Higher order singletons & Partially massless fields

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Outline

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1. **Higher-spin holography**

   Conjecture [Sezgin-Sundell/Klebanov-Polyakov '02]
   free/critical large-N vector model $\leftrightarrow$ Vasiliev's higher-spin gravity

   Evidences
   - **kinematical** (free singleton)
     - *spectrum* [Flato-Fronsdal theorem '78]
       tensor product of two scalar singletons $\leftrightarrow$ tower of massless bosons
     - *symmetry* [Eastwood theorem '02]
       free scalar singleton symmetry algebra $\leftrightarrow$ bosonic higher-spin algebra
   - **dynamical** (vector model)
     - *correlators*
       - match of 3-point functions [Giombi-Yin '10]
       - Coleman-Mandula-like theorem
         [Maldacena-Zhiboedov '11, Todorov '12, Stanev '13, ...]
       - unfolding
         [Colombo-Sundell '12, Didenko-Skvortsov '12, Gelfond-Vasiliev '13]
     - *free energy & Casimir energy*
       - (vanishing) sum of 1-loop vacuum-bubbles for the higher-spin multiplet
         [Giombi-Klebanov '13, + Safdi '14, + Tseytlin '14]

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1. **Higher-spin holography**
2. **Higher-order generalizations** of higher-spin holography to multicritical vector models at isotropic Lifshitz points provide a realization of the (A)dS/CFT dictionary between partially massless fields [Deser-Waldron '01] and partially conserved currents [Dolan-Nappi-Witten '01] of all spins.

   The ambient approach (manifestly $\mathfrak{o}(d, 2)$ covariant) to free:

   - **Higher-order** singletons
   - **Partially** massless fields & conserved currents
   - **Higher-depth** shadow fields & conformal Killing tensors

   allows to check standard kinematical evidences (match of spectrum and symmetries) and the existence of corresponding nonlinear Vasiliev's equations.

Higher-spin holography

Growing body of evidence [Petkou-Sezgin-Sundell, '03; Giombi-Yin, '10; ...] that [Sezgin-Sundell/Klebanov-Polyakov conjecture, '03]:
the bulk dual of the singlet sector of the free/critical O(N) models in 3 dimensions should be Vasiliev's minimal higher-spin gravity on AdS_4**.**

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the bulk dual of the singlet sector of the free/critical O(N) models in 3 dimensions should be Vasiliev's minimal higher-spin gravity on AdS_4.

Remarks: Vasiliev's equations are simpler in dimensions 4 (or less) for technical reasons (twistor realizations of higher-spin superalgebras).
The same is true for CFTs in dimensions 3 (or less).
Thus most works and generalizations are restricted to AdS_4/CFT_3 (or AdS_3/CFT_2).
Nevertheless, *bosonic* higher-spin gravity has been constructed for *any* dimension (and for any internal classical compact group) [Vasiliev, '03].

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Nevertheless, *bosonic* higher-spin gravity has been constructed for *any* dimension (and for any internal classical compact group) [Vasiliev, '03].
So the natural generalization to any dimension is:
the bulk dual of the singlet sector of large-N vector models in $d > 2$ dimensions should be Vasiliev's higher-spin gravity on AdS_{d+1}.

Large-N vector model

Large-N vector multiplet $\vec{\phi} = (\phi^1, \dots, \phi^N)$

Kinetic term $\vec{\phi}^* \cdot \Box \vec{\phi}$

$\Longrightarrow$ Engineering scale dimensions of elementary fields $\vec{\phi}$:

$$\Delta^{\text{elementary}} = \frac{d}{2} - 1$$

Interaction term "Double-trace" quartic interaction $\frac{\lambda}{N}(\vec{\phi}^* \cdot \vec{\phi})^2$

$\Longrightarrow$ Bare and dressed (large-N approx) scale dimensions of composite field $\vec{\phi}^* \cdot \vec{\phi}$ at the fixed points:

$$\Delta^{\text{free}} = 2\,\Delta^{\text{elementary}} = d - 2\,, \qquad \Delta^{\text{int}} = 2$$

Singlet sector "Single-trace" bilinear operators $(\partial \dots \partial \vec{\phi}^*) \cdot (\partial \dots \partial \vec{\phi})$

Higher-spin holography: AdS/CFT dictionary

Before reviewing the kinematical evidences in favor of the conjecture, let us introduce some higher-spin terminology for the catalogue of (free) fields appearing in the higher-spin holography:

- Boundary: large-N vector

  Scalar singleton (elementary field)
- Boundary: singlet sector

  Conserved current (composite field)
  Shadow field (background field)
- Bulk: massless fields

