B \right)^2 \right\rangle = \int \prod_{i=1}^{4} d^3\vec{x}_i \frac{(\Omega(\vec{x}_1)\Omega(\vec{x}_2))^{\Delta_A - 3} (\Omega(\vec{x}_3)\Omega(\vec{x}_4))^{\Delta_B - 3}}{x_{12}^{2\Delta_A} x_{34}^{2\Delta_B}} G$$

with $\Omega(\vec{x}) = 1 + \frac{x^2}{4r^2}$.

- This non-conformal integral can be written as

$$\left\langle \left(\int A \right)^2 \left(\int B \right)^2 \right\rangle \propto \int dU\, dV\, \bar{D}_{3-\Delta_A, 3-\Delta_A, 3-\Delta_B, 3-\Delta_B}(U, V) \frac{G(U, V)}{U^{\Delta_A}}.$$

where $\bar{D}$ function is the (Euclidean) AdS contact Witten diagram for the 4-pt function of ops of dim $3 - \Delta_A$, $3 - \Delta_A$, $3 - \Delta_B$, $3 - \Delta_B$.

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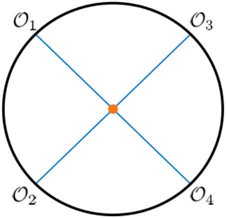
$\bar{D}$ function



$$= D_{r_1,r_2,r_3,r_4}(\vec{x}_i) = \int \frac{dz_0 d^d\vec{z}}{z_0^{d+1}} \prod_{i=1}^4 G_{B\partial}^{r_i}(z_0,\vec{z};\vec{x}_i)$$

$$G_{B\partial}^{r_i}(z_0,\vec{z};\vec{x}_i) = \left(\frac{z_0}{z_0^2 + (\vec{z}-\vec{x}_i)^2}\right)^{r_i}$$

The $\bar{D}$ function is defined as

$$\bar{D}_{r_1,r_2,r_3,r_4}(U,V) = \frac{x_{13}^{\frac{1}{2}\sum_{i=1}^4 r_i - r_4} x_{24}^{r_2}}{x_{14}^{\frac{1}{2}\sum_{i=1}^4 r_i - r_1 - r_4} x_{34}^{\frac{1}{2}\sum_{i=1}^4 r_i - r_3 - r_4}} \frac{2\prod_{i=1}^4 \Gamma(r_i)}{\pi^{\frac{d}{2}} \Gamma\left(\frac{-d+\sum_{i=1}^4 r_i}{2}\right)} D_{r_1,r_2,r_3,r_4}(x_i)$$

- The reason why $\bar{D}$ functions appear in the integrated 4-pt functions on S^3 is that $SO(4,1)/SO(4) = \mathbb{H}^4$.

Summary of computation

- Superconformal Ward id + asymptotic growth in Mellin space + crossing symmetry + analytic structure of Mellin tree amplitudes $\Longrightarrow$ determine Mellin amplitudes M_{SSSS}, M_{SSPP}, M_{PPPP} in $1/c_T$ expansion up to a few undetermined constants at each order
- SUSic localization $\Longrightarrow Z_{S^3}(m_1, m_2) \Longrightarrow F(m_1, m_2)$ in $1/c_T$ expansion $\Longrightarrow$ 2nd and 4th derivatives of $F(m_1, m_2)$ in $1/c_T$ expansion $\Longrightarrow$ conditions on integrated correlators $\langle SSSS \rangle$, $\langle SSPP \rangle$, $\langle PPPP \rangle$.
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- The flat space limit implies $f_{R^4}(s,t) = \frac{stu}{3 \cdot 2^7}$ and $f_{D^4R^4} = 0$, as expected.
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Beyond f_{R^4} and $f_{D^4 R^4}$?

