Is String Interaction the Origin of Quantum Mechanics?

Moyal $\star$ formulation of SFT (MSFT)

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Outline

- Moyal $\star$ product and QM
- Moyal $\star$ formulation of SFT (MSFT)
- Stringy Origin of QM (mini-MSFT)

Based on I. Bars and D. Rychkov,

arXiv:1407.4699, Background Independent MSFT

arXiv:1407.4699, Stringy Origin of QM

Moyal Star and Quantum Mechanics

- Basic ingredient of QM, a very successful but myterious rule

$$\left[\hat{X}^M, \hat{P}_N\right] = i\hbar\delta^M_N \text{ for all dofs.}$$

Operators & representations, Hilbert spc etc. follows. Also product

$$\hat{A}_{12}\left(\hat{X}, \hat{P}\right) = \hat{A}_1\left(\hat{X}, \hat{P}\right)\hat{A}_2\left(\hat{X}, \hat{P}\right)$$

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- Another formalism for QM - classical phase space + Moyal $\star$

Weyl, Wigner, Moyal,...-see summary in sec.III 1407.4699

image $A(X,P)=\int dy e^{iPy}\langle X-\frac{y}{2}|\hat{A}|X+\frac{y}{2}\rangle$ & reverse $\hat{A}=\int dXdP\ A(X,P)\ :\delta\left(X-\hat{X}\right)\delta\left(P-\hat{P}\right):$

integral repr. same exp

$$\begin{aligned} A_{12}\left(X,P\right) &= A_1\left(X,P\right)\star A_2\left(X,P\right) \\ \left(\star = e^{\text{Poisson}}\right) &: \quad \star = \exp\left(\frac{i\hbar}{2}\left(\overleftarrow{\partial}_{X^M}\overrightarrow{\partial}_{P_M} - \overrightarrow{\partial}_{X^M}\overleftarrow{\partial}_{P_M}\right)\right) \end{aligned}$$

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- Check: $\left[X^M, P_N\right]_\star = \left(X^M\star P_N - P_N\star X^M\right) = i\hbar\delta^M_N$

Note: QM requires ALL PHASE SPACE $\left(X^M, P_M\right)$ in $\star$.

This is NOT what we do in MSFT.

Moyal Star for String Joining/Splitting

- Witten's cubic open SFT: $S = \int \left(\frac{1}{2}\psi \star \hat{Q}\psi + \frac{g}{3}\psi \star \psi \star \psi\right)$
  matrix-like product, so $\psi \sim$ like matrix representation of QM operator
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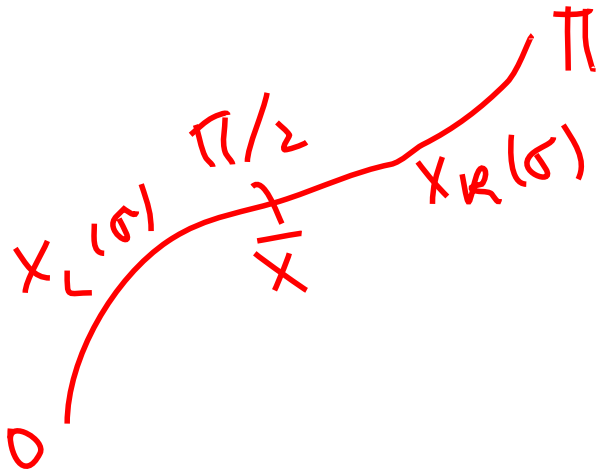
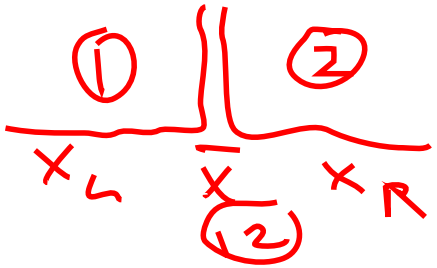


$$\Psi(X(\sigma)) = \Psi(x_L(\sigma), \bar{x}, x_R(\sigma))$$

$$\sim M_{x_L, x_R}(\bar{x})$$

$$\Psi_{12}(x_L, x_R) = \int Dz\, \Psi_1(x_L, z)\, \Psi_2(z, x_R)$$



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  Midpoint drops: $\overleftarrow{\partial}_{p_{-M}\left(\frac{\pi}{2}\right)}\overrightarrow{\partial}_{x_{+}^{M}\left(\frac{\pi}{2},\varepsilon\right)} \longrightarrow \frac{\pi}{2}$ excluded automatically!

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- $\star$ **background independent**: X^M, P_M upper/lower M, for any CFT.
- $\star$ is SUSY $\mathrm{OSp}(d|2)$: $M = (\mu, b, c)$ matter and ghosts.