  "Normalizable" solutions (Dirichlet)
  "Non-normalizable" solutions (Neumann)

Singleton as conformal scalar field

The group-theoretical definition of the free on-shell conformal scalar field with canonical dimension is:

Free scalar singleton: $\mathcal{D}(\frac{d}{2}-1,0)$ *is the space of solutions of the d'Alembert equation in* d *dimensions,*

$$\Box\phi(x)=0$$

where $\phi(x)$ *has conformal weight* $\frac{d}{2}-1$*, that carries a unitary irreducible representation of the conformal algebra* $\mathfrak{o}(d,2)$ *, whose generators are realized as first-order differential operators (e.g.* $x^a\partial_a+\frac{d}{2}-1$ *for dilatations).*

Traceless conserved currents as conformal fields

Conserved current: $\mathcal{D}(d+s-2,s)$ *is the space spanned by the conformal primary field that is a traceless divergenceless symmetric tensor field of conformal weight* $d+s-2$ *and rank* s*,*

$$\partial^{a_1} j_{a_1\ldots a_s}(x) = 0\,, \quad \eta^{a_1 a_2} j_{a_1 a_2 \ldots a_s}(x) = 0\,,$$

together with all its descendants, that carries a unitary irreducible representation of the conformal algebra $\mathfrak{o}(d,2)$*.*

Higher-spin conserved currents

Set of symmetric currents of all ranks (Berends, Burgers, van Dam; 1986)

$$j_{a_1 \dots a_s}(x) \;=\; i^s\, \phi^*(x)\; \overleftrightarrow{\partial}_{a_1} \cdots \overleftrightarrow{\partial}_{a_s}\; \phi(x)$$

$$(a = 0, \dots, d-1)$$

Features:

- Real
- Bilinear in the scalar field ϕ and its conjugate ϕ^*
- Number of derivatives = Rank
- Conserved (on-shell) for $s \geqslant 1$

$$\partial^{a_1} j_{a_1 \dots a_s}(x) \;=\; 0$$

where the equality is valid "on the mass shell", i.e. modulo $\Box\phi(x) = 0$.

Traceless conserved currents

When the scalar field is massless, a set of symmetric currents which are conserved & traceless exists in any dimension (D. Anselmi; 1998).

Remarks:

Such bilinear currents are the conformal primaries appearing in the higher-spin holographic conjecture.

Shadow fields as background fields

In order to define the generating functional of current correlators, one introduces the pairing

$$\int d^d x \, j^{a_1 a_2 \dots a_s}(x) \, h_{a_1 a_2 \dots a_s}(x)$$

which is invariant under the infinitesimal Weyl and Fronsdal like transformations of the background field ("source")

$$\delta h_{a_1 a_2 \dots a_s} = \partial_{(a_1} \epsilon_{a_2 \dots a_s)} - \eta_{(a_1 a_2} \alpha_{a_3 \dots a_s)}$$

when the currents are conserved and traceless.

Shadow field: *conformal primary field that is a symmetric tensor field of conformal weight $2 - s$ and rank s, quotiented by the gauge transformation*

$$\delta h_{a_1 a_2 \dots a_s} = \partial_{(a_1} \epsilon_{a_2 \dots a_s)} - \eta_{(a_1 a_2} \alpha_{a_3 \dots a_s)} \, .$$

Higher-spin holography: AdS/CFT dictionary

Massless field: *on-shell AdS gauge field which is a symmetric tensor field* $\varphi_{\mu_1\ldots\mu_s}$ *on* AdS_{d+1}, *with critical mass*

$$[\nabla^2 - (s-2)(s+d-2) + s]\varphi_{\mu_1\ldots\mu_s} = 0$$

(in the TT gauge) and with (residual) Fronsdal gauge transformations

$$\delta\varphi_{\mu_1\mu_2\ldots\mu_s} = \nabla_{(\mu_1}\epsilon_{\mu_2\ldots\mu_s)}$$

Higher-spin holography: AdS/CFT dictionary

Δ	on-shell bulk field	conformal boundary field
$d+s-2$	massless field (Dirichlet)	conserved current
$2-s$	massless field (Neumann)	shadow field

AdS/CFT dictionary for higher-spin holography

Higher-spin holography: kinematical evidences

In this talk, we restrict ourselves to the kinematical evidences for the duality but higher-spin symmetries are so huge that "kinematical" considerations essentially fix the theory uniquely (interactions in the bulk, correlators on the boundary).

