Tachyon Vacuum in Superstring Field Theory

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1 Intro

- Since Schnabl constructed his analytic solution for the tachyon vacuum of open bosonic SFT back in 2005, its been a central problem to construct an analogous solution on a non-BPS D-brane or brane-antibrane pair in superstring field theory.
- There are quite a few formulations of open superstring field theory floating out there, but the king of them all is Berkovits' Wess-Zumino-Witten-like formulation.
- This formulation utilizes the large Hilbert space (including states proportional to the zero mode of the ξ ghost obtained upon bosonization of the $\beta\gamma$ superconformal ghosts) and is nonpolynomial. The equations of motion are

$$\eta\big(g^{-1}Qg\big) = 0 \tag{1.1}$$

  where g is a ghost and picture number zero string field with a star algebra inverse g^{-1}, satisfying $g^{-1}g = 1$, where 1 is the identity string field.
- A solution for the tachyon vacuum, however, has been very difficult to find.
- To construct the tachyon vacuum for the open bosonic string, it is sufficient to have three string fields, K, B, c.
- At ghost number 1, the most general state in this subalgebra takes the form

$$\int_0^\infty dt_1dt_2dt_3\, f(t_1,t_2,t_3)\,\Omega^{t_1}cB\Omega^{t_2}c\Omega^{t_3} \tag{1.2}$$

  Therefore finding the tachyon vacuum only requires specifying an appropriate function of three variables $f(t_1,t_2,t_3)$.
- The KBc subalgebra, however, is not rich enough to describe tachyon condensation on a non-BPS D-brane in superstring theory. At the very least, we need string fields which can give an expectation value to the zero momentum tachyon. In the Berkovits theory, the zero momentum tachyon is represented by the string field

$$\zeta = \gamma^{-1}c \tag{1.3}$$

  where

$$\gamma^{-1} = e^{-\phi}\xi \tag{1.4}$$

  is the inverse of the gamma ghost.

- Taking BRST variations it turns out we need γ as well, so the relevant subalgebra of states for describing tachyon condensation on a non-BPS D-brane is generated by five string fields $K, B, c, \gamma, \gamma^{-1}$.

- Though this looks like only a minor addition to K, B, c, the corresponding subalgebra of states is infinitely more complicated.

- At ghost number zero, for example, we can have states of the form

$$\int_0^\infty dt_1 dt_2 ... dt_{2n+1}\, f(t_1, t_2, ..., t_{2n+1})\, \Omega^{t_1}\gamma\Omega^{t_2}\gamma^{-1}\Omega^{t_3}\gamma\, ...\, \gamma^{-1}\Omega^{t_{2n+1}} \tag{1.5}$$

  Thus it seems we need a function of an infinite number of variables to specify the tachyon vacuum in the Berkovits' theory.

- Though it is actually pretty easy to find a solution at the purely symbolic level, because of the seemingly unbounded complexity of the correlators involved it was unclear whether such symbolic solutions could be given a well-defined meaning as a string field, for example in the Fock space expansion.

- Today we are going to present a solution to this problem.

2 Why we need the solution

- Before we commit ourselves to attempting to solve what looks like a very complicated problem, we should ask why we want to find the tachyon vacuum solution in Berkovits' SFT at all.

- Obviously, it is important to have a proof of Sen's conjectures in superstring field theory, but in bits and pieces one might argue that this problem has been solved for a long time now.

- It turns out that there another formulation of superstring field theory based on a Chern-Simons-like action, analogous to Witten's open bosonic string field theory.

- In this approach, the string field Ψ has picture 0, ghost number 1, and lives in the small Hilbert space: $\eta\Psi = 0$. It satisfies the Chern-Simons equations of motion

$$Q\Psi + \Psi^2 = 0 \tag{2.1}$$

  The only difference from the bosonic string is that Ψ is a state in an $\mathcal{N} = 1$ matter+ghost boundary superconformal field theory, rather than the usual $\mathcal{N} = 0$ matter+ghost BCFT of the bosonic string.

- It turns out that Schnabl's solution for the bosonic string can be generalized, with few changes, to a solution to the Chern-Simons equations of motion for the superstring.
- One can easily show that the superstring version of Schnabl's solution has the correct energy, trivial cohomology, and the expected coupling to closed string states.
- However, one problem with the Chern-Simons formulation is that the action requires picture changing operators at the midpoint, which is widely seen to be problematic. But consider the following.
- Given a Berkovits solution g, we automatically generate a solution to the Chern-Simons equations of motion by taking

  $$\Psi = g^{-1}Qg \tag{2.2}$$

  In particular, we can construct a g such that the corresponding Ψ is the superstring analogue of Schnabl's solution.
- This means that the only information about the Berkovits tachyon vacuum g we need to prove Sen's conjectures comes from the combination $\Psi = g^{-1}Qg$.
- So why do we need an explicit analytic solution for g, when $\Psi = g^{-1}Qg$ seems to suffice and is easy to obtain?
- The answer to this question comes from a puzzling property of the superstring analogue of Schnabl's solution: it does not generate an expectation value for the tachyon on a non-BPS D-brane.
- To see why this is strange, recall the classic picture of the tachyon effective potential on a non-BPS D-brane.
- We expect a smooth double-well potential, with two global minima representing the closed string vacuum, and one local maximum representing the unstable non-BPS D-brane.
- When the tachyon of the non-BPS D-brane rolls off the local maximum, it should come to rest at the minimum with finite tachyon expectation value.
- But for the superstring analogue of Schnabl's solution this doesn't happen. Instead, the tachyon rolls off the maximum, and then must somehow roll backwards "underneath" the local maximum to find a minimum with vanishing tachyon expectation value.
- This indicates that the tachyon effective potential is not a valid concept in the Chern-Simons-like superstring field theory.

