Observables for identity-based tachyon vacuum solutions

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This talk is based on the work in collaboration with

Isao Kishimoto (Niigata University)

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in preparation (arXiv:1408.****)

cf. "Gauge Invariant Overlaps for Identity-Based Marginal Solutions", Isao Kishimoto, Tomohiko Takahashi, Prog. Theor. Exp. Phys. (2013) 093B07.

"Comments on Observables for Identity-Based Marginal Solutions in Berkovits' Superstring Field Theory",
Isao Kishimoto, Tomohiko Takahashi, JHEP07(2014)031.

Introduction

Identity-based tachyon vacuum solution has been constructed by TT-Tanimoto (02), Kishimoto-TT (02).

In SFT2009, we talked about numerical analysis of vacuum structure around the identity-based solution.

In SFT2010, we reported about a vacuum loop amplitude around the identity-based tachyon vacuum.

In SFT2011, we proposed the existence of homotopy operators at the identity-based tachyon vacuum.

All these results provide evidence that, despite "identity-based", it is a correct solution.

However, it is difficult to evaluate observables for the identity-based solutions directly.

As in Kishimoto's talk, we reported about the direct evaluation of observables for the identity-based marginal solutions, which are other types of identity-based solutions.

The important point is a gauge equivalent relation among the identity-based solutions and the Erler-Schnabl type solutions constructed by the KBc algebra and the modified KBc algebra.

A new solution constructed by Maccaferri in 2014 is algebraically identical to the Kiermaier-Okawa-Solar solution and it is gauge equivalent to the identity-based marginal solution.

In this talk, we will apply these procedure to evaluate observables for the identity-based tachyon vacuum solution.

Cubic bosonic open string field theory

Action

$$S[\Psi;\,Q_B] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi * Q_B\Psi + \frac{1}{3}\Psi * \Psi * \Psi\right)$$

string field: $\Psi[X(\sigma),\cdots]$
(functional of string coordinates $X^\mu(\sigma)$)

Q_B: Kato-Ogawa BRST operator

Gauge symmetry

The action is invariant under the gauge transformation

$$\Psi' = e^{-\Lambda} * Q_B e^{\Lambda} + e^{-\Lambda} * \Psi * e^{\Lambda}$$

$$e^{\Lambda} = I + \Lambda + \frac{1}{2!}\Lambda * \Lambda + \frac{1}{3!}\Lambda * \Lambda * \Lambda + \cdots$$

I : identity string field, $\quad I * A = A * I = A$ for $\forall\ A$

Equations of motion

$$Q_B \Psi + \Psi * \Psi = 0$$

This is analogous to the 3-dim CS eom. $F = dA + A \wedge A = 0$.
So, a solution is given as the flat connection, $\Psi = g^{-1} * Q_B g$.

Identity based solutions

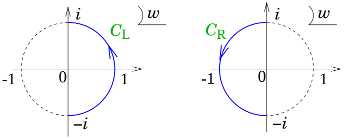
TT-Tanimoto (02), Kishimoto-TT (02), TT-Zeze (03)

$$\Psi_0 = Q_L(e^h - 1)\, I - C_L((\partial h)^2 e^h)\, I$$

where we have defined the operators Q_L and C_L as

$$Q_L(f) = \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i}\, f(z)\, J_{\mathrm{B}}(z), \quad C_L(g) = \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i}\, g(z)\, c(z).$$

The function $h(z)$ has to satisfy $h(-1/z) = h(z)$ and $h(\pm i) = 0$ in order that Ψ_0 is a classical solution. Moreover, the reality condition of Ψ_0 imposes $(h(z))^* = h(1/z^*)$.



Hereafter, let us consider an identity based solution derived from the function,

$$
\begin{aligned}
h(z) &= \log\left(1+\frac{a}{2}\left(z+\frac{1}{z}\right)^2\right) \\
&= -\log(1-Z(a))^2 - \sum_{n=1}^{\infty}\frac{(-1)^n}{n}Z(a)(z^{2n}+z^{-2n}),
\end{aligned}
$$

where $Z(a) = (1+a-\sqrt{1+2a})/a$.

For the solution to be well defined, the parameter a is larger than or equal to $-1/2$.

This function generates the simplest expression of the theory expanded around the identity based solution.