Examples:

- Coleman-Mandula-like theorem
  (d=3: Maldacena & Zhiboedov; 2011, d=4: Stanev; 2013)
- Higher-spin algebra and cubic vertex
  (Boulanger, Ponomarev, Skvortsov, Taronna; 2013)

Rigid symmetries: reducibility parameters

A key element in higher-spin holography is the correspondence between **reducibility parameters** and **rigid symmetries**, e.g.

- Reducibility parameter for a background field: $\delta h_{a_1 \ldots a_s} = 0$
  $\leftrightarrow$ Rigid symmetry of the conformal scalar field
- Reducibility parameter for an AdS gauge field: $\delta \varphi_{\mu_1 \ldots \mu_s} = 0$
  $\leftrightarrow$ Vacuum symmetry of the bulk theory

Rigid symmetries: reducibility parameters

Conformal Killing tensor field: (Geroch, 1969)

$$\partial_{(a_1}\epsilon_{a_2\dots a_s)} = \eta_{(a_1 a_2}\alpha_{a_3\dots a_s)}\,, \quad \eta^{a_1 a_2}\epsilon_{a_1 a_2\dots a_s} = 0$$

$\leftrightarrow$ Reducibility parameter for a shadow field: $\delta h_{a_1 a_2\dots a_s} = 0$

AdS Killing tensor field: (Takeuchi, 1983)

$$\nabla_{(\mu_1}\epsilon_{\mu_2\dots\mu_s)} = 0$$

$\leftrightarrow$ Reducibility parameter for a massless field: $\delta\varphi_{\mu_1\mu_2\dots\mu_s} = 0$

Lemma: *1-to-1 correspondence between conformal and AdS Killing tensor fields.*

Corollary: *1-to-1 correspondence between reducibility parameters of shadow and massless fields.*

Rigid symmetries: space

Theorem: *(Eastwood, 2002) The space of infinitesimal symmetry generators of the wave equation $\Box\phi = 0$, (i.e. differential operators $\hat{A}$ such that $\Box\hat{A} = \hat{B}\,\Box$ and modulo trivial generators $\hat{A} = \hat{C}\,\Box$) is isomorphic to the space of conformal Killing tensors.*

$\Longrightarrow$ Match of symmetries

Rigid symmetries: algebra

Theorem: *(Eastwood, 2002) The algebra of infinitesimal symmetry generators of the wave equation $\Box\phi = 0$ (i.e. differential operators $\hat{A}$ such that $\Box\hat{A} = \hat{B}\,\Box$ and modulo trivial generators $\hat{A} = \hat{C}\,\Box$) is the enveloping algebra of $\mathfrak{o}(d,2)$ generators, quotiented by the ideal corresponding to the wave operator $\Box$.*

Vasiliev's equations of bosonic higher-spin gravity around AdS_{d+1} (Vasiliev, 2003) are based on (a suitable real form of) the symmetry algebra of the free scalar singleton.

Singleton tensor product decomposition

Theorem: *(d=3: Flato & Fronsdal; 1978, d>3: Vasiliev, 2004)*
The tensor product of two scalar singletons decomposes as the following sum

$$\mathcal{D}\Big(\frac{d}{2}-1,0\Big)\otimes\mathcal{D}\Big(\frac{d}{2}-1,0\Big)=\bigoplus_{s=0}^{\infty}\mathcal{D}(d+s-2,s)$$

of the unitary irreducible $\mathfrak{o}(d,2)$*-modules describing the conformal currents of all ranks* $s\in\mathbb{N}$.

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Remarks: The $\mathfrak{o}(d,2)$-modules on the right-hand-side

- are the Noether currents associated to the symmetry algebra of the free singleton.
- can also be interpreted as on-shell bulk massless fields which are, in this sense, composite fields made of two singletons $\Leftrightarrow$ "doubletons".

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of the unitary irreducible $\mathfrak{o}(d,2)$*-modules describing the conformal currents of all ranks* $s\in\mathbb{N}$.

Remarks: The right-hand-side is the spectrum of Vasiliev's equations of bosonic higher-spin gravity (Vasiliev, 2003).

$\Longrightarrow$ Match of spectrum

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of the unitary irreducible $\mathfrak{o}(d,2)$*-modules describing the conformal currents of all ranks* $s\in\mathbb{N}$*.*

Remarks: In a sense, the spectrum of Vasiliev's bosonic higher-spin gravity is a single Regge trajectory (actually, even less) of AdS massless fields.

Higher-order generalizations

Higher-spin holography: Generalizations

Other free gauge fields of any spin exist in (A)dS: partially massless fields (Deser, Waldron; 2001).

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An (A)dS/CFT dictionary has been proposed for these free fields (Dolan, Nappi, Witten; 2001, Deser, Waldron; 2001).