- This is unfortunate since the presumed form of the tachyon effective potential is central to our understanding of the physics of tachyon condensation.

- For example, in the field theory of a non-BPS Dp-brane we expect to find a classical solution representing a BPS D$(p-1)$-brane.

- Classically, this solution is visualized as a kink interpolating between the two minima of the tachyon effective potential. As anyone who'se read Sydney Coleman's lectures knows, the kink solution in a double-well potential is a topological soliton. The topological charge of the kink can be identified as the Ramond-Ramond charge of the BPS D$(p-1)$-brane.

- In the Chern-Simons formulation, there is no double-well potential, so it is not clear where the topological charge of the D$(p-1)$-brane is coming from.

- This suggests that to get a full understanding of the relation between tachyon condensation and D-brane charges, we must find the full analytic solution for tachyon condensation in Berkovits' superstring field theory.

- This is what we turn to now.

3 Solution

- To construct the tachyon vacuum, we follow a recipe which has in fact been known for a long time.

- First we start with a solution Ψ of the Chern-Simons-like equations of motion of the superstring. Then we find the corresponding Berkovits g by solving the equation

$$\Psi = g^{-1}Qg \tag{3.1}$$

- Multiplying both sides of this equation by g it can be recast in the form

$$\begin{aligned} 0 &= Qg - g\Psi \\ &= Q_{0\Psi}g \end{aligned} \tag{3.2}$$

  where $Q_{0\Psi}$ is the kinetic operator for a stretched string between the perturbative vacuum 0 and the Chern-Simons-like solution Ψ.

- The operator $Q_{0\Psi}$ is nilpotent, and in fact, in the large Hilbert space, it has no cohomology. Therefore we immediately conclude that g must take the form

$$g = Q_{0\Psi}\beta \tag{3.3}$$

  for some ghost number -1 field β.

- This construction of g is subject to a consistency condition, however. If possible, we must choose β in such a way that $g = Q_{0\Psi}\beta$ has a star algebra inverse g^{-1}.

- With this preparation, the analytic tachyon vacuum of the Berkovits theory follows from the choices

$$\Psi = (c + Q(Bc))\frac{1}{1+K} \tag{3.4}$$

$$\beta = \alpha + q\,\zeta\frac{B}{1+K} \tag{3.5}$$

- Ψ is the "simple" solution for tachyon condensation introduced by M. Schnabl and myself. We can think of it as a streamlined version of Schnabl's solution.

- The field β is composed of two terms. The first, $\alpha = -\gamma^{-2}c$, is the homotopy operator for Q in the large Hilbert space which satisfying $Q\alpha = 1$. The second term gives an expectation value to the zero momentum tachyon ζ, and is proportional to a constant q which serves the role of a gauge parameter in the solution.

- It turns out that for g to have an inverse, q cannot be zero. That is, the tachyon field must acquire an expectation value at the tachyon vacuum.

- Typically, this is where the construction of a tachyon vacuum in the Berkovits theory gets stuck. While choosing Ψ and β gives a relatively straightforward expression for g, one does not, typically, have more than a symbolic definition of g^{-1}.

- In fact, when I found this solution I had already pretty much given up hope on this problem. Rather, I was trying to prove Sen's conjectures without having a fully explicit solution.

- However, this choice of Ψ and β produces some miraculous simplifications which at first were difficult to notice.

- To see what happens, consider string fields of the form

$$M = -\gamma BX_1\zeta + cBX_2 + \gamma BY_1 - cBY_2\zeta \tag{3.6}$$

  where X_1, X_2, Y_1, Y_2 are string fields which commute with B.

- It turns out that string fields of this form multiply exactly like 2×2 matrices, with entries

$$M = \begin{pmatrix} X_1 & Y_1 \\ Y_2 & X_2 \end{pmatrix} \tag{3.7}$$

- With this choice of Ψ and β, it turns out that the solution can be written as the product of a string of 2×2 matrices in this sense:

$$g = \begin{pmatrix} 1 & 0 \\ -q & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & q^2 + K + qV \end{pmatrix} \begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{1+K} \end{pmatrix} \tag{3.8}$$

- Here $V = \frac{1}{2}\gamma^{-1}\partial c$.