The solution is parametrized by a; $\Psi_0(a) \quad (a > -1/2)$.

The solution has the following properties:

i) It has a well-defined universal Fock space expression.

$$|\Psi_0(a)\rangle = \varphi_0(a)\, c_1 \,|0\rangle + v_0(a)\, c_1 L^X_{-2}\,|0\rangle + u_0(a)\, c_{-1}\,|0\rangle + \cdots$$

ii) It can be represented as a trivial pure gauge configuration for $a > -1/2$, but it can't at $a = -1/2$.

$$\Psi_0(a) = g(a) * Q_B g^{-1}(a)$$

but $g(a)$ is singular at $a = -1/2$! if acting on the Fock space.

The solution at $a = -1/2$ may be given as a kind of singular gauge transform of the trivial configuration.

iii) No open strings on the vacuum of $a = -1/2$

If we expand the string field around the identity based solution as $\Psi = \Psi_0(a) + \Phi$, the action for the fluctuation field Φ is given as

$$S[\Phi;\, Q'(a)] = -\frac{1}{g^2}\int \left(\frac{1}{2}\Phi * Q'(a)\Phi + \frac{1}{3}\Phi * \Phi * \Phi\right),$$

where the kinetic operator is given by

$$Q'(a) = (1+a)Q_B + \frac{a}{2}(Q_2 + Q_{-2}) + 4aZ(a)c_0 - 2aZ(a)^2(c_2 + c_{-2}) \\ -2a(1 - Z(a)^2)\sum_{n=2}^{\infty}(-1)^n Z(a)^{n-1}(c_{2n} + c_{-2n}).$$

To find spectrum on this vacuum, we have to consider the cohomology of the new BRST charge.

We proved the following facts:

1. For $a > -1/2$, the new BRST charge gives rise to the cohomology which has one-to-one correspondence to the cohomology of the original BRST charge.

(The spectrum is unchanged.)

2. At $a = -1/2$, the new BRST charge has vanishing cohomology in the Hilbert space with the ghost number one.

Kishimoto-Takahashi (02)

3. At $a = -1/2$, we find a homotopy operator if extending the space of string fields outside the single Fock space.

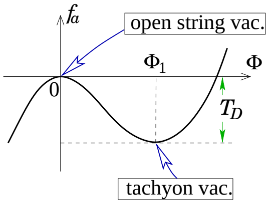
Inatomi-Kishimoto-Takahashi (11)

Therefore, we have no open string excitation in the theory expanded around $a = -1/2$ solution.

iv). Numerical analysis up to level 26 strongly suggests that the expanded theory has the vacuum structure as follows;

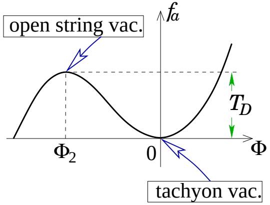
Kishimoto-Takahashi (09), Kishimoto (10)

For $a > -1/2$
(around a trivial pure gauge)



stable solution Φ_1
tachyon vacuum

At $a = -1/2$
(on the tachyon vacuum)



unstable solution Φ_2
perturbative open string vacuum

From all these results i)-iv), it is reasonable to expect that the identity based solution $\Psi_0(a)$ can be regarded as

$a > -1/2 \;\cdots\;$ trivial pure gauge solution

$a = -1/2 \;\cdots\;$ tachyon vacuum solution

Action

Expanding the string field Ψ around the solution as $\Psi = \Psi_0 + \Phi$, we find an action for fluctuation:

$$S[\Psi;\, Q_{\rm B}] \;\;=\;\; S[\Psi_0;\, Q_{\rm B}] + S[\Phi;\, Q'],$$

where the kinetic operator Q' is given by

$$Q' \;\;=\;\; Q(e^h) - C((\partial h)^2 e^h).$$

The operators $Q(f)$ and $C(f)$ are defined as

$$Q(f) = \oint \frac{dz}{2\pi i}\, f(z)\, J_{\rm B}(z), \quad C(g) = \oint \frac{dz}{2\pi i}\, g(z)\, c(z).$$

modified KBc algebra

First, we construct a modified KBc algebra associated with the deformed BRST operator Q':

