Is String Interaction the Origin of Quantum Mechanics?

Moyal ∗ formulation of SFT (MSFT)

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Moyal ★ product and QM

Moyal ★ formulation of SFT (MSFT)

Stringy Origin of QM (mini-MSFT)

Based on I. Bars and D. Rychkov,

arXiv:1407.4699, Background Independent MSFT

arXiv:1407.4699, Stringy Origin of QM
Basic ingredient of QM, a very successful but mysterious rule

\[ [\hat{X}^M, \hat{P}_N] = i\hbar \delta^M_N \text{ for all dofs.} \]

Operators & representations, Hilbert spc etc. follows. Also product

\[ \hat{A}_{12} (\hat{X}, \hat{P}) = \hat{A}_1 (\hat{X}, \hat{P}) \hat{A}_2 (\hat{X}, \hat{P}) \]
Moyal Star and Quantum Mechanics

- Basic ingredient of QM, a very successful but mysterious rule

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\]

- Another formalism for QM - classical phase space + Moyal $\star$

Weyl, Wigner, Moyal,...-see summary in sec.III 1407.4699

image \( A(X,P) = \int d\text{ye}^{iPy} \langle X-\frac{Y}{2} | \hat{A} | X+\frac{Y}{2} \rangle \) & reverse \( \hat{A} = \int dXdP \ A(X,P) : \delta(X-\hat{X}) \delta(P-\hat{P}) : \)

integral repr. same exp

\[
A_{12} (X, P) = A_1 (X, P) \star A_2 (X, P)
\]

\[
(\star = e^{\text{Poisson}}) \quad : \quad \star = \exp \left( \frac{i \hbar}{2} \left( \overleftarrow{\partial} X^M \overrightarrow{\partial} P_M - \overrightarrow{\partial} X^M \overleftarrow{\partial} P_M \right) \right)
\]
Basic ingredient of QM, a very successful but mysterious rule

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\left[ \hat{X}^M, \hat{P}_N \right] = i\hbar \delta^M_N \quad \text{for all dofs.}
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Another formalism for QM - classical phase space + Moyal ★
Weyl, Wigner, Moyal,...-see summary in sec.III 1407.4699
image \(A(X,P) = \int \text{dye}^{iPy} \langle X - \frac{V}{2} | \hat{A} | X + \frac{V}{2} \rangle\) & reverse \(\hat{A} = \int \text{d}X\text{d}P \ A(X,P) : \delta(\hat{X} - \hat{X}) \delta(\hat{P} - \hat{P})::\int\text{e}^{\text{integral repr. same exp}}\)

\[
A_{12} (X, P) = A_1 (X, P) \ast A_2 (X, P)
\]

\[
(\ast = e^{\text{Poisson}}) : \ast = \exp \left( \frac{i\hbar}{2} \left( \overleftarrow{\partial} X^M \overrightarrow{\partial} P_M - \overrightarrow{\partial} X^M \overleftarrow{\partial} P_M \right) \right)
\]

Check:

\[
[ X^M, P_N ]_\ast = (X^M \ast P_N - P_N \ast X^M) = i\hbar \delta^M_N
\]

Note: QM requires ALL PHASE SPACE \((X^M, P_M)\) in ★.
This is NOT what we do in MSFT.
Witten’s cubic open SFT: 

\[ S = \int \left( \frac{1}{2} \psi \ast \hat{Q} \psi + \frac{\xi}{3} \psi \ast \psi \ast \psi \right) \]

matrix-like product, so \( \psi \) \~ like matrix representation of QM operator

Use Weyl’s map to find Moyal \( \ast \) version of this theory (IB 2001)

\[ \psi(X(\sigma)) = \psi(\chi_L(\sigma), \chi, \chi_R(\sigma)) \]

\[ \cong \mathcal{M}(\chi) \]

\[ \psi_{12}(\chi_L, \chi_R) = \int \mathcal{D}^2 \psi_1(\chi_L, \bar{z}) \psi_2(z, \chi_R) \]

\[ \text{I. Bars (USC)} \]

\[ \text{MSFT and Stringy Origin of QM} \]

Aug. 2014
Witten’s cubic open SFT: \( S = \int \left( \frac{1}{2} \psi \star \hat{Q} \psi + \frac{\alpha}{3} \psi \star \psi \star \psi \right) \)

