QCD@LHC:
challenges and opportunities
in heavy flavor production

Laura Reina

• The **Large Hadron Collider (LHC)** is testing new ground and will answer some of the fundamental open questions of Particle Physics:
  
  → Electroweak (EW) symmetry breaking: Higgs mechanism?
  
  → New Physics (NP) in the TeV range?
  
  → …

• The incredible physics potential of the LHC relies on our ability of providing **very accurate QCD predictions**:

  → **Discovery**: precise prediction of signals/backgrounds;

  → **Identification**: precise extraction of parameters ($\alpha_s, m_t, M_H, y_{t,b}, M_X, y_X, \ldots$);

  → **Precision**: $\sigma_{W/Z}$ as parton luminosity monitors (PDF’s), …

• Heavy Quark production w/o associated particles crucial to control:
  
  → top/bottom-quark properties;

  → signatures involving hard (b)-jets, multi-leptons and missing $E_T$
  (background to new physics signatures).

Think of: $t\bar{t}, t\bar{t} + H, b\bar{b} + H, b\bar{b} + W/Z, t\bar{t} + W/Z, t\bar{t}b\bar{b}, t\bar{t}WW/ZZ, \ldots$
Outline

• Overview of QCD benchmarks for hadronic colliders.

• Focusing on Heavy Quark physics:
  → toward a precise prediction of $Q\bar{Q}$ production;
  → heavy quark production with Higgs bosons: $Ht\bar{t}$, $Hb\bar{b}$;
  → heavy quark production with weak gauge bosons: $Wb\bar{b}$, $Zb\bar{b}$;

  physical impact, theoretical progress and perspectives.

• Conclusions and outlook.
State of the art of QCD calculations
for hadronic processes

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(from N. Glover, updated)

Green light $\rightarrow$ Done!

Red light $\rightarrow$ Still work in progress!

NLO: $V + 4j$, $VV + 2j$, $t\bar{t} + 2j$, $t\bar{t} + b\bar{b}$, $Ht\bar{t}/b\bar{b}$, $V + b\bar{b}/t\bar{t}$, $VVV$, $H + 2j$,…

(so many people!)

NNLO: since ’07 steady progress towards $q\bar{q}, gg \rightarrow Q\bar{Q}$ at NNLO
(Czakon, Mitov, Moch, Beneke, Falgari, Schwinn, Bonciani, Ferroglia, Gehrmann, …)

plus NNLO PDFs.
Why pushing the Loop Order . . .

- **Stability and predictivity of theoretical results**, since less sensitivity to unphysical renormalization/factorization scales. First reliable normalization of total cross-sections and distributions. Crucial for:
  - → precision measurements ($M_W, m_t, M_H, y_{b,t}, \ldots$);
  - → searches of new physics (precise modeling of signal and background);
  - → reducing systematic errors in selection/analysis of data.

- **Physics richness**: more channels and more partons in final state, i.e. more structure to better model (in perturbative region):
  - → differential cross-sections, exclusive observables;
  - → jet formation/merging and hadronization;
  - → initial state radiation.

- **First step towards matching with algorithms that resum particular sets of large corrections in the perturbative expansion**: resummed calculations, parton shower Monte Carlo programs.
• NLO: challenges have largely been faced and enormous progress has been made:
  
  → traditional approach (FD’s) made more efficient to handle high multiplicity;
  
  → new techniques based on unitarity methods and recursion relations offers a powerful and promising alternative, particularly suited for automation;
  
  → interface with parton shower MC well advanced (MC@NLO, POWHEG, Sherpa)

• When is NLO not enough?

  → When NLO corrections are large, to tests the convergence of the perturbative expansion. This may happen when:
    ▶ processes involve multiple scales, leading to large logarithms of the ratio(s) of scales;
    ▶ new parton level subprocesses first appear at NLO;
    ▶ new dynamics first appear at NLO;
    ▶ . . .

  → When truly high precision is needed (very often the case!).

