The Bootstrap Program for Defect CFT

Pedro Liendo

October 17 2018

String Seminars in Trieste, ICTP/SISSA.

Motivation

Renormalization group flow

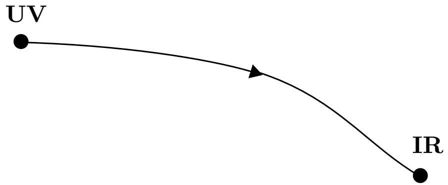

Figure: Renormalization group (RG) flow.

Renormalization group flow

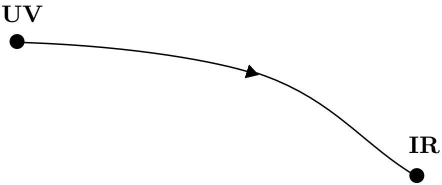

Figure: Renormalization group (RG) flow.

Figure: Kenneth G. Wilson (1936-2013).

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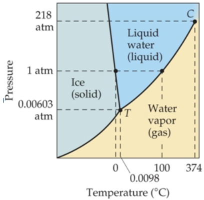
- Critical phenomena.
  Gas-liquid, order-disorder, superfluids.



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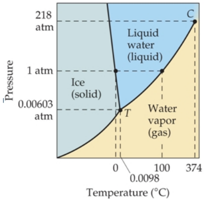

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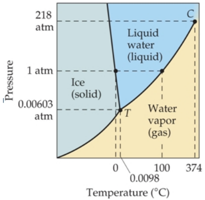

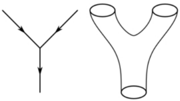

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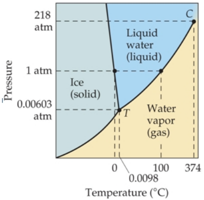

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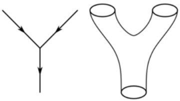

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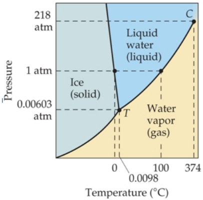

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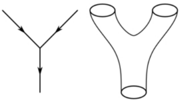

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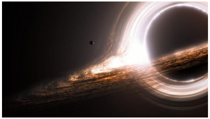
  Black holes, quantum gravity.



Bootstrap basics

The conformal algebra

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Conformal generators

$$\{\mathcal{P}_{\alpha\dot{\alpha}}\,, \mathcal{K}^{\dot{\alpha}\alpha}\,, \mathcal{M}_{\alpha}^{\ \beta}\,, \bar{\mathcal{M}}^{\dot{\alpha}}_{\ \dot{\beta}}\,, \mathcal{D}\}$$

$$\mathcal{O} \to \{\Delta, j, \bar{j}\}$$

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Operators are organized in conformal families

$$\begin{aligned} Primary: &\quad \mathcal{K}^{\dot{\alpha}\alpha} O(0) = 0 \\ Descendants: &\quad \mathcal{P}^{k}_{\alpha\dot{\alpha}} O(0) \end{aligned}$$

CFT correlators

The conformal algebra puts tight restrictions on correlation functions

$$\langle \phi_1(x_1)\phi_2(x_2)\rangle = \begin{cases} \frac{1}{|x_1-x_2|^{2\Delta_\phi}} & \text{if } \Delta_1 = \Delta_2 \\ 0 & \text{if } \Delta_1 \neq \Delta_2 \end{cases},$$

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \frac{C_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}|x_{13}|^{\Delta_1+\Delta_3-\Delta_2}}.$$

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The collection $\{C, \Delta\}$ is the CFT data.

The four-point function is not completely fixed, for identical fields.

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{|x_{12}|^{2\Delta_\phi}|x_{34}|^{2\Delta_\phi}},$$

Operator Product Expansion

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Four-point functions can then be expanded

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \frac{1}{|x_{12}|^{2\Delta_\phi} |x_{34}|^{2\Delta_\phi}} (1 + \sum_{\mathcal{O}} C_{\mathcal{O}}^2 g_{\mathcal{O}}(u, v))$$

where the "conformal block" $g_{\mathcal{O}}(u, v)$ is known (Dolan-Osborn).

Crossing Symmetry

Four-point functions satisfy *crossing symmetry*:

$$v^{\Delta_\phi}\Big(1+\sum_{\mathcal{O}} C_{\mathcal{O}}^2 g_{\mathcal{O}}(u,v)\Big) = u^{\Delta_\phi}\Big(1+\sum_{\mathcal{O}} C_{\mathcal{O}}^2 g_{\mathcal{O}}(v,u)\Big)$$

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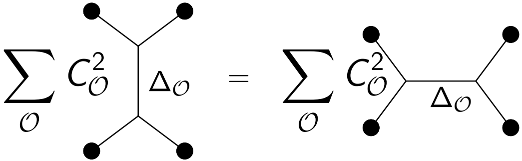
It can be represented pictorially,



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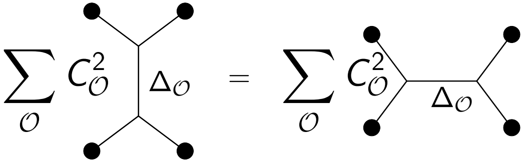

Very constraining system for the CFT data.