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Use Weyl’s map to find Moyal \( \star \) version of this theory (IB 2001)

New \( \sigma \)-basis (2014) - a much improved version of MSFT:

Full:
\[
\begin{align*}
x_{\pm M} (\sigma) &= \frac{1}{2} \left( X^M (\sigma) \pm X^M (\pi - \sigma) \right) \\
p_{\pm M} (\sigma) &= \frac{1}{2} \left( P_M (\sigma) \pm P_M (\pi - \sigma) \right)
\end{align*}
\]

\( \psi (X (\sigma)) \Rightarrow A (x_+ (\sigma, \epsilon), p_- (\sigma)), \text{ half-phase-space, } A_{12} = A_1 \star A_2 \)
Witten’s cubic open SFT: \( S = \int \left( \frac{1}{2} \psi \ast \hat{Q} \psi + \frac{g}{3} \psi \ast \psi \ast \psi \right) \)

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\begin{align*}
\chi^M_\pm (\sigma) &= \frac{1}{2} \left( X^M(\sigma) \pm X^M(\pi - \sigma) \right) \\
\rho^M_\pm (\sigma) &= \frac{1}{2} \left( P^M(\sigma) \pm P^M(\pi - \sigma) \right)
\end{align*}
\]

\( \psi(X(\sigma)) \Rightarrow A(x_+ (\sigma, \epsilon), p_- (\sigma)) \), half-phase-space, \( A_{12} = A_1 \ast A_2 \)

New star in \( \sigma \)-basis, midpoint not separate dof (huge improvement)

\[
\star = e^{\frac{i\theta}{4} \int_0^\pi d\sigma \, \text{sign} \left( \frac{\pi}{2} - \sigma \right) \left( \overleftarrow{\partial} p_{-M}(\sigma) \overrightarrow{\partial} x^M_+ (\sigma, \epsilon) - \overleftarrow{\partial} p_{-M}(\sigma) \overrightarrow{\partial} x^M_+ (\sigma, \epsilon) \right)}
\]

Midpoint drops: \( \overleftarrow{\partial} p_{-M} \left( \frac{\pi}{2} \right) \overrightarrow{\partial} x^M_+ \left( \frac{\pi}{2}, \epsilon \right) \rightarrow \frac{\pi}{2} \) excluded automatically!
Moyal Star for String Joining/Splitting

- Witten’s cubic open SFT: $S = \int \left( \frac{1}{2} \psi \star \hat{Q} \psi + \frac{g}{3} \psi \star \psi \star \psi \right)$
  - matrix-like product, so $\psi \sim$ like matrix representation of QM operator
  - Use Weyl’s map to find Moyal $\star$ version of this theory (IB 2001)

- New $\sigma$-basis (2014) - a much improved version of MSFT:
  - Full: $\left\{ \begin{array}{c} x_\pm^M (\sigma) = \frac{1}{2} \left( X^M (\sigma) \pm X^M (\pi - \sigma) \right) \\ p_\pm^M (\sigma) = \frac{1}{2} \left( P^M (\sigma) \pm P^M (\pi - \sigma) \right) \end{array} \right\}$
    - $(+): x_e^M \cos(e \sigma), \ e=0,2,4,...$
    - $(-): p_o^M \cos(o \sigma), \ o=1,3,5,...$
  - $\psi (X (\sigma)) \Rightarrow A (x^+_+ (\sigma, \varepsilon), p^- (\sigma)), \text{half-phase-space}, \ A_{12} = A_1 \star A_2$

- New star in $\sigma$-basis, midpoint not separate dof (huge improvement)
  - $\star = e^{i \theta / 4} \int_0^\pi d\sigma \ \text{sign} \left( \frac{\pi}{2} - \sigma \right) \left( \overleftarrow{\partial}_{p^- M (\sigma)} \overrightarrow{\partial}_{x^+_+ (\sigma, \varepsilon)} - \overleftarrow{\partial}_{p^- M (\sigma)} \overrightarrow{\partial}_{x^+_+ (\sigma, \varepsilon)} \right),$
  - Midpoint drops: $\overleftarrow{\partial}_{p^- M \left( \frac{\pi}{2} \right)} \overrightarrow{\partial}_{x^+_+ \left( \frac{\pi}{2}, \varepsilon \right)} \rightarrow \frac{\pi}{2} \text{ excluded automatically!}$