Higher-spin holography: Generalizations

Other free gauge fields of any spin exist in (A)dS: partially massless fields (Deser, Waldron; 2001).

An (A)dS/CFT dictionary has been proposed for these free fields (Dolan, Nappi, Witten; 2001, Deser, Waldron; 2001).

$\Longrightarrow$ look for a dynamical realization of this (A)dS/CFT dictionary between partially massless fields and partially conserved currents of all spins by generalizing the higher-spin holography.

Higher-spin holography: Generalizations

***Partially* massless field:** *on-shell AdS gauge field which is a symmetric tensor field* $\varphi_{\mu_1\ldots\mu_s}$ *on* AdS_{d+1}*, with tuned mass*

$$[\nabla^2 - (s-t-1)(s+t+d-3) + s]\varphi_{\mu_1\ldots\mu_s} = 0$$

(in the TT gauge) and with (residual) depth-t Fronsdal-like gauge transformations

$$\delta\varphi_{\mu_1\mu_2\ldots\mu_s} = \nabla_{(\mu_1}\ldots\nabla_{\mu_t}\epsilon_{\mu_{t+1}\ldots\mu_s)} + \ldots$$

Remark: On AdS_4, the partially massless fields have helicities $\pm s$, $\pm(s-1)$, ..., $\pm(s-t+1)$.

Higher-spin holography: Generalizations

Partially conserved current: *$\mathcal{D}(d+s-t-1,s)$ is the space spanned by the conformal primary field that is a symmetric tensor field of conformal weight $d+s-t-1$ and rank s, such that*

$$\partial^{a_1}\ldots\partial^{a_t} j_{a_1\ldots a_t\ldots a_s}(x) = 0\,, \quad \eta^{a_1 a_2} j_{a_1 a_2\ldots a_s}(x)\, q = 0\,,$$

together with all its descendants, that carries a non-unitary irreducible representation of the conformal algebra $\mathfrak{o}(d,2)$.

Higher-spin holography: Generalizations

***Higher-depth* shadow field:** *conformal primary field that is a symmetric tensor field of conformal weight $1+t-s$ and rank s, quotiented by the gauge transformation*

$$\delta h_{a_1 a_2 \ldots a_s} = \partial_{(a_1} \ldots \partial_{a_t} \epsilon_{a_{t+1} \ldots a_s)} - \eta_{(a_1 a_2} \alpha_{a_3 \ldots a_s)} \,.$$

Higher-spin holography: Generalization

Δ	on-shell bulk field	conformal boundary field
$d+s-t-1$	partially massless field (Dirichlet)	partially conserved current
$1+t-s$	partially massless field (Neumann)	higher-depth shadow field

AdS/CFT dictionary for higher-depth holography

Higher-spin holography: Generalization

Proposal: A plausible boundary dual of a tower of partially massless fields is the higher-order scalar singleton.

Higher-spin holography: Generalization

Proposal: A plausible boundary dual of a tower of partially massless fields is the higher-order scalar singleton.

Free higher-order scalar singleton: *$\mathcal{D}(\frac{d}{2}-p,0)$ is the space of solutions of the d'Alembert equation in d dimensions,*

$$\Box^p \phi(x) = 0$$

where $\phi(x)$ has conformal weight $\frac{d}{2}-p$, that carries a non-unitary irreducible representation of the conformal algebra $\mathfrak{o}(d,2)$, whose generators are realized as first-order differential operators.

Multicritical large-N vector model

Kinetic term $\vec{\phi}^{*} \cdot \Box^{p} \vec{\phi}$

Interaction term $\frac{\lambda}{N}(\vec{\phi}^{*} \cdot \vec{\phi})^{2}$

Remarks: Multicritical fixed point called "isotropic Lifshitz point"

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Simplest case: p=2

Renormalisation Group theory (Hornreich, Luban, Shtrikman, 1975)

Experimental observation (Ternary mixture of homopolymer & diblock copolymer, 1995)

Multicritical large-N vector model

Kinetic term $\vec{\phi}^* \cdot \Box^p \vec{\phi}$

Interaction term $\frac{\lambda}{N}(\vec{\phi}^* \cdot \vec{\phi})^2$

Remarks: Other deformations might be relevant. For simplicity, we concentrated on the quartic interaction with lowest (bare) scale dimension. For the usual Lifshitz point ($p = 2$), in dimensions $6 \leqslant d \leqslant 8$ it is indeed the only relevant deformation.