Bootstrap techniques

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- **The numerical bootstrap.** Powerful numerical techniques that constrain the low-lying spectrum.
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- **The lightcone bootstrap.** Analytic constraints for operators with high spin.
  (Poland, Kaplan, Komargodski, Fitzpatrick, Simmons-Duffin, Caron-Huot)
- **Solvable truncation.** In supersymmetric theories there is a solvable truncation of the crossing equations.
  (C. Beem, M. Lemos, PL, W. Peelaers, L. Rastelli, B. van Rees.)

Defect CFT

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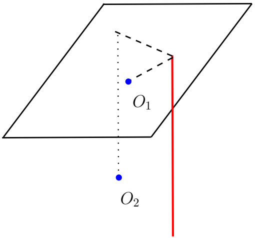

Figure: Local operatos in the presence of a defect.

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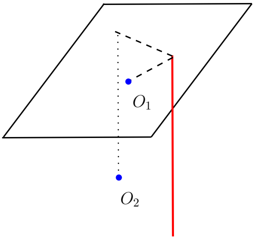

Figure: Local operatos in the presence of a defect.

We have $SO(1, d+1) \to SO(1, p+1) \times SO(q)$ where $q + p = d$.

Defect CFT correlators

The $SO(1, p+1) \times SO(q)$ symmetry preserved by the defect implies that one-point functions are non-zero:

$$\langle \mathcal{O}(x) \rangle = \frac{a_{\mathcal{O}}}{(x^i)^{\Delta}} \,.$$

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Remark. Compare with the four-point function in the bulk CFT.

Two-point function configuration

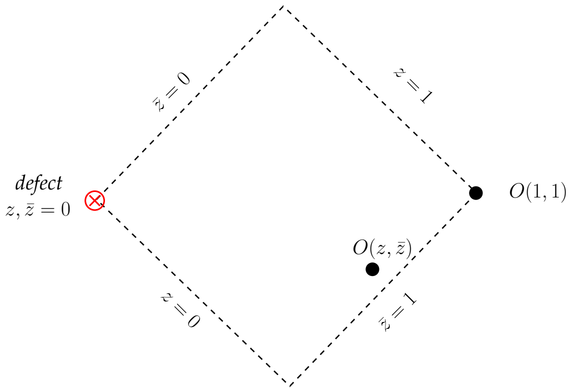

Figure: Configuration of the system in the plane orthogonal to the defect.

Bulk OPE

Bulk channel: We had

$$\phi(x)\phi(0) \sim \sum_{\mathcal{O}} C_{\phi\phi O} d(x, \partial) O(0)\,.$$

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The expansion for the two-point function is

$$\langle \phi(x_1)\phi(x_2) \rangle = \left(\frac{(1-z)(1-\bar{z})}{(z\bar{z})^{1/2}} \right)^{-\Delta_\phi} \sum_{\Delta, J} C_{\phi\phi O} \, a_O \, f_{\Delta,J}(z, \bar{z})$$

where the sum goes over the bulk spectrum.

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Defect channel:
We can also write a bulk operator as a sum of defect operators

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Plugging this expansion into the two-point function,

$$\langle \phi(x_1)\phi(x_2) \rangle = \sum_{\hat{\Delta},s} (b_{\phi\widehat{O}})^2 \, \widehat{f}_{\hat{\Delta},s}(z,\bar{z}) \, .$$

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Equality of both expansions implies

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Pictorially

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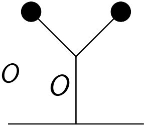

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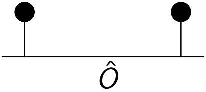

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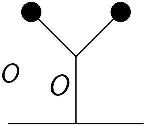

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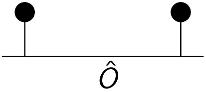

The defect blocks are known in closed-form.

[Billo, Goncalvez, Lauria, Meineri (2016)]

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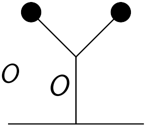

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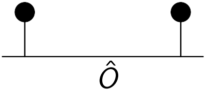

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The bulk blocks are Calogero-Sutherland wave-functions.