- $\theta = \hbar$, AHA!! - this is an INDUCED QM - we call it iQM.
  - NOT the canonical $e^{(\text{Poisson})}$. Not QM, but string joining/splitting!
Moyal Star for String Joining/Splitting

- Witten’s cubic open SFT: $S = \int \left( \frac{1}{2} \psi \star \hat{Q} \psi + \frac{2}{3} \psi \star \psi \star \psi \right)$

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  \end{align*} \]

  (+): $x_e^M \cos(e\sigma), e=0,2,4,\ldots$

  (-): $p_o^M \cos(o\sigma), o=1,3,5,\ldots$

  $\psi (X (\sigma)) \Rightarrow A \left( x_+ (\sigma, \varepsilon), p_- (\sigma) \right)$, half-phase-space, $A_{12} = A_1 \star A_2$

- New star in $\sigma$-basis, midpoint not separate dof (huge improvement)

  $$\star = e^{i\theta \frac{4}{\pi} \int_0^\pi d\sigma \, \text{sign} \left( \frac{\pi}{2} - \sigma \right) \left( \frac{\partial}{\partial p^M_-(\sigma)} \frac{\partial}{\partial x^M_+(\sigma, \varepsilon)} - \frac{\partial}{\partial p^M_-(\sigma)} \frac{\partial}{\partial x^M_+(\sigma, \varepsilon)} \right)}$$

  Midpoint drops: $\frac{\partial}{\partial p^M_-(\frac{\pi}{2})} \frac{\partial}{\partial x^M_+(\frac{\pi}{2}, \varepsilon)} \rightarrow \frac{\pi}{2}$ excluded automatically!

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  NOT the canonical $e^{\text{(Poisson)}}$. Not QM, but string joining/splitting!

- $\star$ background independent: $X^M, P^M$ upper/lower $M$, for any CFT.
Moyal Star for String Joining/Splitting

- Witten’s cubic open SFT: \( S = \int \left( \frac{1}{2} \psi \star \hat{Q} \psi + \frac{g}{3} \psi \star \psi \star \psi \right) \)
  matrix-like product, so \( \psi \sim \) like matrix representation of QM operator
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  \(+\): \( x^M_e \cos(e\sigma), \ e=0,2,4,\cdots \)
  \(-\): \( p^M_o \cos(o\sigma), \ o=1,3,5,\cdots \)
  \( \psi(X(\sigma)) \Rightarrow A(x_+(\sigma, \varepsilon), p_-(\sigma)) \), half-phase-space, \( A_{12} = A_1 \star A_2 \)

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  \]
  Midpoint drops: \( \overleftarrow{\partial}_{p^M_-} \varepsilon_x^M(\pi/2, \varepsilon) \rightarrow \frac{\pi}{2} \) excluded automatically!

- \( \theta = \hbar \), AHA!! - this is an INDUCED QM - we call it iQM.
  NOT the canonical \( e^{(\text{Poisson})} \). Not QM, but string joining/splitting!

- **background independent**: \( X^M, P^M \) upper/lower \( M \), for any CFT.

- \( \star \) is SUSY OSp\((d|2)\) : \( M = (\mu, b, c) \) matter and ghosts.
More Properties of new MSFT

- Regulator $x^M_+ (\sigma, \varepsilon) = e^{-\varepsilon|\partial \sigma|} x^M_+ (\sigma) \rightarrow x_0 + \sqrt{2} \sum_{e \geq 2} e^{-\varepsilon e} x_e \cos e\sigma$.

In computations $\partial x^M_+ (\sigma) x^N_+ (\sigma', \varepsilon)$ or $e^{-\varepsilon|\partial \sigma|} \partial^N_{p-M(\sigma)} p_{-N} (\sigma')$, only regulated $\delta_\varepsilon (\sigma, \sigma')$, everything finite, no midpoint associativity anomalies, reliable computations. $|\partial \sigma| \equiv \sqrt{-\partial^2_\sigma}$
More Properties of new MSFT

- Regulator \( x_+^M(\sigma, \epsilon) = e^{-\epsilon|\partial\sigma|} x_+^M(\sigma) \to x_0 + \sqrt{2} \sum_{e \geq 2} e^{-\epsilon e} x_e \cos e\sigma \).