$$K' = Q'B,\ Q'K' = 0,\ Q'c = cK'c,\ B^2 = 0,\ c^2 = 0,\ \{B, c\} = 1,$$

where B and c are the same string fields in the conventional KBc algebra, and K' is given by

$$K' = \frac{\pi}{2}(K_1')_L I, \quad (K_1')_L = \{Q',\, (B_1)_L\}.$$

The operator $(K_1')_L$ is explicitly calculated as

$$\begin{aligned}(K_1')_L \quad &= \int_{C_{\rm left}} \frac{dz}{2\pi i}(1+z^2)\left\{e^{h(z)}T(z) + (\partial h)e^{h(z)}j_{\rm gh}(z)\right.\\ &\quad \left. + \left(\frac{3}{2}\partial^2 h + \frac{1}{2}(\partial h)^2\right)e^{h(z)}\right\},\end{aligned}$$

K', B, c and Q' have the same algebraic structure as that of the KBc algebra with Q_B.

However, there are exceptional cases that the algebra is more simplified. (cf. Ishibashi san's talk)

For $h_a(z)$, we find

$$e^{h_a(z)} = \frac{1}{(1-Z(a))^2}\{z^2 + Z(a)\}\{z^{-2} + Z(a)\},$$
$$Z(a) = (1 + a - \sqrt{1+2a})/a.$$

$e^{h_a(z)}$ has zeros at $\pm\sqrt{-Z(a)}$ and $\pm 1/\sqrt{-Z(a)}$.

When the parameter a approaches $-1/2$ from positive infinity, $Z(a)$ runs from 1 to -1.

So, the zeros are on $z = \pm 1$ for $a = -1/2$ and then the solution becomes the tachyon vacuum solution.

simplified algebra

Let us consider the relation $Q'c = cK'c$. This relation is derived from

$$\begin{aligned}\{Q(e^h),\, c(z)\} &= e^{h(z)} c\partial c(z), \\ \left[K_1',\, c(z)\right] &= -\partial\{(1+z^2)\} e^{h(z)} c(z) + (1+z^2) e^{h(z)} \partial c(z),\end{aligned}$$

For $a = -1/2$, since the function $e^{h_a(z)}$ has zeros on $z = \pm 1$, $Q(e^{h_a})$ and $c(1)$ anticommute with each other. Similarly, K_1' and $c(1)$ commute.

Consequently, we find simplified algebra only in the $a = -1/2$ case, namely in the theory around the tachyon vacuum solution:

$$K' = Q'B, \quad Q'K' = 0, \quad Q'c = 0,$$
$$B^2 = 0, \quad c^2 = 0, \quad Bc + cB = 1.$$

classical solutions

The equation of motion in the theory around the identity-based solution is given by

$$Q'(a)\Phi + \Phi^2 = 0.$$

We can find various classical solutions by substituting K' into K of solutions in the conventional theory.

In the conventional theory, a general solution using the KBc algebra is written as

$$\Psi_0(K,B,c) \;=\; \mathcal{A}(K)\,c\,\mathcal{B}(K)+\mathcal{C}(K)\,c\,\mathcal{D}(K)\,c\,\mathcal{E}(K)B.$$

Then, a classical solution in the expanded theory is constructed as

$$\Phi_0(K',B,c) \;=\; \mathcal{A}(K')\,c\,\mathcal{B}(K')+\mathcal{C}(K')\,c\,\mathcal{D}(K')\,c\,\mathcal{E}(K')B.$$

Here, we emphasize that in the case that the algebra is simplified (K' commutes with c), the solution has a simpler expression:

Ishibashi san's talk

$$\Phi_0(K', c) = \mathcal{F}(K')c,$$

where $\mathcal{F}(K') = \mathcal{A}(K')\mathcal{B}(K')$ and the second term in $\Phi_0(K', B, c)$ vanishes due to $c^2 = 0$.

the operator $\tilde{q}(h_a)$

To evaluate observables for the $K'Bc$ solutions, we will consider the transformation from K' to K.

