The Ramond Sector of Heterotic String Field Theory

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1. Introduction

♠ WZW-type Superstring field theory

♢ A formulation with *no explicit picture changing operator* and working well for NS sector:

- ∘ Open superstring [Berkovits (1995)]
- ∘ Hetetrotic string [Okawa and Zwiebach (2004), Berkovits, Okawa and Zwiebach (2004)]

However,

♢ Difficult to construct an action for R sector

For Open superstring:

- ∘ EOMs (non-covariant action) [Berkovits (2001)]
- ∘ An action with a constraint [Michishita (2005)]

We construct the EOMs for heterotic string field theory

CONTENTS

2. NS EOM [Berkovits Okawa Zwiebach(2004)]

- Closed string products :

The n-string product is defined by linear maps:

$$\{B_1, \cdots, B_n\} \longrightarrow [B_1, \cdots, B_n], \tag{1}$$

with $|[B_1, \cdots, B_n]| = \sum_i |B_i| + 1$ and $(G, P) = (\sum_i G_i - 2n + 3, \sum_i P_i)$.

- Fundamental relations (L_∞-algebra) :

$$\begin{aligned} 0 =& Q[B_1, \cdots, B_n] + \sum_{i=1}^{n} (-1)^{(|B_1|+\cdots+|B_{i-1}|)} [B_1, \cdots, QB_i, \cdots, B_n] \\ &+ \sum_{\substack{\{i_l, j_k\} \\ l+k=n}} \sigma(i_l, j_k)[B_{i_1}, \cdots, B_{i_l}, [B_{j_1}, \cdots, B_{j_k}]]. \end{aligned} \tag{2}$$

- Derivations :

Operator $X = \eta$ or δ satisfies $[Q, X\} = 0$ and

$$X[B_1, \cdots, B_n] = (-1)^X \sum_{i=1}^{n} (-1)^{X(B_1+\cdots+B_{i-1})} [B_1, \cdots, XB_i, \cdots, B_n]. \quad (3)$$

- NS String Field :

V is Grassmann *odd* with $(G, P) = (1, 0)$.

- Pure-gauge string field: $B_Q = G(V)$ [cf. $A_Q = e^{-\Phi}(Qe^{\Phi})$ in OSSFT]

$$G(V) = QV + \frac{\kappa}{2}[V, QV] + \frac{\kappa^2}{3!}\Big([V, (QV)^2] + [V, [V, QV]]\Big) \cdots ,$$

satisfying (pure gauge solution of *closed bosonic* string EOM)

$$QG(V) + \sum_{n=2}^{\infty} \frac{\kappa^{n-1}}{n!} [(G(V))^n] \equiv 0.$$

• Shifted quantities :

$$Q_G B = QB + \sum_{n=1}^{\infty} \frac{\kappa^n}{n!}[G^n, B],$$

$$[B_1, \cdots, B_n]_G = \sum_{m=0}^{\infty} \frac{\kappa^m}{m!}[G^m, B_1, \cdots, B_n],$$

satisfying the same form of relations with (2).

In contrast, X neither commutes with Q_G nor acts as derivation on $[\cdots]_G$.

$$Q_G(XB) - (-1)^X X(Q_G B) = -\kappa[XG, B]_G, \tag{4}$$

$$X[B_1, \cdots, B_n]_G = \sum_{i=1}^{n} (-1)^{X(1+B_1+\cdots+B_n)}[B_1, \cdots, XB_i, \cdots, B_n]_G + (-1)^X \kappa[XG, B_1, \cdots, B_n]_G. \tag{5}$$

- Useful (MC) equations :

$$\delta G(V) = Q_G B_\delta(V), \tag{6a}$$
$$\eta G(V) = Q_G B_\eta(V). \tag{6b}$$

- NS EOM :

$$\eta G(V) = 0. \tag{7}$$

- Gauge symmetry :