More Properties of new MSFT

- Regulator $x_+^M(\sigma,\varepsilon) = e^{-\varepsilon|\partial_\sigma|} x_+^M(\sigma) \to x_0 + \sqrt{2}\sum_{e\geq 2} e^{-\varepsilon e} x_e \cos e\sigma$. In computations $\partial_{x_+^M(\sigma)} x_+^N(\sigma',\varepsilon)$ or $e^{-\varepsilon|\partial_\sigma|}\partial^N_{p_{-M}(\sigma)} p_{-N}(\sigma')$, only **regulated** $\delta_\varepsilon(\sigma,\sigma')$, everything finite, no midpoint associativity anomalies, reliable computations. $|\partial_\sigma| \equiv \sqrt{-\partial_\sigma^2}$

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- D-brane boundary conditions at end points $\sigma = 0, \pi$. **Regulated** delta functions $\delta_\varepsilon^{\pm(nn \text{ or } dd)}(\sigma,\sigma')$ that satisfy boundary conditions, Neumann-Neumann (nn), Dirichlet-Dirichlet (dd), symmetric or antisymmetric $(\pm)$ relative to midpoint.

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- Representation of $\hat{Q}_{BRST} A = \{Q, A\}_\star$, explicit FIELD $Q(x_+, p_-)$ ANY CFT $\hat{Q}(\hat{X},\hat{P}) \to Q(x_+,p_-)_\star$ same function in half phase space

$$Q_{\substack{flat\\CFT}} = \int_0^{\pi/2} \frac{d\sigma}{2} \left[\begin{array}{c} p_b\left(\pi^2 p_-^\mu p_{-\mu} + x_+'^\mu x_{+\mu}' + i\pi^2 p_{-c} p_{-b} + i x_+'^b x_+'^c\right) \\ -i\left(|\partial_\sigma|^{-2} x_+'^c\right)\left(2x_+'^\mu p_{-\mu} + x_+'^c p_{-c} + x_+'^b p_{-b}\right) \end{array} \right]$$

satisfies $Q(x_+,p_-) \star Q(x_+,p_-) = 0$, a FIELD like $A(x_+,p_-)$.

MSFT Action

- Action. Midpoint insertion, $\partial_{\bar{b}} A = \partial_{x^b(\pi/2)} A$, to balance ghost No.

$$S(A) = -\int \left(\overset{+1}{A} \star \overset{-1}{Q} \star \overset{+1}{A} + \frac{g_0}{3} \overset{-1}{A} \star \overset{+1}{\partial_{\bar{b}}} \overset{-1}{A} \star \overset{+1}{\partial_{\bar{b}}} \overset{-1}{A} \right) \quad .$$

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- Has BRST gauge symmetry: $\delta_\Lambda A = [Q, \Lambda]_\star + g_0 \{\partial_{\bar{b}} A, \partial_{\bar{b}} \Lambda\}_\star$

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$$S(A) = -\int^{+1\ -1\ \ +1\ \ -1\ \ \ -1\ \ \ +1-1\ \ \ +1-1} \left(A \star Q \star A + \frac{g_0}{3} A \star \partial_{\bar{b}} A \star \partial_{\bar{b}} A \right) \quad .$$

- Has BRST gauge symmetry: $\delta_\Lambda A = [Q, \Lambda]_\star + g_0 \{\partial_{\bar{b}} A, \partial_{\bar{b}} \Lambda\}_\star$
- Can fix to Siegel gauge, $A_s = x_0^b A_s^{(0)}$ (no more ghost zero modes)

$$S_s = -\int' \left(A_s^{(0)} \star \left(\mathcal{L}_0 - \frac{1}{2} \right) \star A_s^{(0)} + \frac{g_0}{3} A_s^{(0)} \star A_s^{(0)} \star A_s^{(0)} \right)$$

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- Agrees with all previous successful MSFT computations of IB+Matsuo+Kishimoto+Park, including off shell Veneziano amplitude with greater accuracy (IB+Park, 2003).

mini-MSFT and QM: A Model with Only Two Particles

- Two particles named L (left) and R (right), located at $(\vec{x}_L, \vec{x}_R)$.
  Center of mass and relative coordinates

$$R^i = \frac{1}{2}\left(x_L^i + x_R^i\right), \quad r^i = \left(x_L^i - x_R^i\right)$$
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- Parallels between the full MSFT and mini-MSFT:

$$R^i \sim x_+^M(\sigma), \; r^i \sim x_-^M(\sigma), \; P_i \sim p_{+M}(\sigma), \; p_i \sim p_{-M}(\sigma)$$

  and $\hat{H}(\hat{P}, \hat{p}, \hat{R}, \hat{r}) \sim \hat{Q}_{BRST}$ or Virasoro $\hat{L}_0$ - A "kinetic" operator of mini-MSFT.

