Covariant Map between RNS and Pure Spinor Superstring Formalisms

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Related papers: 0712.0324, 1305.0693

Nathan Berkovits

ICTP-SAIFR/IFT-UNESP, Sao Paulo

Introduction

- RNS formalism has elegant worldsheet description but spacetime description is complicated:

  spin fields and picture-changing are needed for Ramond vertex operators, spacetime supersymmetry is difficult to see and requires summing over spin structures, unknown how to describe Ramond-Ramond backgrounds, ...

- Pure Spinor formalism has elegant spacetime description but worldsheet description is complicated:

  no reparameterization invariant worldsheet action, unknown origin of BRST operator, complicated B ghost, cannot describe "noncritical" backgrounds unrelated to d=10 sugra, ...
- Covariant map between two superstring formalisms is useful for better understanding both approaches

- In light-cone gauge, can map: $\psi^j \leftrightarrow \theta^a \quad j,a=1 \text{ to } 8$

$$\psi^J = e^{i\sigma_J}, \quad \psi_J = e^{-i\sigma_J} \quad J=1 \text{ to } 4 \qquad \text{(Witten '83)}$$

$$\theta^J = e^{i\sigma_J - \frac{i}{2}(\sigma_1+\sigma_2+\sigma_3+\sigma_4)}, \quad \theta_J = e^{-i\sigma_J + \frac{i}{2}(\sigma_1+\sigma_2+\sigma_3+\sigma_4)}$$

- U(5)-covariant version of this map: $\psi^m \longleftrightarrow (\theta^\alpha, p_\alpha)$

$$\psi^A = e^{i\sigma_A}, \quad \psi_A = e^{-i\sigma_A} \qquad A=1 \text{ to } 5$$

$$\theta^A = e^{i\sigma_A - \frac{i}{2}(\sigma_1+\sigma_2+\sigma_3+\sigma_4+\sigma_5)} e^{\frac{\phi}{2}}, \quad p_A = e^{-i\sigma_A + \frac{i}{2}(\sigma_1+\sigma_2+\sigma_3+\sigma_4+\sigma_5)} e^{-\frac{\phi}{2}}$$

- Remaining 11 components of $(\theta^\alpha, p_\alpha)$ form a "non-minimal quartet" with pure spinors $(\lambda^\alpha, w_\alpha)$
- Related to U(5) hybrid formalism (NB '99), but covariant d=10 description only at classical level (Tonin '91; Matone et al '02; Sorokin '00)

- Will use alternative map that does not require bosonization and "twists" using RNS γ ghost

$$\theta^A = \gamma\psi^A, \quad p_A = \frac{1}{\gamma}\psi_A \qquad A = 1 \text{ to } 5$$

(Baulieu et al `92, `96, `97; NB `94)

- To d=10 covariantize, use pure spinors $(\lambda^\alpha, \bar\lambda_\alpha) \in \frac{SO(10)}{U(5)} \times \mathcal{C}$

$$\lambda\gamma^m\theta = \gamma\frac{(\lambda\gamma^m\gamma^n\bar\lambda)}{2(\lambda\bar\lambda)}\psi_n, \quad \bar\lambda\gamma_m p = \frac{1}{\gamma}(\bar\lambda\gamma_m\gamma_n\lambda)\psi^n$$

- Map is related to ``non-minimal'' RNS formalism with quartet $(\theta^\alpha, p_\alpha; \Lambda^\alpha, \Omega_\alpha)$ and unconstrained Λ^α
- Λ^α might be related to Grassi et al '01; Aisaka et al '03
- Vertex operators for spinors in non-minimal RNS are not spin fields!

Outline

1) Review of pure spinor formalism $(x^m, \theta^\alpha, p_\alpha; \lambda^\alpha, w_\alpha)$
2) "Non-minimal" RNS formalism with variables

$$(x^m, \psi^m; c, b, \gamma, \beta) \quad \text{and} \quad (\theta^\alpha, p_\alpha; \Lambda^\alpha, \Omega_\alpha)$$