EOM (7) has symmetry under [cf. $A_\delta = e^{-\Phi}(\delta e^{\Phi})$]

$$B_\delta(V) = Q_G \Lambda_0 + \eta \Lambda_1, \tag{8}$$

since

$$\delta(\eta G(V)) = -\kappa[\eta G(V), \eta\Lambda_1]_G. \tag{9}$$

In particular, EOM is *invariant* under Λ_0-gauge tf. because of (6a):

$$\delta_{\Lambda_0} G(V) = Q_G(Q_G \Lambda_0) = 0. \tag{10}$$

• Important identities :

$$Q_G(\eta G) \equiv 0, \tag{11a}$$

$$\eta(\eta G) \equiv 0. \tag{11b}$$

The first identity (11a) is a result of (6b).

3.2 Including R sector [HK arXiv:1312.7197]

- R string field :

  Ψ is Grassmann *odd* with (G,P)=(1,1/2).

- EOMs :

  At the leading order in Ψ (but full order in V), it is natural to assume

$$\eta G(V) + \frac{\kappa}{2}[(\eta\Psi)^2]_G = 0, \tag{12a}$$

$$Q_G \eta\Psi = 0, \tag{12b}$$

which are consistent with (11a) as

$$Q_G\left(\eta G + \frac{\kappa}{2}[(\eta\Psi)^2]_G\right) = -\kappa[\eta\Psi, Q_G\eta\Psi]_G = 0.$$

EOMs (12) have to be consistent also with (11b) but

$$\eta\left(\eta G + \frac{\kappa}{2}[(\eta\Psi)^2]_G\right) = -\frac{\kappa}{2}[(\eta\Psi)^2, \eta G]_G$$
$$= -\frac{\kappa}{2}[(\eta\Psi)^2, \left(\eta G + \frac{\kappa}{2}[(\eta\Psi)^2]_G\right)]_G + \frac{\kappa^2}{4}[(\eta\Psi)^2, [(\eta\Psi)^2]_G]_G$$
$$\neq 0.$$

Thus EOMs (12) have to be corrected. Since RHS is higher-order in Ψ, this is possible order by order in Ψ under the following ansatz.

- G-ansatz :

Recall that the Λ_0-gauge tf.

$$B_\delta(V) = Q_G\Lambda_0, \qquad (13)$$

keeps NS EOM (7) invariant since

$$\delta G(V) = Q_G B_\delta = Q_G(Q_G\Lambda_0). \qquad (14)$$

We assume that this is also true in the full EOMs. That is, the transformation

$$B_\delta(V) = Q_G \Lambda_0, \qquad \delta\Psi = 0, \tag{15}$$

keeps the full EOMs invariant, which is achieved by

G-ansatz

V only appears through $G(V)$ in Q_G and $[\cdots]_G$ in the correction terms.

- **General form** :

From (11a), we can guess that the full EOMs have the form

$$\eta G + \frac{\kappa}{2}[(B_{-\frac{1}{2}})^2]_G + Q_G B_{-1} = 0, \tag{16a}$$

$$Q_G B_{-\frac{1}{2}} = 0, \tag{16b}$$

with

$$B_{-1/2} = \eta\Psi + \mathcal{O}(\Psi^3), \qquad B_{-1} = \mathcal{O}(\Psi^4). \tag{17}$$

We can detemine $B_{-1/2}$ and B_{-1} order by order in Ψ from

$$\eta\left(\eta G+\frac{\kappa}{2}[(B_{-\frac{1}{2}})^2]_G+Q_G B_{-1}\right)=0. \tag{18}$$

For example, in the next order in Ψ, the possible terms are

$$B^{(3)}_{-\frac{1}{2}}=\alpha\kappa^2[\Psi,(\eta\Psi)^2]_G, \qquad B^{(4)}_{-1}=\beta\kappa^3[\Psi,(\eta\Psi)^3]_G. \tag{19}$$