  → When a really reliable error estimate is needed.
$Q\bar{Q}$ production at the Tevatron and LHC

- **NNLO**: 2-loop corrections to both $q\bar{q} \rightarrow Q\bar{Q}$ and $gg \rightarrow Q\bar{Q}$ calculated in the limit $m_Q^2 \gg \hat{s}, -\hat{t}$, step by step. Czakon, Mitov, Moch (since 07-08)

- **NNLO**: full 2-loop corrections to $q\bar{q} \rightarrow Q\bar{Q}$ calculated numerically. Czakon (08)

- **NNLO**: partial 2-loop corrections to $q\bar{q} \rightarrow Q\bar{Q}$ calculated analytically. Bonciani, Ferroglia, Gerhmann, Maitre, Studerus, (08-09)

- **NNLL-NLO**: resumming soft threshold corrections at NNLL, Cacciari, Czakon, Mangano, Mitov, Nason (11)

(Using results derived by Beneke, Czakon, Falgari, Mitov, Schwinn (09))
Cacciari, Czakon, Mangano, Mitov, Nason (arXiv:1111.5869)

- scale+PDF uncertainty: 10 – 15% (conservative b/c incomplete NNLO)
- comparable to current experimental uncertainty: precision requires NNLO.
$Q\bar{Q}$ associated production with a Higgs boson

- **Motivations**
  - $Ht\bar{t}$: important channel when $H \to \gamma\gamma, b\bar{b}, \tau^+ \tau^-$;
  - $Ht\bar{t}$: instrumental to Higgs couplings determination;

- **Interesting aspects of the NLO calculation.**

- **Latest studies:** LHC Higgs Cross Section Working Group.
LHC, $pp \rightarrow t\bar{t}H$: NLO cross section

Dawson, Jackson, Orr, L.R., Wackeroth
Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas

- Independent calculations show full agreement.
- Theoretical uncertainty from scale dependence, $\alpha_s$, PDFs: below 15%.
- Several crucial backgrounds: $t\bar{t} + j$ (NLO, Dittmaier, Uwer, Weinzierl), $t\bar{t}b\bar{b}$ (Dittmaier et al., Bevilacqua et al.), $t\bar{t} + 2j$ (Bevilacqua et al.).
LHC, $pp \to t\bar{t}H$: interface with parton-shower MC

Frederix, Garzelli, Kardos, Papadopoulos, Trócsányi

- Very good agreement between MC@NLO and POWHEG.
- Provided state-of-the-art tools to experimental analyses.
- Need to focus on backgrounds, in particular $t\bar{t} + b\bar{b}$ and $t\bar{t} + jj$. 
$p\bar{p}, pp \rightarrow H + b$ jets: exclusive vs inclusive cross section

- **b-quarks identification** requires tagging ($p_T^b$ and $\eta^b$ cuts): **exclusive** (1 b-, 2 b-tags) vs **inclusive** (1 b-, 0 b-tags) cross section.

- **Exclusive modes** have smaller cross section, but also smaller background and they measure the bottom-quark Yukawa coupling unambiguously.

- **Inclusive modes** enhanced by **large collinear** $\ln(\mu_H^2/m_b^2)$ arising in the PS integration of untagged $b$-quarks in $gg \rightarrow b\bar{b}H$

  \[ b(x, \mu) = \frac{\alpha_s(\mu)}{2\pi} \log \left( \frac{\mu^2}{m_b^2} \right) \int_x^1 \frac{dy}{y} P_{qg} \left( \frac{x}{y} \right) g(y, \mu) \]

  - large collinear logs ($g \rightarrow b\bar{b}$)
  - regulated by $m_b$

They can be resummed by introducing a $b$-quark PDF:
• Semi-inclusive and inclusive cross sections: **2 approaches**

  → Use $q\bar{q}, gg \rightarrow b\bar{b}h$ (at NLO) → **4FNS**
  
imposing tagging cuts on only one or no final state $b$ quarks.
  