3) Map from non-minimal RNS BRST to pure spinor BRST

$$Q = Q_{RNS} + \int \Lambda^\alpha p_\alpha = e^R \int (\lambda^\alpha d_\alpha + \bar{w}^\alpha r_\alpha + \widehat{\gamma} b + u_m \Psi_+^m) e^{-R}$$

$$\Lambda^\alpha = \lambda^\alpha + \frac{1}{2(\lambda\bar{\lambda})} u_m (\gamma^m \bar{\lambda})^\alpha, \quad \widehat{\gamma} = \gamma^2, \quad \Psi_+^m \equiv \frac{(\bar{\lambda}\gamma^m \gamma^n \lambda)}{2\gamma(\lambda\bar{\lambda})} \psi_n$$

4) Gauge-fix non-minimal RNS vertex operators to either RNS or pure spinor vertex operators
5) Relation of RNS and pure spinor amplitude prescriptions
6) Conclusions and open questions

Review of pure spinor formalism

$$S = \int d^2z\left(\frac{1}{2}\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + w_\alpha \bar{\partial}\lambda^\alpha + \bar{w}^\alpha \bar{\partial}\bar{\lambda}_\alpha + s^\alpha \bar{\partial} r_\alpha\right)$$

Pure spinor constraints: $\lambda\gamma^m\lambda = 0, \quad \bar{\lambda}\gamma^m\bar{\lambda} = 0, \quad \bar{\lambda}\gamma^m r = 0$

$$Q = \int dz(\lambda^\alpha d_\alpha + r_\alpha \bar{w}^\alpha)$$

$$d_\alpha = p_\alpha - \frac{1}{2}\left(\partial x^m + \frac{1}{4}(\theta\gamma^m\partial\theta)\right)(\gamma_m\theta)_\alpha$$

$$\Pi^m = \partial x^m + \frac{1}{2}(\theta\gamma^m\partial\theta)$$

$$d_\alpha(y)d_\beta(z) \longrightarrow -(y-z)^{-1}\gamma^m_{\alpha\beta}\Pi_m$$

Super-Yang-Mills vertex operator: $V = \lambda^\alpha A_\alpha(x,\theta)$

$QV = 0, \quad \delta V = Q\Omega$ ⇒ $A_\alpha(x,\theta) = a_m(x)(\gamma^m\theta)_\alpha + \xi^\beta(x)(\gamma^m\theta)_\alpha(\gamma_m\theta)_\beta + \ldots$

- N-point tree amplitude prescription

$$A = \langle V_1(z_1)V_2(z_2)V_3(z_3)\prod_{r=4}^{N}\int dz_r U_r(z_r)\rangle$$

$$U = \partial\theta^\alpha A_\alpha + \Pi^m A_m + d_\alpha W^\alpha + \frac{1}{2}(\lambda\gamma^{mn}w)F_{mn}, \quad QU = \partial V$$

$$A_m = a_m + ..., \quad W^\alpha = \xi^\alpha + ..., \quad F_{mn} = \partial_{[m}a_{n]} + ...$$

- N-point g-loop amplitude prescription

$$A = \prod_{s=1}^{3g-3}\int d\tau_s \langle\prod_{r=1}^{N}\int dz_r U_r(z_r) \ \int dy_s \mu_s(y_s)B(y_s)\rangle$$

$$\{Q, B(y)\} = T(y) = \frac{1}{2}\partial x^m \partial x_m + p_\alpha \partial\theta^\alpha + w_\alpha \partial\lambda^\alpha + s^\alpha \partial r_\alpha + \bar{w}^\alpha \partial\bar{\lambda}_\alpha$$ → arXiv:1305.0693

$$B = \Pi^m \bar{\Gamma}_m - \frac{(\lambda\gamma^{mn}r)}{4(\lambda\bar{\lambda})}\bar{\Gamma}_m\bar{\Gamma}_n + s^\alpha \partial\bar{\lambda}_\alpha + w_\alpha \partial\theta^\alpha - \frac{(w\gamma_m\bar{\lambda})(\lambda\gamma^m\theta)}{2(\lambda\bar{\lambda})} \qquad \bar{\Gamma}^m \equiv \frac{(\bar{\lambda}\gamma^m d)}{2(\lambda\bar{\lambda})} - \frac{(\bar{\lambda}\gamma^{mnp}r)(\lambda\gamma_{np}w)}{4(\lambda\bar{\lambda})^2}$$