(18) gives

$$\begin{aligned}0=&\eta\left(\frac{\kappa}{2}[(\eta\Psi)^2]_G+\alpha\kappa^3[\eta\Psi,[\Psi,(\eta\Psi)^2]_G]_G+Q_G\left(\beta\kappa^3[\Psi,(\eta\Psi)^3]_G\right)\right)\\ =&(\alpha-4\beta)\kappa^3[\eta\Psi,[(\eta\Psi)^3]_G]_G+\frac{\kappa^3}{4}(1-24\beta)[(\eta\Psi)^2,[(\eta\Psi)^2]_G]_G+\cdots\ .\end{aligned} \tag{20}$$

Hence $\alpha = 1/3!$ and $\beta = 1/4!$. EOMs at the next-leading order in Ψ are

$$\eta G + \frac{\kappa}{2}[(\eta\Psi)^2]_G + \frac{\kappa^3}{3!}[\eta\Psi, [\Psi, (\eta\Psi)^2]_G]_G + Q_G\left(\frac{\kappa^3}{4!}[\Psi, (\eta\Psi)^3]_G\right) = 0, \quad (21)$$

$$Q_G\left(\eta\Psi + \frac{\kappa^2}{3!}[\Psi, (\eta\Psi)^2]_G\right) = 0. \quad (22)$$

We can also determine the gauge transformations.

3.3 First-order EOMs [HK arXiv:1407.0801]

- **First-order EOMs:**

EOMs are also written in the first-order (w.r.t Q and η) form as

$$\hat{Q}\hat{B}+\sum_{m=2}^{\infty}\frac{\kappa^{(m-1)}}{m!}[\hat{B}^m]=0, \tag{23}$$

with

$$\hat{Q}=Q+\eta, \qquad \hat{B}=\sum_{n=0}^{\infty}B_{-n/2}. \tag{24}$$

Eq. (23) holds at each picture number independently, and thus an infinite number of equations for an infinite number of *independent* string fields $B_{-n/2}$ with $n=0,1,2,\cdots$.

Eq. (23) satisfies the consistency equation (cf. Bosonic SFT)

$$\hat{Q}\left(\hat{Q}\hat{B}+\sum_{n=2}^{\infty}\frac{\kappa^{n-1}}{n!}[\hat{B}^n]\right)=-\sum_{m=1}^{\infty}\frac{\kappa^m}{m!}[\hat{B}^m,\left(\hat{Q}\hat{B}+\sum_{n=2}^{\infty}\frac{\kappa^{n-1}}{n!}[\hat{B}^n]\right)]. \tag{25}$$

Let us show that these first-order equations are equaivalent to the previous EOMs by considering (23) in details.

Expanding (23) into different pictures, we obtain the following equations.

At $P = 0$,

$$QB_0 + \sum_{n=2}^{\infty} \frac{\kappa^{n-1}}{n!}[(B_0)^n] = 0, \tag{26}$$

which can be solved by

$$B_0 = G(V). \tag{27}$$

We substitute this in the following equations and define $\tilde{B} = \sum_{n=1}^{\infty} B_{-n/2}$.

At $P = -1/2$ and -1,

$$Q_G B_{-1/2} = 0, \qquad \eta G + \frac{\kappa}{2}[(B_{-1/2})^2]_G + Q_G B_{-1} = 0. \tag{28}$$

Thses have the same form as the previous EOMs (16).

At $P \leq -3/2$, (Subsidiary equations)

$$\eta B_{-n/2} + Q_G B_{-(n+2)/2} + \sum_{m=2}^{\infty} \frac{\kappa^{m-1}}{m!} [\tilde{B}^m]_G \Big|_{-(n+2)/2} = 0, \quad (n \geq 1). \quad (29)$$

If (29) are solved for $B_{-n/2}$ in terms of Ψ (and implicitly V), eqs. (28) give the previous EOMs.

We can show order by order in Ψ that this is indeed the case under

Ansatz

$B_{-1/2} \neq 0$ but $B_{-n/2} = 0$ $(n \geq 2)$ at the linearized level.