First, we introduce the operator, Kishimoto-TT (02), TT-Zeze (03)

$$\tilde{q}(h) = \oint \frac{dz}{2\pi i} h(z) \left(j_{\mathrm{gh}}(z) - \frac{3}{2} z^{-1} \right),$$

Using the operator, $Q'(a)$ is transformed to Q_B

$$e^{-\tilde{q}(h)} Q' e^{\tilde{q}(h)} = Q_B.$$

So, we can remove the term including the ghost number current:

$$\begin{aligned} e^{-\tilde{q}(h)} (K_1')_L \, e^{\tilde{q}(h)} &= \left\{ Q_B, \, e^{-\tilde{q}(h)} (B_1)_L \, e^{\tilde{q}(h)} \right\} \\ &= \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i} (1+z^2) e^{h(z)} T(z), \end{aligned}$$

conformal transformations

Next, we look for a conformal transformation $z' = f(z)$ which maps from the above to $(K_1)_L$.
Since $T(z)$ is a primary field with the dimension 2, it is mapped to

$$f\left[\int_{C_{\rm left}} \frac{dz}{2\pi i}(1+z^2)e^{h(z)}T(z)\right] = \int_{C'_{\rm left}} \frac{df}{2\pi i}(1+z^2)e^{h(z)}\frac{df}{dz}T(f(z)),$$

where $C'_{\rm left}$ is a integration path in the mapped plane.

If this is equal to $(K_1)_L$, $f(z)$ must satisfy the differential equation

$$\frac{df}{1+f^2} = \frac{dz}{(1+z^2)e^{h(z)}},$$

and $C'_{\rm left}$ must remain the same path along the left half of s string.

differential equations

The differential equation has symmetries:

$$
\begin{aligned}
&1. \quad z \;\to\; -\frac{1}{z},\\
&2. \quad f \;\to\; \frac{af+b}{-bf+a}, \qquad (a^2+b^2=1, \quad a,b\in \mathrm{C})
\end{aligned}
$$

The first is a Z_2 transformation derived from $h(-1/z)=h(z)$,
The second forms the group $SO(2,\mathrm{C})$ in which $f=\pm i$ are fixed points.
Moreover, it can be reduced to the homogeneous equation

$$
\frac{dg}{dz}+\frac{2i}{(1+z^2)e^{h_a(z)}}\,g=0, \qquad f=i\frac{2g+i}{2g-i}.
$$

Using these facts, we can solve the equation.

But, for the tachyon vacuum solution, $e^{h_a(z)}$ has zeros on the unit circle and therefore it is impossible to find a regular map f.

Only around the trivial pure gauge case, we can find a regular solution.

Moreover, we can prove that, under the initial condition $f(1) = 1$, the solution $f(z)$ satisfies the following results:

$$\begin{array}{ll} 1. & |z| = 1 \;\Rightarrow\; |f(z)| = 1 \\ 2. & f:\; C_{\text{left}} \;\twoheadrightarrow\; C_{\text{left}} \\ 3. & f\left(-\dfrac{1}{z}\right) = -\dfrac{1}{f(z)}. \end{array}$$

From 1 and 2, we find that the conformal map by the solution f unchanges the integration path, namely $C'_{\rm left} = C_{\rm left}$.

So we can transform the operator to $(K_1)_L$ by the conformal transformation f.

Moreover, the result 3 indicates that the conformal map $f(z)$ is generated by the operators $K_n = L_n - (-1)^n L_{-n}$.

After all, only around the trivial pure gauge, we can construct the transformation

$$U_f e^{-\tilde{q}(h)}(K'_1)_L \, e^{\tilde{q}(h)} U_f^{-1} \;\; = \;\; (K_1)_L,$$

where

$$U_f = \exp\Big(\sum v_n K_n\Big),$$

with certain parameters v_n.

an example of the transformation

We illustrate the existence of the transformation U_f. Under the initial condition $f(1) = 1$, setting $z = e^{i\sigma}$, the solution is given by

$$f(e^{i\sigma}) \;=\; e^{i\phi(\sigma)}, \quad \phi(\sigma) = \sigma + 2\arctan\frac{g(\sigma)\cos\sigma}{1 + g(\sigma)\sin\sigma},$$

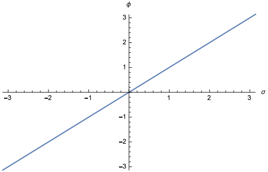
where, for $-1/2 > a \geq 0$ ($-1 < Z(a) \leq 0$), $g(\sigma)$ is given as

$$g(\sigma) \;=\; \tanh\left\{\frac{\sqrt{-Z(a)}}{1+Z(a)}\arctan\left(\frac{2\sqrt{-Z(a)}}{1+Z(a)}\sin\sigma\right)\right\}.$$