  → Use $b$-quark PDF, resumming the large collinear logs → **5FNS**

\[
\begin{align*}
\text{Perturbative series ordered in Leading and SubLeading powers of} & \quad \alpha_s \ln(\mu^2_H/m_b^2) . \\
\text{→ Expect consistence at higher order when comparing} & \quad q\bar{q}, gg \rightarrow b\bar{b}H \\
\text{(NLO) to} & \\
\text{→ } b\bar{b} \rightarrow H \text{ (NNLO) (no } b\text{-tag)} & \\
\quad \text{(R.Harlander, W.Kilgore; D.Dicus, T.Stelzer, Z.Sullivan, S.Willenbrock)} \\
\text{→ } bg \rightarrow bH \text{ (NLO) (one } b\text{-tag)} & \\
\quad \text{(J.Campbell, R.K.Ellis, F.Maltoni, S.Willenbrock)}
\end{align*}
\]
Inclusive cross sections: 4FNS vs 5FNS

$\sigma_{\text{NLO}} [\text{pb}]$ Tevatron, $\sqrt{s} = 1.96$ TeV

$\sigma_{\text{NLO}} [\text{pb}]$ LHC, $\sqrt{s} = 14$ TeV

Dawson, Jackson, L.R., Wackeroth
Motivations:

- important test of QCD
- main background to several important signatures, for example,
  - $WH/ZH$ associated production;
  - single-top production;
  - Higgs production with $H \rightarrow ZZ$;
  - several non-standard model signatures.

Main QCD studies:

- $Wb\bar{b}/Zb\bar{b}$ at NLO, $b$ massless/massive;
- $Wb + \text{jet}, Zb + \text{jet}$: 4FNS vs 5FNS at NLO;
- interface of NLO $Wb\bar{b}/Zb\bar{b}$ with parton shower Monte Carlo programs;
- $Wb\bar{b} + j$ at NLO, $b$ massive: one-loop contributions.

Comparison with Tevatron and LHC data
Associated production of SM Higgs with weak vector bosons

→ NNLO QCD corrections have been calculated for the signal [O.Brien, A.Djouadi and R.Harlander, 2004]

→ $O(\alpha)$ EW corrections have been calculated for the signal [M.L.Ciccolini, S.Dittmaier and M.Kramer, 2003]

→ Results for $WH/ZH$ associated production, Summer 2011

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diagram.png}
\end{figure}
$W + b$ jets

Studied at NLO in QCD/measured in experiments:

- $W + 2b$ jets ($m_b \neq 0$):
  - Badger, Campbell, Ellis, arXiv:1011.6647 (with $W \rightarrow l\nu$)
  - Oleari, L. R., arXiv.1105.4488 $\rightarrow$ POWHEG
  $\rightarrow$ MC@NLO

- $W + 2b +$ jet:
  - L.R., Schutzmeier, arXiv:1110.4438 (one-loop corrections)

- $W + 2$ jets with at least one $b$ jet:
  - Campbell, Ellis, Febres Cordero, Maltoni, L.R., Wackeroth, Willenbrock,
    arXiv:0809.3003
  - the CDF collaboration, arXiv:0909.1505,
  - the ATLAS collaboration, arXiv:1109.1470,
    Campbell, Caola, Febres Cordero, L.R., Wackeroth, arXiv:1107.3714
$Z + b$ jets

Studied at NLO in QCD/measured in experiments:

- $Z + 2b$ jets ($m_b \neq 0$):
  - Oleari, L.R., in preparation $\rightarrow$ POWHEG
    $\rightarrow$ MC@NLO

- $Z + 1b$ jet, $Z + 2$ jets with at least one $b$ jet:
  - Campbell, Ellis, Maltoni, Willenbrock, hep-ph/0312024
  - Campbell, Ellis, Maltoni, Willenbrock, hep-ph/0510362
  - the CDF collaboration, hep-ex/0812.4458
  - the D0 collaboration, arXiv:1010.6203
  - the ATLAS collaboration, arXiv:1109.1403
$W/Z + 2b$ jets: based on $Wb\bar{b}/Zb\bar{b}$