Non-minimal RNS formalism

1) Add "non-minimal" quartet $(\theta^\alpha, p_\alpha; \Lambda^\alpha, \Omega_\alpha)$

to usual RNS variables $(x^m, \psi^m; c, b, \gamma, \beta)$

$$Q = \int dz[\Lambda^\alpha p_\alpha + cT + \gamma\psi^m \partial x_m + \gamma^2(b + \Omega_\alpha \partial\theta^\alpha) - bc\partial c]$$

$$T = T_{RNS} - p_\alpha \partial\theta^\alpha - \Omega_\alpha \partial\Lambda^\alpha$$

2) Perform similarity transformation $Q \to e^{-R} Q e^R$

$$R = \int dz \frac{1}{2\gamma}(\Lambda\gamma^m\theta)\psi_m$$

$$Q = \int dz[\Lambda^\alpha d_\alpha + \frac{1}{2\gamma}(\Lambda\gamma^m\Lambda)\psi_m + cT + \gamma\psi^m \Pi_m + \gamma^2(b + \Omega_\alpha \partial\theta^\alpha) - bc\partial c]$$

- BRST operator now has manifest spacetime susy!

$\frac{1}{\gamma}$ dependence in Q??

3) Define pure spinors λ^α and $\bar\lambda_\alpha$ by expressing

$$\Lambda^\alpha = \lambda^\alpha + \frac{1}{2(\lambda\bar\lambda)} u^m (\gamma_m \bar\lambda)^\alpha$$

Can gauge-fix $\bar\lambda_\alpha$ and 5 components of u^m

$$Q = \int dz [\bar w^\alpha r_\alpha + \Lambda^\alpha d_\alpha + \frac{1}{2\gamma}(\Lambda\gamma^m\Lambda)\psi_m + cT + \gamma\Pi_m\psi^m + \gamma^2(b + \Omega_\alpha\partial\theta^\alpha + s^\alpha\partial\bar\lambda_\alpha) - bc\partial c]$$

$$= \int dz [\bar w^\alpha r_\alpha + \lambda^\alpha d_\alpha + cT + \gamma\frac{(\lambda\gamma^m\bar\gamma^n\bar\lambda)}{2(\lambda\bar\lambda)}\Pi_m\psi_n + \gamma^2(b - B) - bc\partial c + u_m(\bar\Gamma^m - \frac{(\bar\lambda\gamma^m\gamma^n\lambda)}{2\gamma(\lambda\bar\lambda)}\psi_n)]$$

$$= e^{-U}\int dz [\bar w^\alpha r_\alpha + \lambda^\alpha d_\alpha + \gamma^2 b - u_m\frac{(\bar\lambda\gamma^m\gamma^n\lambda)}{2\gamma(\lambda\bar\lambda)}\psi_n]\ \ e^U$$

$$U = \int dz [cB + \gamma\bar\Gamma^m\psi_m]$$

$$B = \Pi^m\bar\Gamma_m - \frac{(\lambda\gamma^{mn}r)}{4(\lambda\bar\lambda)}\bar\Gamma_m\bar\Gamma_n + s^\alpha\partial\bar\lambda_\alpha + w_\alpha\partial\theta^\alpha - \frac{(w\gamma_m\bar\lambda)(\lambda\gamma^m\theta)}{2(\lambda\bar\lambda)} \qquad \bar\Gamma^m \equiv \frac{(\bar\lambda\gamma^m d)}{2(\lambda\bar\lambda)} - \frac{(\bar\lambda\gamma^{mnp}r)(\lambda\gamma_{np}w)}{4(\lambda\bar\lambda)^2}$$

4) After similarity transformation by $U = \int dz[cB + \gamma\bar{\Gamma}^m\psi_m]$ non-minimal RNS BRST operator is mapped to

$$Q = \int dz[\lambda^\alpha d_\alpha + \bar{w}^\alpha r_\alpha + \widehat{\gamma} b - u_m \Psi^m_+]$$

$$\widehat{\gamma} \equiv \gamma^2, \quad \Psi^m_+ \equiv \frac{(\bar{\lambda}\gamma^m\gamma^n\lambda)}{2\gamma(\lambda\bar{\lambda})}\psi_n, \quad \Psi^m_- \equiv \frac{\gamma(\lambda\gamma^m\gamma^n\bar{\lambda})}{2(\lambda\bar{\lambda})}\psi_n$$

- Similarity transformation by U maps $b \to b + B$ where B is the composite pure spinor b ghost

