SISSA ISAS

SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES
I-34014 Trieste ITALY - Via Beirut 4 - Tel. [+]39-40-37871 - Telex:460269 SISSA I - Fax: [+]39-40-3787528

October 2005 – Entrance Examination: Statistical and Biological Physics

Solve **one** of the following problems (no extra credit is given for attempts to solve more than one problem). Write out solutions clearly and concisely. State each approximation used. Diagrams welcome. Number page, problem, and question clearly. Do not write your name on the problem sheet, but use extra envelope.

Problem 1

Consider a collection of 2 large spheres and n "small" spheres. The centres of the spheres can stay only on the vertices of a regular polygon of N sides, each of length a. The diameter of each of the two (identical) large spheres is $1.8a$ while that of the (identical) small spheres if $a/2$. The spheres are impenetrable therefore, due to excluded volume effects: (i) no two spheres can occupy the same site (vertex); (ii) two large spheres cannot occupy neighbouring sites; (iii) no small sphere can occupy a site adjacent to a large one.

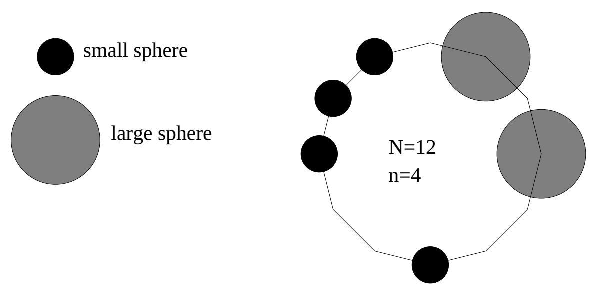


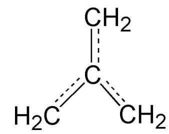
Figure 1: Example of allowed configuration of two large spheres and $n = 4$ small spheres on a ring of $N = 12$ vertices.

Assume that the number of sites, N, is even and that $N >> n >> 1$.

1. Calculate the number of microstates of the system when the centres of the large spheres are at separation $2a$, $3a$, $4a$, ... $N/2a$. (The separation is the shortest distance along the ring going either clockwise or counter-clockwise).
2. Assuming the equiprobability of the microstates, calculate the probability that the large spheres are at separation $2a$, $3a$, $4a$... . What is their most probable separation? What is the least probable separation?
3. Discuss how the average separation of the large spheres depends on n at fixed N. How would it be affected by temperature in the canonical ensemble? Explain and comment your results.
4. Calculate the partition function of the system, $\mathcal{Z}$, and estimate $\partial \ln \mathcal{Z}/\partial n$ and $\partial \ln \mathcal{Z}/\partial N$. Discuss the physical significance of your results.

Problem 2

1. Calculate the Hückel π-electron energy and the π-orbital coefficients of trimethylenemethane:



What is the π ground state electronic configuration?

2. Calculate the π system delocalization energy of trimethylenemethane relative to two isolated ethylene molecules.

3. Which are the symmetry elements of this molecule? Which is its symmetry point group?

4. Classify the π-orbitals according to their symmetry.

5. Which are the allowed π-π^* electronic transitions from the ground state? Consider only the electric dipole induced transitions.

Problem 3

Address one of the following questions if you intend to follow an Experimental Curriculum in Statistical and Biological Physics.

1. Discuss the physical principle of electron transfer in organic and biological systems.

2. Discuss the physical principles and the limits of two complementary nanoscopic probes for biological and soft matter investigation

3. Discuss the structural measurements techniques and properties of membranes and membrane-like systems (such as Langmuirm Langmuir-Blodgett and Self-Assembled monolayers).