For $0 > a$ ($0 < Z(a) < 1$), $g(\sigma)$ is given as

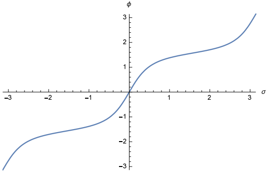
$$g(\sigma) \;=\; -\tanh\left\{\frac{\sqrt{Z(a)}}{1+Z(a)}\text{arctanh}\left(\frac{2\sqrt{Z(a)}}{1+Z(a)}\sin\sigma\right)\right\}.$$

$Z = 0$

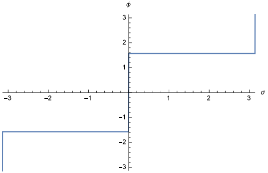


$\phi(\sigma) = \sigma$

$Z = -1/4$



$Z = -1$ $(a = -1/2)$



no regular mapping!

For $a > -1/2$, $\phi(\sigma)$ is a monotonically increasing function. And $\phi(\pm\pi/2) = \pm\pi/2$. So, $C'_{\rm left}$ is unchanged from $C_{\rm left}$.

Here, we should emphasize that the transformation exits only in the case $a > -1/2$ and it doesn't at $a = -1/2$.

The circle-to-circle correspondence for the integration path is broken down at $a = -1/2$. In fact, taking the limit $a \to -1/2$, the phase $\phi(\sigma)$ approaches a step function:

$$\lim_{a\to -1/2} \phi(\sigma) = \begin{cases} \dfrac{\pi}{2} & (0 < \sigma < \pi) \\ -\dfrac{\pi}{2} & (-\pi < \sigma < 0) \end{cases}$$

Then, we can not transform $(K_1')_L$ to $(K_1)_L$ by a regular conformal map at $a = -1/2$.

Observables around the trivial pure gauge

We find that $c(1)$ and $(B_1)_L$ are invariant by the similarity transformation. Using $e^{-\tilde{q}(h)}c(z)e^{\tilde{q}(h)} = e^{-h(z)}c(z)$,

$$U_f e^{-\tilde{q}(h)}c(z)e^{\tilde{q}(h)}U_f^{-1} = e^{h(z)}\left(\frac{df(z)}{dz}\right)^{-1} c(f(z)).$$

From the differential equation, we have

$$= \frac{1+z^2}{1+f(z)^2}\, c(f(z)),$$

and then $c(1)$ is invariant under the transformation ($f(1) = 1$). With regard to $(B_1)_L$, the invariance can be easily seen by using $e^{-\tilde{q}(h)}b(z)e^{\tilde{q}(h)} = e^{h(z)}b(z)$.

Now that the similarity transformation of $(K_1')_L$, $(B_1)_L$ and $c(1)$ is established, we can transform the $K'Bc$ solution to the KBc solution.
An important point is that the generators $\tilde{q}(h)$ and K_n are derivations.
Then, we can find the transformation from the $K'Bc$ algebra to the KBc algebra:

$$K' = e^{\tilde{q}(h)} U_f^{-1} K, \quad B = e^{\tilde{q}(h)} U_f^{-1} B, \quad c = e^{\tilde{q}(h)} U_f^{-1} c.$$

Noting that U_f and $e^{\tilde{q}(h)}$ are given as an exponentials of derivations, we find that the KBc solution is given as:

$$\Phi_0(K', B, c) = e^{\tilde{q}(h)}\, U_f^{-1} \Psi_0(K, B, c).$$

vacuum energy

Using $\Phi_0(K',B,c) = e^{\tilde{q}(h)}\, U_f^{-1}\Psi_0(K,B,c)$, the action for Φ_0 is given by

$$S[\Phi_0(K',B,c);\, Q'] = S[\Psi_0(K,B,c);\, U_f Q_{\rm B} U_f^{-1}],$$

where we have used $Q' = e^{\tilde{q}} Q_B e^{-\tilde{q}}$. Since U_f is generated by the Virasoro operators, $U_f Q_{\rm B} U_f^{-1}$ is equal to $Q_{\rm B}$.