5) "Dynamically twist" $\psi^m \to (\Psi^m_+, \Psi^m_-)$, $(\gamma, \beta) \to (\widehat{\gamma}, \widehat{\beta})$
Shift of central charge: $+5 \to -10$ $\quad +11 \to +26$

- $(\widehat{\gamma}, \widehat{\beta}; c, b; u_m, v^m; \Psi^m_+, \Psi^m_-)$ decouple leaving pure spinor BRST operator $Q = \int dz(\lambda^\alpha d_\alpha + \bar{w}^\alpha r_\alpha)$

Vertex Operators

- Non-minimal RNS BRST operator is

$$Q = \int dz[\Lambda^\alpha d_\alpha + \frac{1}{2\gamma}(\Lambda\gamma^m\Lambda)\psi_m + cT + \gamma\psi^m\Pi_m + \gamma^2(b + \Omega_\alpha\partial\theta^\alpha) - bc\partial c]$$

- Massless super-Yang-Mills vertex operator is

$$V = \Lambda^\alpha A_\alpha(x,\theta) - \gamma\psi^m A_m(x,\theta) - \gamma^2\Omega_\alpha W^\alpha(x,\theta)$$

$$+c[\partial\theta^\alpha A_\alpha + \Pi^m A_m + d_\alpha W^\alpha + (\psi^m\psi^n + \frac{1}{2}\Lambda\gamma^{mn}\Omega)F_{mn} - \gamma\psi^m\Omega_\alpha\partial_m W^\alpha]$$

- In gauge $\theta^\alpha = \Lambda^\alpha = 0$, reduces to NS vertex operator $V = \gamma\psi^m a_m(x) + c(\partial x^m a_m + \psi^m\psi^n\partial_{[m}a_{n]})$
- In gauge $u^m = \Psi_+^m = \widehat{\gamma} = c = 0$, reduces to pure spinor vertex operator $V = \lambda^\alpha A_\alpha(x,\theta)$ with integrated operator $U = \partial\theta^\alpha A_\alpha + \Pi^m A_m + d_\alpha W^\alpha + \frac{1}{2}(\lambda\gamma^{mn}w)F_{mn}$

- GSO(+) RNS states do not contain $\frac{1}{\gamma}$ dependence in the zero picture
- GSO(-) states and GSO(+) states in other pictures appear to require $\frac{1}{\gamma}$ dependence
- Spacetime susy acts covariantly on GSO(+) states
- Relation of Ramond states with usual Ramond vertex operators involving spin fields is unclear
- As in pure spinor and GS formalisms, Ramond-Ramond backgrounds are constructed by taking left-right product of integrated super-Yang-Mills vertex operators $\int d^2z|\partial\theta^\alpha A_\alpha + \Pi^m A_m + d_\alpha W^\alpha + (\psi^m\psi^n + \frac{1}{2}\Lambda\gamma^{mn}\Omega)F_{mn} - \gamma\psi^m\Omega_\alpha\partial_m W^\alpha|^2$

and adding to superstring worldsheet action

Relating the amplitude prescriptions

- String theories with chiral bosons require BRST-invariant regulators for functional integration over noncompact bosonic zero modes
- Regulator cancels infinities from bosonic zero modes with zeros from fermionic zero modes
- Ex. 1: RNS picture-changing operators

$$Z_\beta = \{Q, \xi\} = \delta(\beta)(\partial x^m \psi_m + ...), \quad Y_\gamma = c\delta'(\gamma) = c\partial\xi e^{-2\phi}$$

- Ex. 2: Pure spinor regulator

$$\mathcal{N}_{\lambda,\bar{\lambda},w,\bar{w}} = e^{\{Q,\chi\}} = e^{-\lambda^\alpha \bar{\lambda}_\alpha - r_\alpha \theta^\alpha - w_\alpha \bar{w}^\alpha - s^\alpha d_\alpha} \quad \chi = -\bar{\lambda}_\alpha \theta^\alpha - w_\alpha s^\alpha$$

- Amplitude is independent of choice of regulator

- Claim: Up to global issues, RNS and pure spinor amplitude prescriptions coincide after including BRST-invariant regulators for chiral bosons