  In computations \( \partial x_+^M(\sigma) x_+^N(\sigma', \epsilon) \) or \( e^{-\epsilon|\partial\sigma|} \partial^N p_{-M(\sigma)} p_{-N}(\sigma') \), only regulated \( \delta_\epsilon(\sigma, \sigma') \), everything finite, no midpoint associativity anomalies, reliable computations. \[ |\partial\sigma| \equiv \sqrt{-\partial^2_\sigma} \]

- D-brane boundary conditions at end points \( \sigma = 0, \pi \). Regulated delta functions \( \delta^\pm(\text{nn or dd})(\sigma, \sigma') \) that satisfy boundary conditions, Neumann-Neumann (nn), Dirichlet-Dirichlet (dd), symmetric or antisymmetric (±) relative to midpoint.
More Properties of new MSFT

- Regulator \( x_+^M (\sigma, \varepsilon) = e^{-\varepsilon |\partial \sigma|} x_+^M (\sigma) \rightarrow x_0 + \sqrt{2} \sum_{e \geq 2} e^{-\varepsilon e} x_e \cos e\sigma \). In computations \( \partial x_+^M (\sigma) x_+^N (\sigma', \varepsilon) \) or \( e^{-\varepsilon |\partial \sigma|} \partial p_{-M (\sigma)} p_{-N} (\sigma') \), only regulated \( \delta_\varepsilon (\sigma, \sigma') \), everything finite, no midpoint associativity anomalies, reliable computations. \( |\partial \sigma| \equiv \sqrt{-\partial^2 \sigma} \)

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- Representation of \( \hat{Q}_{BRST} A = \{ Q, A \}_* \), explicit FIELD \( Q (x_+, p_-) \)

ANY CFT \( \hat{Q} (\hat{X}, \hat{P}) \rightarrow Q (x_+, p_) \), same function in half phase space

\[
Q_{\text{flat}}^{CFT} = \int_0^{\pi/2} d\sigma \left[ \frac{p_b \left( \pi^2 p_{-\mu} + x_+^{l\mu} + i \pi^2 p_{-c} + i x_+^{l b} x_+^{l c} \right)}{2} - i \left( |\partial \sigma|^{-2} x_+^{l c} \right) \right.
\]

\[
\left. 2 x_+^{l\mu} + x_+^{l c} + x_+^{l b} p_{-b} \right]
\]

satisfies \( Q (x_+, p_-) \star Q (x_+, p_-) = 0 \), a FIELD like \( A (x_+, p_-) \).
Action. Midpoint insertion, $\partial \bar{b} A = \partial_x \pi_{/2} A$, to balance ghost No.

$$S (A) = - \int \left( A \star Q \star A + \frac{g_0}{3} A \star \partial \bar{b} A \star \partial \bar{b} A \right) .$$
- Action. Midpoint insertion, $\partial \bar{b} A = \partial_x (\pi/2) A$, to balance ghost No.

$$S (A) = - \int \left( A \ast Q \ast A + \frac{g_0}{3} A \ast \partial \bar{b} A \ast \partial \bar{b} A \right).$$

- Has BRST gauge symmetry: $\delta_\Lambda A = [Q, \Lambda]_\ast + g_0 \{\partial \bar{b} A, \partial \bar{b} \Lambda\}_\ast$
Action. Midpoint insertion, $\partial_b A = \partial_{x^b(\pi/2)} A$, to balance ghost No.

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Has BRST gauge symmetry: $\delta_\Lambda A = [Q, \Lambda]_\star + g_0 \{ \partial_b A, \partial_b \Lambda \}_\star$

Can fix to Siegel gauge, $A_s = x^b A_s^{(0)}$ (no more ghost zero modes)

$$S_s = - \int' \left( A_s^{(0)} \star \left( \mathcal{L}_0 - \frac{1}{2} \right) \star A_s^{(0)} + \frac{g_0}{3} A_s^{(0)} \star A_s^{(0)} \star A_s^{(0)} \right).$$
MSFT Action

- Action. Midpoint insertion, $\partial_{\bar{b}}A = \partial_{x^b(\pi/2)}A$, to balance ghost No.