**LO Feynman diagrams:**

$q\bar{q}' \rightarrow Wb\bar{b}$ and $q\bar{q} \rightarrow Zb\bar{b}$

**Subprocesses at LO:**

$\rightarrow Wb\bar{b}: \ q\bar{q}' \rightarrow Wb\bar{b}$

$\rightarrow Zb\bar{b}: \ q\bar{q} \rightarrow Zb\bar{b}$ and $gg \rightarrow Zb\bar{b}$

$q\bar{q} \rightarrow Zb\bar{b}$

$gg \rightarrow Zb\bar{b}$
Virtual corrections: $\hat{\sigma}_{ij}^{\text{virt}}$

$$
\hat{\sigma}_{ij}^{\text{virt}} = \hat{\sigma}(ij \to W/Z \, \bar{b}b) = \int d(PS_3) \sum \left( A_0 A_{1-\text{loop}}^\dagger + A_{1-\text{loop}}^\dagger A_0 \right)
$$

where, using traditional Feynman-diagram techniques,

$$
A_{1-\text{loop}} = \sum_{D_i} A_{D_i} \quad (D_i \to \text{Feynman diagrams})
$$

or using new techniques based in unitarity properties of scattering amplitudes,

$$
A_{1-\text{loop}} = \sum_i d_i I_4^i + \sum_i c_i I_3^i + \sum_i b_i I_2^i + \sum_i a_i I_1^i + R
$$

given $I_{4,3,2,1}^i \to \text{4-point, \ldots, 1-point scalar integrals}$.

Notice:

$\rightarrow$ Use dimensional regularization to regularize UV and IR divergencies.

$\rightarrow$ UV divergencies are canceled by a suitable set of counterterms.

$\rightarrow$ IR divergencies will cancel with $\hat{\sigma}_{ij}^{\text{real}}$. 
Real corrections: $\hat{\sigma}_{ij}^{\text{real}}$

\[
\hat{\sigma}_{ij}^{\text{real}} = \hat{\sigma}(ij \to W/Z \ b\bar{b} + k) = \int d(PS_4) \sum |A_{\text{real}}(ij \to W/Z \ b\bar{b} + k)|^2
\]

\[\rightarrow\] IR divergencies associated with the integration over the PS of the extra parton, can be extracted using standard techniques

\[\rightarrow\] Phase Space Slicing method with \textit{one or two cutoffs};

\[\rightarrow\] Dipole Subtraction method.

\[\rightarrow\] Non-singular (hard) phase space region: integrated numerically using Monte Carlo techniques.
Theoretical uncertainty meaningfully defined at NLO:

- residual renormalization/factorization scale dependence;
- large variations due to particular NLO effects;
- parton distribution function dependence;
- parametric dependencies (ex.: $m_b$)

In the following (unless differently specified):

- $b$-jets defined using:
  - $k_T$ algorithm with $R=0.7$;
  - $p_T$ and $\eta$ cuts.
- used CTEQ6M or CTEQ6.6 parton distributions functions;
- $m_b = 4.7$ GeV.
- **inclusive**: 2 $b$ jets (+ light jet);
- **exclusive**: 2 $b$ jets only.
Tevatron: scale dependence and theoretical uncertainty at NLO

\[ \mu_f / \mu_0 \]

\[ \sigma_{\text{total}} \ (\text{pb}) \]

- \( \mu_0 = M_W/2 + m_b \)
- \( R = 0.7 \)
- \( |\eta| < 2 \)
- \( p_T > 15 \text{ GeV} \)

- \( Wb\bar{b}: \) Tevatron (PRD 74 (2006) 034007)
- \( Zb\bar{b}: \) Tevatron (PRD 78 (2008) 074014)

\[ \Rightarrow \] Bands obtained by varying both \( \mu_R \) and \( \mu_F \) between \( \mu_0/2 \) and \( 4\mu_0 \) (with \( \mu_0 = m_b + M_V/2 \) (\( V = W, Z \))).

