PhD Entrance Examination
Physics and Chemistry of Biological Systems
September 2021

IMPORTANT GUIDELINES

- Solve **three** of the following problems.
- No extra credit is given for attempts to solve more than three problems.
- Do not write your name on the problem sheet nor use any mark that can identify you, as this would invalidate your exam.
- Write out solutions clearly and concisely. State each approximation used. Diagrams welcome.
- Number page, problem, and question clearly.
- All essays/solutions should be written in English.

Exercise 1

A molecule can be found in three different conformations (A, B, and C). The free-energy differences between these conformations are $\Delta G_{AB} = G(B) - G(A) =$ 4 kJ/mol and $\Delta G_{BC} = G(C) - G(B) = 16$ kJ/mol, so that conformation A is the one with the lowest free energy. The distance between the two ends of the molecule in conformations A, B, and C is 1nm, 2nm and 0.6 nm respectively.

- Which is the population of each of the three conformations at equilibrium if temperature $T = 0$? Which is the average end-to-end distance in that case?
- Which is the population of each of the three conformations at equilibrium in the limit of an infinitely large T? Which is the average end-to-end distance in that case?
- Plot the population of the three conformations and the average value of the end-to-end distance as a function of the temperature of the system. Consider $k_B T = 2.5 kJ/mol$

Exercise 2

Consider a system with three states, which can perform transitions between each other with the rate constants represented in this scheme:

$$\mathrm{A} \underset{k_{AB}=1\ \mathrm{s}^{-1}}{\overset{k_{BA}=2\ \mathrm{s}^{-1}}{\leftrightarrows}} \mathrm{B} \underset{k_{BC}=2\ \mathrm{s}^{-1}}{\overset{k_{CB}=1\ \mathrm{s}^{-1}}{\leftrightarrows}} \mathrm{C}$$

The probability of observing the system in state A at time t is given by

$$p_A(t) = p_A^{eq} + C_1 \exp(-t/\tau_1) + C_2 \exp(-t/\tau_2)$$

where p_A^{eq} is the equilibrium probability of observing state A, and C_1 and C_2 are constants depending on the initial condition.

1. Compute the equilibrium probability of observing A
2. Compute the relaxation times τ_1 and τ_2.

Exercise 3

The one-dimensional diffusive motion of a particle of mass μ and friction γ under gravity is described by the Langevin equation:

$$\gamma \frac{dz(t)}{dt} = -\mu g + \eta(t)\,, \quad (1)$$

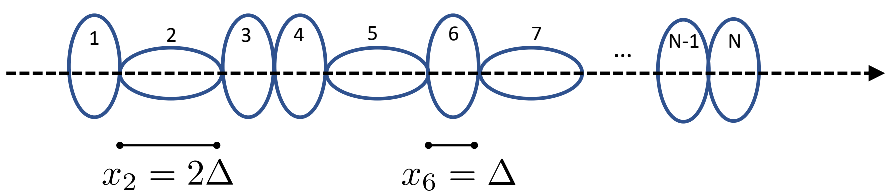
where g is the gravitational acceleration, and the stochastic force η obeys standard correlations $\langle \eta(t) \rangle = 0$ and $\langle \eta(t')\eta(t) \rangle = 2\,\kappa_B T\,\gamma\,\delta(t'-t)$ with κ_B the Boltzmann constant and T the absolute temperature.

Answer the following questions:

1. Use dimensional analysis to write the appropriate time scales $\tau_g(\delta z)$ and $\tau_d(\delta z)$ associated to, respectively, motion by gravity and by diffusion on a length scale δz.
2. For a given T and δz, for which values of the mass μ is diffusion negligible compared to gravity-induced motion?
3. Show that (2) applies to grains of salt ($\mu_{\text{grain}} \approx 10^{-7}$ kg) at room temperature ($\kappa_B T \approx 4 \cdot 10^{-21}$ N·m, $g \approx 10$ m/s^2) in a glass of water ($\delta z \approx 10^{-1}$m).
4. In water, salt dissolves in ions. For the typical ion mass $\mu_{\text{ion}} \approx 10^{-26}$ kg is gravity still important?

Exercise 4

Consider a directed chain of N catenated rings in one dimension. The rings have two possible orientations: vertical or horizontal. As shown in the sketch, the longitudinal size of the ith ring is $x_i = \Delta$ for vertical orientation and $x_i = 2\Delta$ for horizontal orientation. The rings are randomly oriented, and their longitudinal overlap is negligible.



1. Calculate the average end-to-end extension of the chain, $R_{ee} = \sum_{i=1}^{N} x_i$.
2. Write down the (formal) expression for $P(R_{ee} = m\Delta)$, the probability that the end-to-end distance is equal to $m\Delta$, where $N \leq m \leq 2N$. Sketch the $P(R_{ee})$ curve.
3. Consider now the normalised end-to-end distance, R_{ee}/N. Sketch and discuss how its probability distribution, $P(R_{ee}/N)$, varies with N, and comment the $N \to \infty$ limit.

Exercise 5

The hydrophobic effect, which prevents oil and water from mixing, is one of the most central driving forces for processes in chemistry and biology. In this question, you will explore some ideas related to this phenomenon:

1. Consider two oily molecules such as methane interacting with each other. What types of interaction forces exist between the two molecules? Make a sketch of the interaction potential between the two molecules as a function of the distance r between them.

2. Relative to thermal energy, the strength of the interaction between the two methane molecules is quite weak. Why does the hydrophobic effect then lead to phenomena such as oil and water not mixing?

3. If we were to mutate methane into a methanol molecule, how would the interaction forces change with the surrounding water?

Exercise 6

The freezing point of a water solution with concentration 0.350 M (molarity, moles per liter of solution) of an acid HA having molecular weight $MW = 148.3$ g/mol is T_{fr}=-0.908 °C. The density of the solution is $d = 1.025$ g/ml, the cryoscopic constant of water is $K_{cr} = 1.853$ K Kg/mol and the ideal gas constant R is 0.08206 l atm K^{-1} mol^{-1} . Find:

1. the degree of dissociation of HA

2. the osmotic pressure at 28.0 °C

Assume that the degree of dissociation remains constant from -0.908 °C to 28.0 °C.

To solve the exercise consider the freezing point depression law: $\Delta T_{fr} = iK_{cr}m$ where i is the van't Hoff factor and m is the concentration in molality (moles per 1000 g of solvent).

The van't Hoff factor is defined as $i = 1 + \alpha(\nu - 1)$ where α is the degree of dissociation and ν is the number of ions in which the compound dissociates.