$$S(A) = - \int (A \ast Q \ast A + \frac{g_0}{3} A \ast \partial_{\bar{b}}A \ast \partial_{\bar{b}}A) .$$

- Has BRST gauge symmetry: $\delta_A A = [Q, \Lambda]_\ast + g_0 \{\partial_{\bar{b}}A, \partial_{\bar{b}}\Lambda\}_\ast$

- Can fix to Siegel gauge, $A_s = x_0^b A_s^{(0)}$ (no more ghost zero modes)

$$S_s = - \int' \left( A_s^{(0)} \ast \left( L_0 - \frac{1}{2} \right) \ast A_s^{(0)} + \frac{g_0}{3} A_s^{(0)} \ast A_s^{(0)} \ast A_s^{(0)} \right)$$

- Agrees with all previous successful MSFT computations of IB+Matsuo+Kishimoto+Park, including off shell Veneziano amplitude with greater accuracy (IB+Park, 2003).
Two particles named $L$ (left) and $R$ (right), located at $(\vec{x}_L, \vec{x}_R)$. Center of mass and relative coordinates

$$R^i = \frac{1}{2} \left( x_L^i + x_R^i \right), \quad r^i = (x_L^i - x_R^i)$$
$$P_i = (p_{iL} + p_{iR}), \quad p_i = \frac{1}{2} (p_{iL} - p_{iR})$$

Hamiltonian $H(P, p, R, r)$ unimportant for now.
Two particles named \( L \) (left) and \( R \) (right), located at \((\vec{x}_L, \vec{x}_R)\).

Center of mass and relative coordinates

\[
R^i = \frac{1}{2} (x^i_L + x^i_R), \quad r^i = (x^i_L - x^i_R)
\]

\[
P_i = (p_iL + p_iR), \quad p_i = \frac{1}{2} (p_iL - p_iR)
\]

Hamiltonian \( H(P, p, R, r) \) unimportant for now.

Parallels between the full MSFT and mini-MSFT:

\[
R^i \sim x^M_+ (\sigma), \quad r^i \sim x^M_- (\sigma), \quad P_i \sim p_{+M} (\sigma), \quad p_i \sim p_{-M} (\sigma)
\]

and \( \hat{H}(\hat{P}, \hat{p}, \hat{R}, \hat{r}) \sim \hat{Q}_{BRST} \) or Virasoro \( \hat{L}_0 \) - A “kinetic” operator of mini-MSFT.
Two particles named $L$ (left) and $R$ (right), located at $(\vec{x}_L, \vec{x}_R)$. Center of mass and relative coordinates

$$R^i = \frac{1}{2} (x^i_L + x^i_R), \quad r^i = (x^i_L - x^i_R)$$

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Parallels between the full MSFT and mini-MSFT:

$$R^i \sim x^M_+ (\sigma), \quad r^i \sim x^M_- (\sigma), \quad P_i \sim p_+M (\sigma), \quad p_i \sim p_-M (\sigma)$$

and $\hat{H}(\hat{P}, \hat{p}, \hat{R}, \hat{r}) \sim \hat{Q}_{BRST}$ or Virasoro $\hat{L}_0$ - A “kinetic” operator of mini-MSFT.

Field in position space = probability amplitude,

$\psi (x_L, x_R) = \langle x_L, x_R | \psi \rangle$. Consider the Fourrier transform

$$\psi (x_L, x_R) = \psi (R, r) \quad \text{Fourrier} \quad (r, p) \quad A(R, p) = \langle R, p | \psi \rangle. \quad (1)$$

where $\langle R, p |$ is also a complete eigenbasis - a half-phase-space.
Matrix-like product of fields in position space. Like quarks (0-branes) at the ends of a string. Computing the probability amplitude of joining two strings

\[ \psi_{12}(x_L, x_R) = \int_{-\infty}^{\infty} d^n z \, \psi_1(x_L, z) \psi_2(z, x_R), \]

**Physically different but mathematically analogous to the usual Moyal product. Note usual Moyal would give**

\[ R_i, p_j \sim \text{usual} = 0. \]
Star Product in mini-MSFT

- Matrix-like product of fields in position space. Like quarks (0-branes) at the ends of a string. Computing the probability amplitude of joining two strings

\[ \psi_{12}(x_L, x_R) = \int_{-\infty}^{\infty} d^n z \, \psi_1(x_L, z) \, \psi_2(z, x_R), \]