Then the nontrivial subsidiary equation at the linearized level is only

$$\eta B^{(1)}_{-1/2} = 0, \tag{30}$$

which can be solved using Ψ as

$$B^{(1)}_{-1/2} = \eta\Psi. \tag{31}$$

Eqs. (28) now become the leading order EOMs (12):

$$Q_G\eta\Psi = 0, \qquad \eta G + \frac{\kappa}{2}[(\eta\Psi)^2]_G = 0. \tag{32}$$

- Next order :

The linearized solution (31) make all the $B_{-n/2}$ non-trivial through the next order subsidiary equations:

$$\eta B^{(n+2)}_{-n/2} = -\frac{\kappa^{n+1}}{(n+2)!}[(\eta\Psi)^{n+2}]_G, \qquad (n \geq 1). \tag{33}$$

can be solved by

$$B^{(n+2)}_{-n/2} = \frac{\kappa^{n+1}}{(n+2)!}[\Psi, (\eta\Psi)^{n+1}]_G, \qquad (n \geq 1), \tag{34}$$

under EOMs (28) up to the η-exact terms which can be gauged away using the invariance under the following gauge invariance.

• Gauge symmetry :

EOM (23) is invariant under

$$\delta\hat{B} = \hat{Q}\hat{\sigma} + \sum_{m=2}^{\infty} \frac{\kappa^{m-1}}{m!}[\hat{B}^m, \hat{\sigma}], \tag{35}$$

with $\hat{\sigma} = \sum_{n=-1}^{\infty} \sigma_{-n/2}$. Parameters $\sigma_{-n/2}$ are $independent$, except for

$$\sigma_{1/2} = Q_G \Lambda_{1/2}, \tag{36}$$

which is constrained by $P = 1/2$ component of (35): $0 = Q_G\sigma_{1/2}$.

The ambiguity from η-exact terms can be gauged away using $\sigma_{-n/2}$ with $n \geq 0$ since (35) has the form

$$\delta B_{-n/2} = \eta\sigma_{-(n-2)/2} + \cdots \,, \qquad (n \geq 1). \tag{37}$$

Note: Ambiguity of $B_{-1/2}$ cannot be gauged away but absorbed into the redefiniton of Ψ.

• Next to next order :

The subsidiary equations at next-to-next order are given as

$$\eta B^{(n+4)}_{-n/2} = -Q_G B^{(n+4)}_{-(n+2)/2} - \sum_{m=1}^{n+1} [(\eta\Psi)^m, B^{(n-m+4)}_{-(n-m+2)/2}]_G, \tag{38}$$

where $B^{(n+2)}_{-n/2}$ is given by (34).

Eqs. (38) can be solved as

$$B_{-n/2}^{(n+4)} = -\frac{\kappa^{n+3}}{(n+4)!}[\Psi, Q_G\Psi, (\eta\Psi)^{n+2}]_G - \frac{\kappa^{n+3}}{(n+4)!}(n+3)[\Psi, (\eta\Psi)^{n+1}, [\Psi, \eta\Psi]_G]_G + \frac{\kappa^{n+3}}{(n+4)!}\sum_{k=0}^{n}\binom{n+3}{k}[\Psi, (\eta\Psi)^k, [\Psi, (\eta\Psi)^{n+2-k}]_G]_G, \quad (39)$$

where the ambiguity from η-exact terms again gauged away.

It has been proved that the procedure can be repeated order by order in Ψ, and eqs. (28) explicitely give the *second-order* EOMs (16).

3.4 Gauge symmetry

The symmetry of the second-order EOMs can also be obtained from (35) as follows.

Expanding (35) into different picture numbers, we obtain, at $P = 0$,

$$\delta B_0 = Q_G \left(\sigma_0 - \kappa[B_{-1/2}, \Lambda_{1/2}]_G\right) - \kappa[Q_G B_{-1/2}, \Lambda_{1/2}]_G. \tag{40}$$

This gives the transformation of V as

$$B_\delta(V) = \sigma_0 - \kappa[B_{-1/2}, \Lambda_{1/2}]_G + Q_G \Lambda_0. \tag{41}$$

Here, however, σ_0 is not independent parameter, as will be shown shortly, since it is needed to fix the ambiguity.