- Can one go beyond reconstructing f_{R^4} and $f_{D^4 R^4}$?
- More SUSic localization results for ABJM theory are available, e.g. partition function on a squashed sphere.
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$$\langle SSSS \rangle = \frac{1}{x_{12}^2 x_{34}^2} \sum_{\text{multiplets } \mathcal{M}} \lambda_{\mathcal{M}}^2 g_{\mathcal{M}}(U, V)$$

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Topological sector

- 3d $\mathcal{N} = 4$ SCFTs have a 1d topological sector [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13; Chester, Lee, SSP, Yacoby '14; Dedushenko, SSP, Yacoby '16] defined on a line $(0, 0, x)$ in $\mathbb{R}^3$.
- $\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$ depends only on the ordering of x_i on the line.
- Ops in 1d are 3d 1/2-BPS operators $\mathcal{O}(\vec{x})$ placed at $\vec{x} = (0, 0, x)$ and contracted with x-dependent R-symmetry polarizations.
- The operators $\mathcal{O}(x)$ are in the cohomology of a supercharge $\mathbb{Q} =$ "$Q + S$" cohomology s.t. translations in x are $\mathbb{Q}$-exact.
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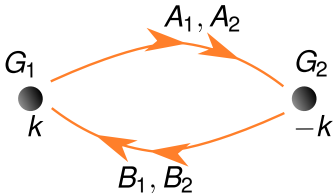
Known $\mathcal{N} = 8$ SCFTs

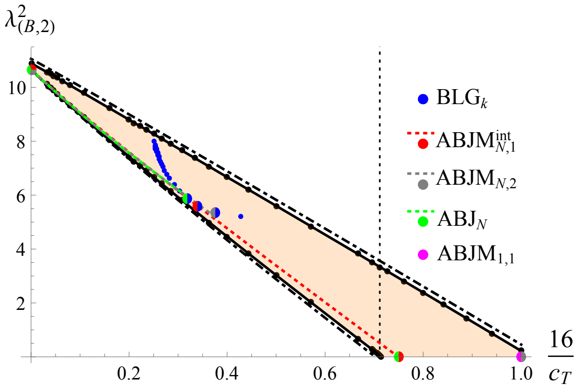
A few families of $\mathcal{N} = 8$ SCFTs:



- With holographic duals:
  - $\text{ABJM}_{N,1}$: $U(N)_1 \times U(N)_{-1} \quad \longleftrightarrow \quad AdS_4 \times S^7$.
  - $\text{ABJM}_{N,2}$: $U(N)_2 \times U(N)_{-2} \quad \longleftrightarrow \quad AdS_4 \times S^7/\mathbb{Z}_2$.
  - $\text{ABJ}_{N,2}$: $U(N)_2 \times U(N+1)_{-2} \quad \longleftrightarrow \quad AdS_4 \times S^7/\mathbb{Z}_2$.
- Without known holographic duals:
  - BLG_k: $SU(2)_k \times SU(2)_{-k}$.

Bootstrap bounds [Agmon, Chester, SSP '17]

- Bounds from conformal bootstrap applying to all $\mathcal{N} = 8$ SCFTs.



- SUGRA (leading large c_T) saturates bootstrap bounds.
- Conjecture: $\text{ABJM}_{N,1}$ or $\text{ABJM}_{N,2}$ or $\text{ABJ}_{N,2}$ saturate bound at all N in the limit of infinite precision.

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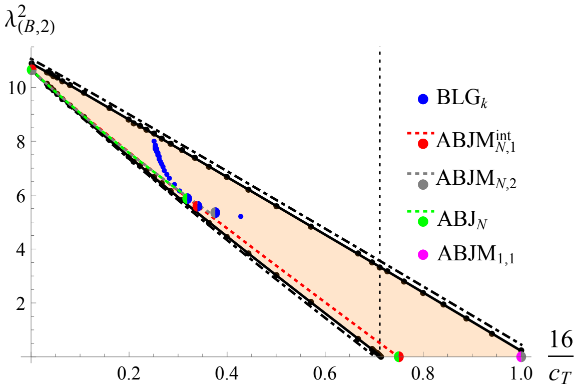