- Equivalent $\star$ product in the half-phase-space.

\[
A_{12}(R, p) = A_1(R, p) \exp \left( \frac{i\theta}{2} \left( \overleftarrow{\partial}_R \overrightarrow{\partial}_p - \overrightarrow{\partial}_R \overleftarrow{\partial}_p \right) \right) A_2(R, p)
\]

\[
= A_1 A_2 + \frac{i\theta}{2} \left( \frac{\partial A_1}{\partial R^i} \frac{\partial A_2}{\partial p_i} - \frac{\partial A_1}{\partial p_i} \frac{\partial A_2}{\partial R^i} \right) + \ldots
\]

\[
[R^i, p_j]_\star = i\theta \delta^i_j \quad \text{AHA} \quad \text{iQM joining/splitting, NOT QM}
\]

Physically different but mathematically analogous to the usual Moyal product. Note usual Moyal would give $[R^i, p_j]_{\star \text{usual}} = 0$. 

I. Bars (USC)
Map between operators in QM and their representative in iQM - an elegant and intuitive property of MSFT.

\[
\begin{align*}
\hat{x}_L^i A &= R^i \star A \\
\hat{p}_{iL} A &= p_i \star A \\
\hat{x}_R^i A &= A \star R^i \\
\hat{p}_{iR} A &= -A \star p_i
\end{align*}
\]

for particle \( L \) the \( \star \) product from left

for particle \( R \) the \( \star \) product from right

only \( \star \) products between \textit{two fields in the half-phase-space}. 
Map between operators in QM and their representative in iQM - an elegant and intuitive property of MSFT.

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\end{align*}
\]

for particle \(L\) the \(\star\) product from left

for particle \(R\) the \(\star\) product from right

only \(\star\) products between \textit{two fields in the half-phase-space}.

Compute commutators, use associativity of \(\star\)

\[
\begin{align*}
[\hat{x}_L^i, \hat{p}_{Lj}] A &= [R^i, p_j]_\star A = i\theta \delta^i_j A, \\
[\hat{x}_R^i, \hat{p}_{Rj}] A &= A \star [-p_j, R^i]_\star = i\theta \delta^i_j A, \\
[\hat{x}_L^i, \hat{p}_{Rj}] A &= -R^i \star A \star p_j + R^i \star A \star p_j = 0, \\
[\hat{x}_R^i, \hat{p}_{Lj}] A &= p_j \star A \star R^i - p_j \star A \star R^i = 0.
\end{align*}
\]

\textbf{Obtained QM from iQM}! Must identify the parameter \(\theta\) with \(\hbar\).

\(\theta = \hbar\).
All Operators in QM Have iQM Representatives

- Observables of only particle $L$ or only particle $R$ (case full MSFT)
  \[
  \hat{O}_L (\hat{x}_L, \hat{p}_L) A = O_L (R, p) \star A, \quad (\star \text{ from left}).
  \]
  \[
  \hat{O}_R (\hat{x}_R, \hat{p}_R) A = A \star O_R (R, p), \quad (\star \text{ from right}).
  \]
All Operators in QM Have iQM Representatives

- Observables of only particle $L$ or only particle $R$ (case full MSFT)
  \[
  \hat{O}_L (\hat{x}_L, \hat{p}_L) A = O_L (R, p) \star A , \ (\star \text{ from left}).
  \]
  \[
  \hat{O}_R (\hat{x}_R, \hat{p}_R) A = A \star O_R (R, p) , \ (\star \text{ from right}).
  \]

- General QM operator, e.g. arbitrary Hamiltonian $\hat{H}(\hat{x}_L, \hat{p}_L, \hat{x}_R, \hat{p}_R)$
  \[
  \hat{H}(\hat{x}_L, \hat{p}_L, \hat{x}_R, \hat{p}_R) A = H ((R\star), (p\star), (\star R), (\star p)) A (R, p).
  \]
  Keep order of the original operators, and keep order of left $\star$ or right $\star$.
All Operators in QM Have iQM Representatives