Proof: Before twisting, $A_{nonmin} = A_{RNS}$

After twisting, $A_{nonmin} = A_{pure}$

- N-point tree amplitude:

$$A^{before}_{nonmin} = \langle (Y_\Lambda)^{16} (Y_\gamma)^2 V_1 V_2 V_3 \prod_{r=4}^{N} \int dz_r U_r \rangle = A_{RNS}$$

$$(Y_\Lambda)^{16} = [\delta(\Lambda^\alpha)\theta^\alpha]^{16}$$

$$A^{after}_{nonmin} = \langle (Y_u)^5 (Y_{\widehat{\gamma}})^3 \mathcal{N}_{\lambda,\bar{\lambda}} V_1 V_2 V_3 \prod_{r=4}^{N} \int dz_r U_r \rangle = A_{pure}$$

$$(Y_u)^5 = [\delta(u^m)\Psi^m_-]^5$$

$$Y_{\widehat{\gamma}} = \delta(\widehat{\gamma})c$$

- Regulators fix the chiral boson zero modes and absorb the non-minimal fermionic zero modes

- N-point g-loop amplitude:

$$A^{before}_{nonmin} = \int d^{3g-3}\tau \langle (Z_\Omega)^{16g} (Z_\beta)^{2g-2} (Y_\Lambda)^{16} (\int \mu b)^{3g-3} \prod_{r=1}^{N} \int dz_r U_r \rangle = A_{RNS}$$

$$A^{after}_{nonmin} = \int d^{3g-3}\tau \langle (Z_v)^{5g} (Z_{\widehat{\beta}})^{3g-3} (Y_u)^5 \mathcal{N}_{\lambda,\bar\lambda,w,\bar w} (\int \mu b)^{3g-3} \prod_{r=1}^{N} \int dz_r U_r \rangle = A_{pure}$$

$$Z_\Omega = \delta(\Omega_\alpha)[Q,\Omega_\alpha] = \delta(\Omega_\alpha)(d_\alpha + \ldots), \quad Z_\beta = \delta(\beta)[Q,\beta] = \delta(\beta)(\psi_m \partial x^m + \ldots)$$

$$Z_v = \delta(v^m)[Q,v^m] = \delta(v^m)(\Psi^m_+ + \ldots), \quad Z_{\widehat{\beta}} = \delta(\widehat{\beta})[Q,\widehat{\beta}] = \delta(\widehat{\beta})(b - B + \ldots)$$

- Inserting $(Z_{\widehat{\beta}})^{3g-3}$ at same locations as $(\int \mu b)^{3g-3}$ contributes $(\int \mu B)^{3g-3}$ after integrating out the non-minimal $(b, c\, ; \widehat{\beta}, \widehat{\gamma})$ variables.
- Global subtleties coming from non-split RNS supermoduli and $\frac{1}{(\lambda\bar\lambda)}$ poles have been ignored

Conclusions and Open Questions

- Covariant map between RNS and pure spinor formalism relates fermionic vector and spinor by

$$\Psi_+^m \equiv \frac{(\bar\lambda\gamma^m\gamma^n\lambda)}{2\gamma(\lambda\bar\lambda)}\psi_n \longleftrightarrow \bar\Gamma^m \equiv \frac{(\bar\lambda\gamma^m d)}{2(\lambda\bar\lambda)} - \frac{(\bar\lambda\gamma^{mnp}r)(\lambda\gamma_{np}w)}{4(\lambda\bar\lambda)^2}$$

- Non-minimal RNS formalism constructed whose BRST operator has manifest spacetime susy and connects RNS and pure spinor BRST operators
- Non-minimal RNS vertex operators reduce in different gauges to either RNS or pure spinor vertex operators

- Non-minimal RNS BRST operator contains $\frac{1}{\gamma}$ dependence, but GSO(+) vertex operators do not
- Relation of Ramond vertex operators in non-minimal and usual RNS formalisms is unclear
- Amplitudes in non-minimal RNS formalism are related before twisting to RNS amplitudes and after twisting to pure spinor amplitudes
- Twisting procedure maps $(\psi^m; \gamma, \beta) \to (\Psi^m_+, \Psi^m_-; \widehat{\gamma}, \widehat{\beta})$ without shifting the central charge $\widehat{\gamma} = \gamma^2$
- Subtleties associated with non-split RNS supermoduli space and $\frac{1}{(\lambda\bar{\lambda})}$ poles in pure spinor multiloop amplitudes have been ignored