As a result,

the vacuum energy for $\Phi_0(K',B,c)$ is equivalent to that for $\Psi_0(K,B,c)$ in the conventional theory.

gauge invariant observables

The gauge invariant overlap is defined as

$$O_V(\Psi) = \langle I|V(i)|\Psi\rangle,$$

where $V(i)$ is a closed string vertex operator.
$\tilde{q}(h)$ satisfies

$$\langle I|V(i)\,\tilde{q}(h) = 0.$$

In addition, since $U_f V(i) U_f^{-1} = V(f(i)) = V(i)$,

$$\langle I|V(i)\,U_f = \langle I|V(i).$$

So, the overlaps for the $K'Bc$ solution are equivalent to that for the conventional solution:

$$O_V(\Phi_0(K', B, c)) = O_V(\Psi_0(K, B, c)).$$

Observables around the identity-based tachyon vacuum

Let us consider the classical solution in the theory expanded around the identity based tachyon vacuum ($a = -1/2$).
In this case, the modified $K'Bc$ algebra and the classical solution are simplified as mentioned above. $Q'c = 0$ implies that c is a modified BRST closed state. Then, the solution $\Phi_0 = \mathcal{F}(K')c$ is a BRST closed state.
Since Q' has no cohomology at the identity-based tachyon vacuum, the solution can be written as:

$$\Phi_0(K', c) = Q'\chi.$$

We conclude that both of the vacuum energy and the gauge invariant overlaps are zero for the classical solution.

cf. Ishibashi san's talk

a new expression for the identity-based solution

As seen above, in the case that the KBc solution is taken as the tachyon vacuum solution,

$$\Phi_0(K', B, c) = \begin{cases} \text{tachyon vacuum} & (a > -1/2) \\ \text{trivial pure gauge} & (a = -1/2) \end{cases}$$

And, we can find that $\Psi_a = \Psi_0(a) + \Phi_0(K', B, c)$ is a solution of $Q_B \Psi_a + \Psi_a^2 = 0$.

Then, Ψ_a is the tachyon vacuum solution for all a, and so Q_{Ψ_a} has vanishing cohomology.

Differentiating $Q_B\Psi_a + \Psi_a^2 = 0$, we have

$$Q_{\Psi_a} \frac{d}{da}\Psi_a = 0.$$

Since Q_{Ψ_a} has trivial cohomology, by integration, we find

$$\Psi_0(a) + \Phi_0(K', B, c) = \Phi_0(K, B, c) + \int_0^a Q_{\Psi_{a'}} \Lambda_{a'} da',$$

where $\Phi_0(K, B, c)$ is a tachyon vacuum solution like the Erler-Schnabl solution. ($\because\ \Psi(a=0)=0$)

This gives a new expression of the identity-based tachyon vacuum solution.

gauge equivalence

In general, if $\Psi(t)$ and $\Psi\ (= \Psi(t=0))$ are gauge equivalent, we find the following relations:

$$\Psi(t) = g(t)^{-1} Q_B g(t) + g(t)^{-1} \Psi g(t)$$

$$\Psi(t) = \Psi + \int_0^t Q_{\Psi(t')} \Lambda(t') dt'$$

$g(t)$ and $\Lambda(t)$ are connected by

$$g(t) = \mathrm{P} \exp\left(\int_0^t \Lambda(t') dt' \right)$$

Therefore, we conclude that

$\Psi_0(a) + \Phi_0(K', B, c)$ is gauge equivalent to $\Phi_0(K, B, c)$.

From the gauge equivalence, observables for the identity-based tachyon vacuum solution $\Psi_0(-1/2)$ equal to that for the KBc tachyon vacuum solution.

(We note that these can be generalized to the case of general function $h(z)$.)

Conclusions

- We have constructed the modified KBc algebra in the theory around the identity-based solutions.
- The $K'Bc$ algebra is simplified at the identity-based tachyon vacuum.
- The $K'Bc$ solutions can be constructed easily from the $K'Bc$ algebra.
- We have provided a new expression for the identity-based tachyon vacuum solution.
- We can calculate analytically observables for the identity-based tachyon vacuum solution by use of gauge equivalence.