Remaining tfs. at $P \leq -1/2$ can be studied order by order in Ψ.

The ansatz for $B_{-n/2}$ requires that the only σ_0 is non-zero at the linearized level. From (35) at $P=-1$, $0 = \eta\sigma_0^{(0)}$, we obtain, at the leading order in Ψ,

$$\sigma_0^{(0)} = \eta\Lambda_1. \tag{42}$$

Then eq. (35) at $P = -1/2$ gives

$$(\delta B_{-1/2})^{(1)} = \eta\left(Q_G\Lambda_{1/2} - \kappa[\Psi, \eta\Lambda_1]_G\right) - \kappa^2[\eta G, \Psi, \eta\Lambda_1]_G, \tag{43}$$

from which we can read

$$\delta^{(0)}\Psi = Q_G\Lambda_{1/2} - \kappa[\Psi, \eta\Lambda_1]_G + \eta\Lambda_{3/2}. \tag{44}$$

The $\sigma_0^{(0)}$ makes $\sigma_{-n/2}$ non-trivial through (35) at $P \le -1$,

$$\begin{aligned} 0 =& \eta\sigma^{(n)}_{-(n-2)/2} + \frac{\kappa^n}{n!}[(\eta\Psi)^n, \sigma_0^{(0)}]_G \\ =& \eta\left(\sigma^{(n)}_{-(n-2)/2} - \frac{\kappa^n}{n!}[\Psi, (\eta\Psi)^{n-1}, \eta\Lambda_1]_G\right) - \frac{\kappa^n}{n!}[\eta G, \Psi, (\eta\Psi)^{n-1}, \eta\Lambda_1]_G, \end{aligned} \tag{45}$$

which requires

$$\sigma^{(n+2)}_{-n/2} = \frac{\kappa^{n+2}}{(n+2)!}[\Psi, (\eta\Psi)^{n+1}, \eta\Lambda_1]_G, \qquad (n \ge 0). \tag{46}$$

Ambiguity from η-exact terms can again be gauged away using the symmetry of the gauge tfs.

Eq. (35) at $P=-1/2$ gives the next order tf.

$$\delta^{(2)}\bar{B}^{(1)}_{-1/2}+\delta^{(0)}\bar{B}^{(3)}_{-1/2}=(\delta B_{-1/2})^{(3)}+\frac{\kappa^3}{2}[[(\eta\Psi)^2]_G,\Psi,\eta\Lambda_1]_G. \tag{47}$$

We can obtine

$$\begin{aligned}\delta^{(2)}\Psi=&\frac{\kappa^3}{3!}[\Psi,Q_G\Psi,\eta\Psi,\eta\Lambda_1]_G-\frac{\kappa^3}{3!}[\Psi,[\Psi,\eta\Psi,\eta\Lambda_1]_G]_G+\frac{\kappa^3}{3}[[\Psi,\eta\Psi]_G,\Psi,\eta\Lambda_1]_G\\&-\frac{\kappa^2}{3}[\Psi,\eta\Psi,Q_G\Lambda_{\frac{1}{2}}]_G+\frac{\kappa^2}{3!}[\Psi,\eta\Psi,\eta\Lambda_{\frac{3}{2}}]_G.\end{aligned} \tag{48}$$

4. Summary and discussion

Summary

♠ We have constructed EOM and guage tfs. for Heterotic SFT including R sector order by order in Ψ.

♠ Their explicit forms can be systematically obtained using the first-order formulation.

We have also partially constructed

♣ An action with a constraint (in principle),

and explicitly computed

♣ Four point amplitudes with external fermions.

Remaining tasks

★ Constructing an action for the R sector

- Action with the constraint $Q_G \Xi = B_{-1/2}$

  General prescription?

- Action deriving the first-order EOM.

  $\hat{Q}$-cohomology? Democratic formulation?

★ Constructing EOM for the type II superstring