- LO uncertainty \( \sim 40\% \).
- Inclusive NLO uncertainty \( \sim 20\% \).
- Exclusive NLO uncertainty \( \sim 10\% \).
LHC: scale dependence and theoretical uncertainty at NLO

→ NLO corrections very large, particularly for inclusive production;

→ large NLO scale-dependence (LO: 30%, NLO$_{inc}$: 50%, NLO$_{exc}$: 20%), induced by the opening of the $qq(\bar{q}g) \rightarrow Wb\bar{b} + q'(\bar{q}')$ channel;

→ theoretical uncertainty not only given by scale-dependence!

(PRD 80:034015, 2009)
→ NLO corrections still large, particularly for inclusive production;

→ more moderate NLO scale-dependence (LO: 50%, NLO\textsubscript{inc}: 30%, NLO\textsubscript{exc}: 5\%): \(qg(\bar{q}g) \rightarrow Zb\bar{b} + q(\bar{q})\) channel not as dominant.
$W + 1b$ jet vs $W + 2b$ jets in a nutshell:

One or two LO processes, depending on choice of 4FNS vs 5FNS:

$\begin{align*}
q \bar{q}' &\rightarrow W b \bar{b} \text{ at tree level and one loop } (m_b \neq 0) \\
q \bar{q}' &\rightarrow W b \bar{b} g \text{ at tree level } (m_b \neq 0) \\
bq &\rightarrow W b q' \text{ at tree level and one loop } (m_b = 0) \\
bq &\rightarrow W b q' g \text{ and } b g \rightarrow W b q' \bar{q} \text{ at tree level } (m_b = 0) \\
gq &\rightarrow W b b q' \text{ at tree level } (m_b \neq 0) \rightarrow \text{avoiding double counting:}
\end{align*}$

Correspondently, at NLO:

1. $q \bar{q}' \rightarrow W b \bar{b}$ at tree level and one loop $(m_b \neq 0)$
2. $q \bar{q}' \rightarrow W b \bar{b} g$ at tree level $(m_b \neq 0)$
3. $bq \rightarrow W b q'$ at tree level and one loop $(m_b = 0)$
4. $bq \rightarrow W b q' g$ and $b g \rightarrow W b q' \bar{q}$ at tree level $(m_b = 0)$
5. $gq \rightarrow W b b q'$ at tree level $(m_b \neq 0) \rightarrow \text{avoiding double counting:}$

\[ b(x, \mu_F) = \frac{\alpha_s}{2\pi} \ln \frac{\mu^2}{m_b^2} \int_x^1 \frac{d z}{z} P_{qg}(z) g \left( \frac{x}{z}, \mu_F \right) \]

$\triangleright W + 2b$ jets: processes 1 + 2 + 5
$\triangleright W + 2$ jets with at least one $b$ jet: processes 1 + \cdots + 5.
• need to keep \( m_b \neq 0 \) for final state \( b \) quarks (one \( b \) quark has low \( p_T \)):
  first consistent NLO 5FNS calculation.

• four signatures studied: exclusive/inclusive, with single and double-\( b \) jets,
  \begin{itemize}
  \item \( Wb \) exclusive: \( Wb \) only;
  \item \( W(b\bar{b}) \) exclusive: \( W(b\bar{b}) \) only;
  \item \( Wb \) inclusive: \( Wb, Wb + j, Wb\bar{b} \);
  \item \( W(b\bar{b}) \) inclusive: \( W(b\bar{b}) \) and \( W(b\bar{b}) + j \).
  \end{itemize}

• calculate \( \sigma_{\text{event}} \) and \( \sigma_{b-\text{jet}} \) where
  \begin{align*}
  \sigma_{b-\text{jet}} &= \sigma_{\text{event}}(Wb \text{ incl.}) + \sigma_{\text{event}}(Wb\bar{b}) + \sigma_{\text{event}}(W(bb) \text{ incl.}) \\
  &= \sigma_{1j+2j} + \sigma_{\text{event}}(Wb\bar{b})
  \end{align*}

• overall improved scale dependence: NLO corrections to \( gq \rightarrow Wb\bar{b}q' \)
  partially included in 5FNS (see next slide).