- With the help of this matrix structure, the inverse of g can be computed by inspection:

$$g^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1+K \end{pmatrix} \begin{pmatrix} 1 & -q \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{q^2+K+qV} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ q & 1 \end{pmatrix} \tag{3.9}$$

- As a cross-check, note that at $q = 0$ g^{-1} has a factor of $1/K$. As is well known, the field K does not have a star algebra inverse. Thus we see that the tachyon is necessary to have a nonsingular solution for the tachyon vacuum.

- When $q \neq 0$, we can use the Schwinger parameterization to define

$$\frac{1}{q^2 + K + qV} = \int_0^\infty dt\, e^{-q^2 t} e^{-t(K+qV)} \tag{3.10}$$

- The state $e^{-t(K+qV)}$ is essentially a wedge state except that the boundary condition has been modified by a ghost boundary interaction

$$\mathcal{P}e^{-q\int_{\text{boundary}} ds\, \frac{1}{2}\gamma^{-1}\partial c(s)} \tag{3.11}$$

  The appearance of this peculiar boundary interaction was somewhat of a surprise to me, but it appears to play a fundamental role in the solution.

- Though it appears that this boundary interaction places an infinite number of V insertions inside the solution, in fact bc ghost number conservation implies that only a finite number of such insertions contribute to any given correlator.

- Therefore we have a fully explicit and computable solution for the tachyon vacuum in Berkovits' superstring field theory.

4 Kink Charge

- Now let's discuss what the solution has to say about the charge of a BPS D$(p-1)$-brane from the perspective of a non-BPS Dp.

- This example is particularly interesting since the string field theory treatment goes a little beyond the naive picture based on the assumed form of the tachyon effective potential.
- The two global minima of the tachyon effective potential correspond to two tachyon vacuum solutions g and g' related by a sign reversal in the GSO($-$) sector: $g' = (-1)^F g$.
- In the analytic solution, this sign reversal can be described by switching the sign q. That is, once we have chosen a "gauge" where $|q|$ is fixed, the minimum with positive tachyon expectation value represents an analytic solution with $+q > 0$, and the minimum with negative tachyon expectation value represents an analytic solution with $-q < 0$.
- It turns out that the two tachyon vacuum solutions at either side of the effective potential are gauge equivalent. For the analytic solution, the gauge transformation relating them is
$$g' = (g'g^{-1})g \tag{4.1}$$
since the product $g'g^{-1}$ is BRST exact.
- This is perhaps expected, since physically the endpoint of tachyon condensation is the same regardless of which direction the tachyon rolls down the potential.
- But this raises a paradox, since it seems to imply that the BPS D($p-1$) brane is unstable.
- Suppose we form a kink along a noncompact dimension labelled by $x \in \mathbb{R}$. The coordinate x will we transverse to D($p-1$)-brane described by the kink.
- The kink solution $g(x)$ will interpolate from the tachyon vacuum g' to the tachyon vacuum g as x goes from $-\infty \to \infty$:
$$\lim_{x\to-\infty} g(x) = g', \qquad \lim_{x\to\infty} g(x) = g \tag{4.2}$$
- But if g and g' are gauge equivalent, it seems we can turn the kink solution into a lump solution using a gauge transformation which appropriately alters the behaviour towards infinity.
- If this were true, the kink solution would carry no conserved topological charge and would be unstable, in obvious contradiction with the fact that the D($p-1$)-brane is BPS.
- Turning a kink into a lump requires a gauge parameter $V(x)$ satisfying
$$\lim_{x\to-\infty} V(x) = 1, \qquad \lim_{x\to\infty} V(x) = g'g^{-1} \tag{4.3}$$
Apparently $V(x)$ defines a homotopy from $g'g^{-1}$ to the identity.

- This suggests a way out: $g'g^{-1}$ is not homotopic to the identity, and therefore is a *large gauge transformation*. Therefore we cannot turn the kink into a lump, even though the g and g' are gauge equivalent.

- A rigorous understanding of the topology of the space of gauge transformations in the Berkovits theory is far beyond present capabilities. But I can provide some evidence based on the analytic solution that g and g' are topologically disconnected in the gauge orbit.

- We can imagine looking at a subclass of gauge transformations whose only effect on the tachyon vacuum solution is to change the value of q.

- If $g'g^{-1}$ is homotopic to the identity within this subclass of gauge transformations, then we should be able to find a continuous path $q(t), t \in [0, 1]$ with boundary conditions
$$q(0) = -q, \qquad q(1) = q \tag{4.4}$$
such that every tachyon vacuum solution along this path is nonsingular.

- From the expression for g^{-1}, it is clear that constructing a solution for a particular $q(t)$ requires the string field
$$\frac{1}{q(t)^2 + K} \tag{4.5}$$

- It is a well-known fact that well-behaved states in the wedge algebra must (at least) be continuous functions of $K \geq 0$. From this we conclude that $q(t)$ cannot be zero or imaginary.

- But there is no path connecting $-q$ and q which does not pass through the imaginary axis.

- Thus, within the subclass of gauge transformations which change the value of q but otherwise preserve the form of the analytic solution, $g'g^{-1}$ is not homotopic to the identity.

- Thus we are able to gain some concrete insight into the topological origin of D($p-1$)-brane charge.