[Isachenkov, PL, Linke, Schomerus (2018)]

Inversion Formulas

From Euclidean to Lorentzian

The idea... [Caron-Huot (2017)]

$$z = rw\,, \qquad \bar{z} = \frac{r}{w}$$

$$g(r,w) = \int b(\hat{\Delta}, s) h(r,w) \quad \rightarrow \quad b(\hat{\Delta}, s) \sim \int g(r,w) \bar{h}(r,w)$$

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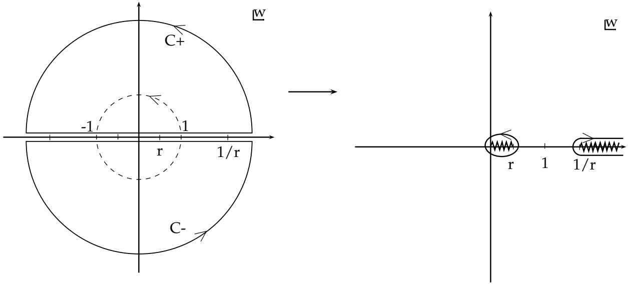

Figure: Contour deformation from Euclidean to Lorentzian configuration.

The lightcone bootstrap

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Higher-twist ($\widehat{\tau} = \widehat{\Delta} - s$) bulk operators are suppressed

$$1 = \lim_{\bar{z} \to 1} \left(\frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}} \right)^{\Delta_\phi} \sum_{\widehat{\tau}, s} (b_{\phi \widehat{O}})^2 \, \widehat{f}_{\widehat{\tau}, s}(z, \bar{z}) \, .$$

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Moreover

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Long story short: we need an *infinite number of defect operators*.

[Lemos, PL, Meineri, Sarkar (2018)]

(Following the lightcone bootstrap)

Universality at large spin

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At large transverse spin s

$$\widehat{\Delta} \sim \Delta_\phi + s \qquad s \to \infty$$

$$b^2_{\phi\hat{O}} \sim s^{\Delta_\phi - 1} \frac{1}{\Gamma(\Delta_\phi)} (\Delta_\phi - \frac{d}{2})! \qquad s \to \infty$$

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At large spin J (consistent with the lightcone bootstrap)

$$\Delta \sim 2\Delta_\phi + J \qquad J \to \infty$$

$$C_{\phi\phi O} a_O \sim \frac{(1 + (-1)^J)}{2^{J+1} (\frac{J}{2})!} \frac{\left(\frac{\Delta_\phi}{2}\right)^2_{\frac{J}{2}}}{J! (2\Delta_\phi + J - 1)_J} \qquad J \to \infty$$

[PL, Linke, Schomerus (to appear)]

More fun stuff

Boundary CFT

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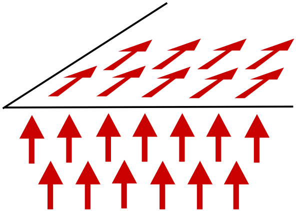

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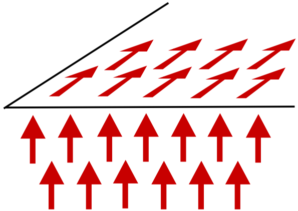
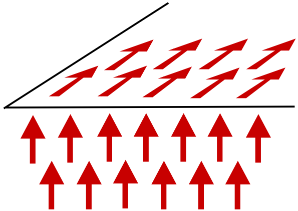


This system could be

$$H = -J_b \sum_{\langle ij \rangle} s_i s_j - J_s \sum_{\substack{\langle ij \rangle \\ \text{surface}}} s_i s_j - H \sum_i s_i - H_1 \sum_{\text{surface}} s_i$$

Boundary CFT

Let's consider



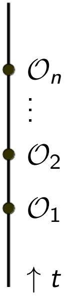
This system could be

$$H = -J_b \sum_{\langle ij \rangle} s_i s_j - J_s \sum_{\substack{\langle ij \rangle \\ \text{surface}}} s_i s_j - H \sum_i s_i - H_1 \sum_{\text{surface}} s_i$$

At criticality we have a BCFT with $SO(d,1)$ symmetry.
This system can be bootstrapped!

[Gliozzi, PL, Meineri, Rago (2015)]

The half-BPS line defect



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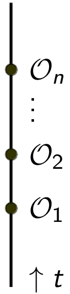
In a $\mathcal{N} = 4$ super Yang-Mills

$$W = \mathrm{tr} P e^{\int dt (iA_t + \phi^6)}$$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_n \rangle = \langle \mathrm{tr} P \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_n e^{\int dt (iA_t + \phi^6)} \rangle$$



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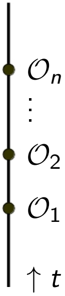

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$$SO(4,2) \to SO(2,1) \times SO(3)$$
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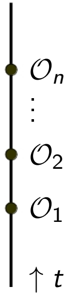

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This system can also be bootstrapped!

[PL, Meneghelli (2016)]
[PL, Meneghelli, Mitev (2018)]

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- We added extended objects! The bootstrap program looks similar to its bulk counterpart.
- On the analytical side we proved universality for defect CFTs at large (transverse) spin.
- We got an idea of the numerical approach to BCFT, and we got a glimpse of a supersymmetric defect.
- **To do**: Develop a numerical algorithm valid for defect CFTs and also non-unitary CFTs. Include spinning objects!

Thank you!