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- Field in position space = probability amplitude, $\psi(x_L, x_R) = \langle x_L, x_R | \psi \rangle$. Consider the Fourrier transform

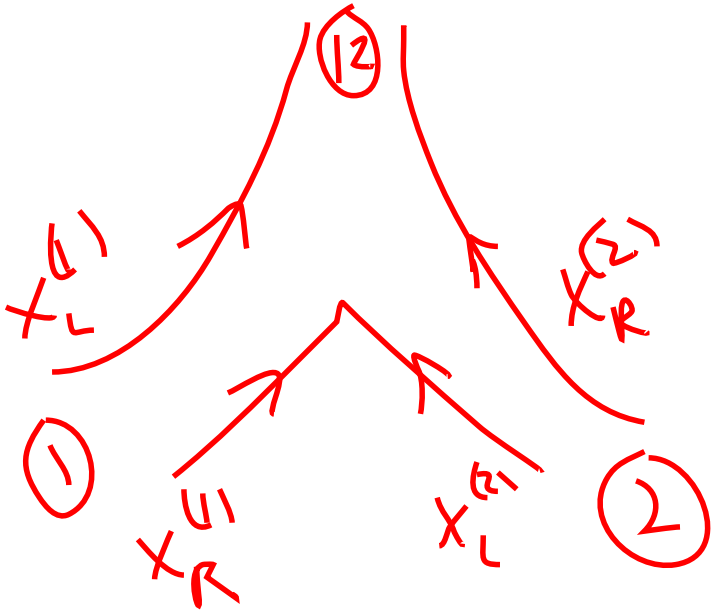
$$\psi(x_L, x_R) = \psi(R, r) \overset{Fourrier}{\leftrightarrow}{}^{(r,p)} A(R, p) = \langle R, p | \psi \rangle. \qquad (1)$$

  where $\langle R, p|$ is also a complete eigenbasis - a **half-phase-space**.

Star Product in mini-MSFT

- Matrix-like product of fields in position space. Like quarks (0-branes) at the ends of a string. Computing the probability amplitude of joining two strings

$$\psi_{12}\left(x_L, x_R\right) = \int_{-\infty}^{\infty} d^n z \; \psi_1\left(x_L, z\right) \psi_2\left(z, x_R\right),$$



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- Equivalent $\star$ product in the half-phase-space.

$$\begin{aligned} A_{12}\left(R,p\right) &= A_1\left(R,p\right) \exp\left(\frac{i\theta}{2}\left(\overleftarrow{\partial}_{R^i}\overrightarrow{\partial}_{p_i} - \overrightarrow{\partial}_{R^i}\overleftarrow{\partial}_{p_i}\right)\right) A_2\left(R,p\right) \\ &= A_1 A_2 + \frac{i\theta}{2}\left(\frac{\partial A_1}{\partial R^i}\frac{\partial A_2}{\partial p_i} - \frac{\partial A_1}{\partial p_i}\frac{\partial A_2}{\partial R^i}\right) + \cdots \\ \left[R^i, p_j\right]_\star &= i\theta\delta^i_j \text{ AHA !!! iQM joining/splitting, NOT QM} \end{aligned}$$

Physically different but mathematically analogous to the usual Moyal product. Note usual Moyal would give $\left[R^i, p_j\right]_{\star\text{usual}} = 0$.

Deriving QM Commutation Rules from iQM

- Map between operators in QM and their representative in iQM - an elegant and intuitive property of MSFT.

$$\left.\begin{array}{l}\hat{x}^i_L A = R^i \star A \\ \hat{p}_{iL} A = p_i \star A\end{array}\right\} \text{ for particle } L \text{ the } \star \text{ product from left}$$

$$\left.\begin{array}{l}\hat{x}^i_R A = A \star R^i \\ \hat{p}_{iR} A = -A \star p_i\end{array}\right\} \text{ for particle } R \text{ the } \star \text{ product from right}$$

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only $\star$ products between *two fields in the half-phase-space*.

- Compute commutators, use associativity of $\star$

$$\begin{aligned}\left[\hat{x}^i_L, \hat{p}_{Lj}\right] A &= \left[R^i, p_j\right]_* \star A = i\theta\delta^i_j A, \\ \left[\hat{x}^i_R, \hat{p}_{Rj}\right] A &= A \star \left[-p_j, R^i\right]_* = i\theta\delta^i_j A, \\ \left[\hat{x}^i_L, \hat{p}_{Rj}\right] A &= -R^i \star A \star p_j + R^i \star A \star p_j = 0, \\ \left[\hat{x}^i_R, \hat{p}_{Lj}\right] A &= p_j \star A \star R^i - p_j \star A \star R^i = 0.\end{aligned}$$

Obtained QM from iQM ! Must identify the parameter θ with $\hbar$.