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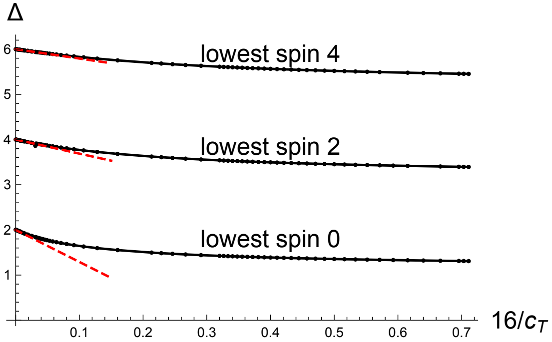
Bound saturation $\Longrightarrow$ read off CFT data

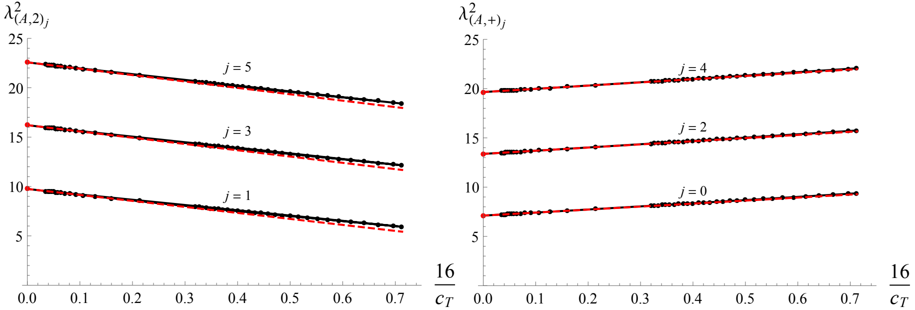
- On the boundary of the bootstrap bounds, the solution to crossing should be unique $\Longrightarrow$ can find $\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle$ and solve for the spectrum !! [Agmon, Chester, SSP '17]



Red lines are leading SUGRA **tree level** results [Zhou '17; Chester '18] .
Lowest operators have the form $S_{IJ}\partial_{\mu_1}\cdots\partial_{\mu_\ell}S^{IJ}$.

$\lambda^2_{(A,2)_j}$ and $\lambda^2_{(A,+)_j}$ from extremal functional

Semishort $(A,2)_j$ and $(A,+)_j$ OPE coefficients for low spin j in terms of $\frac{16}{c_T}$ from extremal functional:



- Red line is tree level SUGRA result [Chester '18] .
- $\lambda^2_{(A,+)_j}$ appears close to linear in $16/c_T$.
- More precision needed.

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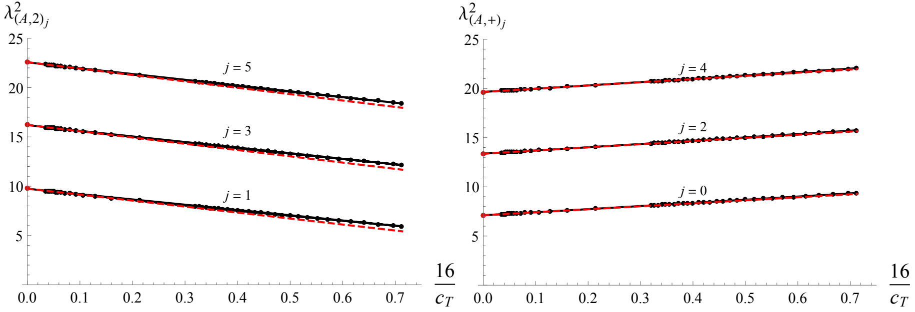

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Conclusion

- A combination of techniques (supersymmetric localization, SUSY Ward identities, Mellin space) can be used to recover graviton scattering amplitudes (at small momentum) from ABJM theory.
- We can reproduce the $f_{R^4}(s,t) = \frac{stu}{3\cdot 2^7}$ term in the flat space 4-graviton scattering amplitude and show that $f_{D^4R^4} = 0$.
- Bootstrap bounds are almost saturated by $\mathcal{N} = 8$ SCFTs with holographic duals.

For the future:

- Generalize to other dimensions, other 4-point function, less SUSY. (See also [Chester, Perlmutter '18] in 6d.)
- Study other SCFTs from which one can compute scattering amplitudes of gauge bosons on branes. (?)
- Loops in AdS.

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