- Observables of only particle $L$ or only particle $R$ (case full MSFT)
  \[ \hat{O}_L (\hat{x}_L, \hat{p}_L) A = O_{L\ast} (R, p) \ast A, \quad (\ast \text{ from left}). \]
  \[ \hat{O}_R (\hat{x}_R, \hat{p}_R) A = A \ast O_{\ast R} (R, p), \quad (\ast \text{ from right}). \]

- General QM operator, e.g. arbitrary Hamiltonian $\hat{H}(\hat{x}_L, \hat{p}_L, \hat{x}_R, \hat{p}_R)$
  \[ \hat{H}(\hat{x}_L, \hat{p}_L, \hat{x}_R, \hat{p}_R) A = H ((R\ast), (p\ast), (\ast R), (\ast p)) A (R, p). \]
  Keep order of the original operators, and keep order of left $\ast$ or right $\ast$

- Example, harmonic oscillator type central force problem
  \[ \hat{H}_1 A = \left[ \frac{1}{2} \left( \hat{p}_L^2 + \hat{p}_R^2 \right) + \frac{\omega^2}{2} (\hat{x}_L - \hat{x}_R)^2 \right] A (R, p) \]
  \[ = \frac{1}{2} \left( \bar{p}^2 + \omega^2 \bar{R}^2 \right) \ast A + \frac{1}{2} A \ast \left( \bar{p}^2 + \omega^2 \bar{R}^2 \right) - \omega^2 \bar{R} \ast A \ast \bar{R} \]
  \[ = \left[ -\frac{1}{4} \hbar^2 \partial_R^2 + p^2 - \frac{\omega^2}{2} \hbar^2 \partial_p^2 \right] A (R, p) \]
QCD in 2D - a cute system in mini-MSFT

2D-QCD at large $\mathcal{N} = 2$-D-strings with quarks (0-branes) at the ends $(x^\mu_L, R)$. Established in 1976, spectrum and interactions (IB). The iQM equivalent for spectrum Eq

$$\hat{H}A = \left[ \frac{m_L^2}{2\hat{p}_L^+} + \frac{m_R^2}{2\hat{p}_R^+} + \gamma |\hat{x}_L^- - \hat{x}_R^-| \right] A(R, p)$$

$$= \frac{m_L^2}{2p} * A + A * \frac{m_R^2}{2p} + \gamma |(R*) - (R)| A(R, p)$$

$$= \frac{m_L^2}{2p} * A(R, p) + A(R, p) * \frac{m_R^2}{2p} + \gamma \hbar \int' \frac{dk}{\pi k^2} A(R, p + k)$$

The last line is equivalent to the ’tHooft integral equation for quark-antiquark bound state.

Interactions between mesons via string joining/splitting in mini-MSFT

$$S = \int d^nR d^n p \left[ \frac{1}{2} A(\hat{H}A) + \frac{g}{3} A \star A \star A + \cdots \right]$$

Feynman graphs analogous to those computed in 1976. Seems like a fun system to play with and generalize. Also use it as a simple model analogous to SFT.
Full MSFT satisfies the same iQM → QM properties. Then this applies to all systems derivable from open SFT, from single particles to complex systems. However, it is not all physics. Still could ask, what about the full M-theory, second quantization etc.? Therefore, let’s say the door is open to understanding the origins of QM from string theory.
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More modest level: mini-MSFT seems fun, and may be useful in its own right to discuss some perturbative and non-perturbative physics in certain circumstances.


I. Bars, “Exact equivalence of chromodynamics to a string theory,” Phys.Rev.Lett 36 (1976) 1521, and references therein to earlier work, see especially Eqs.(9a) and (13a).
The relation between $b, c$ ghosts and $X^b, P_b$ and $X^c, P_c$

\[
\hat{B} (\pm \sigma) = \left( -i \hat{X}^b (\sigma) \pm \pi |\partial_\sigma|^{-2} \partial_\sigma \hat{P}_c (\sigma) \right), \\
\hat{C} (\pm \sigma) = (\pi \hat{P}_b (\sigma) \mp i \partial_\sigma \hat{X}^c (\sigma)) .
\]

$B, C$ are exponential series, with $e^{in\sigma}, -\infty < n < \infty$, but $(X, P)$ are only cosine series $\cos n\sigma$, while their derivatives are $\sin n\sigma$ series.