$$\theta = \hbar.$$

All Operators in QM Have iQM Representatives

- Observables of only particle L or only particle R (case full MSFT)

$$\hat{O}_L\left(\hat{x}_L, \hat{p}_L\right) A = O_{L\star}\left(R, p\right) \star A \ , \quad (\star \text{ from left}).$$
$$\hat{O}_R\left(\hat{x}_R, \hat{p}_R\right) A = A \star O_{\star R}\left(R, p\right) \ , \quad (\star \text{ from right}).$$

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- General QM operator, e.g. arbitrary Hamiltonian $\hat{H}(\hat{x}_L,\hat{p}_L,\hat{x}_R,\hat{p}_R)$

$$\hat{H}(\hat{x}_L,\hat{p}_L,\hat{x}_R,\hat{p}_R)A = H\left(\left(R\star\right),\left(p\star\right),\left(\star R\right),\left(\star p\right)\right)A\left(R,p\right).$$

Keep order of the original operators, and keep order of left $\star$ or right $\star$

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Keep order of the original operators, and keep order of left $\star$ or right $\star$

- Example, harmonic oscillator type central force problem

$$\begin{aligned}\hat{H}_1 A &= \left[\frac{1}{2}\left(\hat{p}_L^2+\hat{p}_R^2\right)+\frac{\omega^2}{2}\left(\hat{x}_L-x_R\right)^2\right]A\left(R,p\right)\\ &= \frac{1}{2}\left(\vec{p}^{\,2}+\omega^2\vec{R}^2\right)\star A+\frac{1}{2}A\star\left(\vec{p}^{\,2}+\omega^2\vec{R}^2\right)-\omega^2\vec{R}\star A\star\vec{R}\\ &= \left[-\frac{1}{4}\hbar^2\partial_R^2+p^2-\frac{\omega^2}{2}\hbar^2\partial_p^2\right]A\left(R,p\right)\end{aligned}$$

String = QCD in 2D - a cute system in mini-MSFT

I.B. Phys.Rev.Lett 36 (1976) 1521

- 2D-QCD at large N = 2D-strings with quarks (0-branes) at the ends $\left(x^{\mu}_{L,R}\right)$.

  Established in 1976, spectrum and interactions (IB). The iQM equivalent for spectrum Eq

$$\begin{aligned}
\hat{H}A &= \left[\frac{m_L^2}{2\hat{p}_L^+} + \frac{m_R^2}{2\hat{p}_R^+} + \gamma\left|\hat{x}_L^- - \hat{x}_R^-\right|\right] A\left(R,p\right) \\
&= \frac{m_L^2}{2p} \star A + A \star \frac{m_R^2}{2p} + \gamma\left|(R\star) - (\star R)\right| A\left(R,p\right) \\
&= \frac{m_L^2}{2p} \star A\left(R,p\right) + A\left(R,p\right) \star \frac{m_R^2}{2p} + \gamma\hbar \int' \frac{dk}{\pi k^2} A\left(R,p+k\right)
\end{aligned}$$

  The last line is equivalent to the 'tHooft integral equation for quark-antiquark bound state.

- Interactions between mesons via string joining/splitting in mini-MSFT

$$S = \int d^n R d^n p \left[\frac{1}{2} A\left(\hat{H}A\right) + \frac{g}{3} A \star A \star A + \cdots\right]$$

  Feynman graphs analogous to those computed in 1976. Seems like a fun system to play with and generalize. Also use it as a simple model analogous to SFT.

Summary

- Full MSFT satisfies the same iQM $\rightarrow$QM properties. Then this applies to all systems derivable from open SFT, from single particles to complex systems. However, it is not all physics. Still could ask, what about the full M-theory, second quantization etc.? Therefore, let's say the door is open to understanding the origins of QM from string theory.

Summary

- Full MSFT satisfies the same iQM $\rightarrow$QM properties. Then this applies to all systems derivable from open SFT, from single particles to complex systems. However, it is not all physics. Still could ask, what about the full M-theory, second quantization etc.? Therefore, let's say the door is open to understanding the origins of QM from string theory.
- More modest level: mini-MSFT seems fun, and may be useful in its own right to discuss some perturbative and non-perturbative physics in certain circumstances.

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The relation between b, c ghosts and X^b, P_b and X^c, P_c

$$\hat{B}(\pm\sigma) = \left(-i\hat{X}^b(\sigma) \pm \pi\left|\partial_\sigma\right|^{-2}\partial_\sigma\hat{P}_c(\sigma)\right),$$
$$\hat{C}(\pm\sigma) = \left(\pi\hat{P}_b(\sigma) \mp i\partial_\sigma\hat{X}^c(\sigma)\right).$$

B, C are exponential series, with $e^{in\sigma}$, $-\infty < n < \infty$, but (X, P) are only *cosine* series $\cos n\sigma$, while their derivatives are $\sin n\sigma$ series.