Singleton tensor product decomposition

Theorem: *(X.B & M. Grigoriev, 2013)*
The tensor product of two higher-order scalar singletons decomposes as the following sum

$$\mathcal{D}\Big(\frac{d}{2}-p,0\Big)\otimes\mathcal{D}\Big(\frac{d}{2}-p,0\Big) = \bigoplus_{s=0}^{\infty}\bigoplus_{k=1}^{p}\mathcal{D}(d+s-2k,s)$$

of the unitary irreducible $\mathfrak{o}(d,2)$-modules describing the partially conserved currents of all ranks $s\in\mathbb{N}$ and all odd depths t $(=2k-1)$ ranging from 1 to $2p-1$.

Remarks: The right-hand-side gives the spectrum of Vasiliev's equations of bosonic higher-spin gravity based on the symmetry algebra of the free higher-order singleton.

$\Longrightarrow$ Match of spectrum

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of the unitary irreducible $\mathfrak{o}(d,2)$*-modules describing the partially conserved currents of all ranks* $s\in\mathbb{N}$ *and all odd depths* t $(=2k-1)$ *ranging from* 1 *to* $2p-1$.

Remarks: The initial proof was a pedestrian check that the $\mathfrak{o}(d)\oplus\mathfrak{o}(2)$ decomposition was identical for both sides.
Characters provide a more systematic tool to prove this result and generalize it to the spinor singleton (T. Basile, X.B, N. Boulanger; 2014).

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Remarks: In a sense, this spectrum is made of p parallel Regge trajectories of partially massless fields (without the partially massless spin-2 field).

Rigid symmetries: reducibility parameters

Generalized conformal Killing tensor field: (Nikitin, 1991)

$$\partial_{(a_1} \dots \partial_{a_t} \epsilon_{a_{t+1} \dots a_s)} = \eta_{(a_1 a_2} \alpha_{a_3 \dots a_s)} \,, \quad \eta^{a_1 a_2} \epsilon_{a_1 a_2 \dots a_s)} = 0$$

$\leftrightarrow$ Reducibility parameter for a spin-s depth-t shadow field: $\delta h_{a_1 a_2 \dots a_s} = 0$

AdS Killing-like tensor field:

$$\nabla_{(\mu_1} \dots \nabla_{\mu_t} \epsilon_{\mu_{t+1} \dots \mu_s)} + \dots = 0$$

$\leftrightarrow$ Reducibility parameter for a spin-s depth-t partially massless tensor field: $\delta \varphi_{\mu_1 \mu_2 \dots \mu_s} = 0$

Proposition: *(X.B & M. Grigoriev, 2013) For any given spin and depth, 1-to-1 correspondence between reducibility parameters of higher-depth shadow fields and partially massless fields.*

Rigid symmetries: space

Theorem: *(p=2: Eastwood, Leistner, 2008; p>2: Gover, Silhan, 2009)* *The space of infinitesimal symmetry generators of the polywave equation $\Box^p \phi = 0$ (i.e. differential operators $\hat{A}$ such that $\Box^p \hat{A} = \hat{B}\,\Box^p$ and modulo trivial generators $\hat{A} = \hat{C}\,\Box^p$) is isomorphic to the space of generalized conformal Killing tensor fields of all odd depths ranging from 1 to $2p-1$.*

$\Longrightarrow$ Match of symmetries

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The algebra of infinitesimal symmetry generators of the polywave equation $\Box^p \phi = 0$ (i.e. differential operators $\hat{A}$ such that $\Box^p \hat{A} = \hat{B}\,\Box^p$ and modulo trivial generators $\hat{A} = \hat{C}\,\Box^p$) is the enveloping algebra of $\mathfrak{o}(d,2)$ generators, quotiented by the ideal corresponding to the polywave operator $\Box^p$.

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Remark: $\exists$ interacting bulk theory

Vasiliev's equations of bosonic higher-spin gravity around AdS_{d+1} remain consistent when based on the symmetry algebra of the free higher-order singleton because it is a suitable quotient of the "off-shell" higher-spin algebra (i.e. the enveloping algebra of $\mathfrak{o}(d,2)$ generators).

Conclusion

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   - *match of spectrum*
     tensor product of two free higher-order singletons $\leftrightarrow$ tower of partially massless fields with odd depths
   - *match of symmetry*
     free higher-order singleton symmetry algebra $\leftrightarrow$ suitable quotient of the off-shell higher-spin algebra
   - *consistency with corresponding Vasiliev's equations*
     solutions of linearized equations $\leftrightarrow$ tower of partially massless fields with odd depths

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     solutions of linearized equations $\leftrightarrow$ tower of partially massless fields with odd depths
3. `Challenge:` check of the duality, at tree level (higher-point large-N correlators) and 1-loop level (vanishing sum of bubbles)