• **Tevatron study** (compared to CDF measurement):
  \( p^j_T > 15 \text{ GeV}, |\eta^j| < 2, \) cone algorithm (\( R = 0.7 \)), CTEQ6M;

• **LHC study** (compared to ATLAS measurement):
  \( p^j_T > 25 \text{ GeV}, |\eta^j| < 2.1, \) anti-\( k_T \) jet algorithm (\( R = 0.4 \)),
  \( 4.2 \text{ GeV} \leq m_b \leq 5 \text{ GeV}, \) CTEQ6.6/MSTW08/NNPDF2.1;
Comparison with CDF measurement: a puzzle?

CDF Note 9321 (arXiv:0909.1505):

\[
\sigma_{b-\text{jet}}(W + b\text{jets}) \cdot Br(W \to l\nu) = 2.74 \pm 0.27\text{(stat)} \pm 0.42\text{(syst)} \text{ pb}
\]

[Neu, Thomson, Heinrich]


\[
\sigma_{b-\text{jet}}(W + b\text{jets}) \cdot Br(W \to l\nu) = 1.22 \pm 0.14 \text{ pb}
\]

[Campbell, Febres Cordero, L.R.]

For comparison:

Badger, Campbell, Ellis: \(0.913 < \sigma_{b-\text{jet}} \cdot Br < 1.389 \text{ pb}\)

ALPGEN prediction: 0.78 pb

PYTHIA prediction: 1.10 pb

- overall consistency of theoretical results;
- important to have a separate analysis from \(D0\)!
Comparison with ATLAS

From our calculation (arXiv:1107.3714):

\[ \sigma_{1j+2j}(W^+ + W^-) = 109.2^{+27.9}_{-16.6} \text{ (scale)}^{+7.4}_{-1.9} \text{ (PDF)}^{+5.7}_{-3.3} (m_b) \text{ pb} \]

More statistics necessary, possible new result by the end of the year.
Further development: $Wb\bar{b}$ implemented in POWHEG and MC@NLO, including $W \rightarrow l\nu_l$ decay.

Distribution sample:

- used (e.g. in ATLAS analysis) to estimate showering and hadronization uncertainties: $\leq 10 - 20\%$;
- $bq \rightarrow bq'W$ process being implemented.

[Oleari, L.R., arXiv.1105.4488]
Further development: towards $Wb\bar{b} + j$ at NLO

[ L.R., Schutzmeier, arXiv:1110.4438]

- One-loop QCD corrections to $qg \rightarrow Wb\bar{b} + q'$:

  ![Diagram](image)

  plus about 300 loop diagrams, keeping full $m_b$ dependence.

- Together with analogous corrections to $qq' \rightarrow Wb\bar{b} + g$ (obtained by crossing), they represent a well-defined piece of the NNLO QCD corrections to $pp, p\bar{p} \rightarrow Wb\bar{b}$: the one-loop virtual corrections from $2 \rightarrow 4$ processes.

- They provide the $O(\alpha_s)$ virtual corrections for $pp, p\bar{p} \rightarrow W + 2b$ jets $+ j$.

- In a fixed-flavor scheme, they also provide the $O(\alpha_s)$ virtual corrections for $pp, p\bar{p} \rightarrow W + b$ jet $+ j$. 
Method’s main characteristics

- Based on traditional Feynman-diagram evaluation.
- New techniques developed to
  - reduce diagram structure to small/minimal set of standard spinor structures using graph techniques;
  - combine different reduction methods to optimize calculation of numerically stable tensor integral coefficient.
- Each level of evaluation automatized within an overall interface (Python) that only takes as input the desired process:
  - diagram generated with a very modified version of QGRAPH;
  - algebraic manipulations to extract first level of SME and tensor structures done with FORM;
  - reduction of tensor integral coefficients and spinor structures use C++;
  - numerical stability checks use library of scalar integrals based on QCDLoop and LoopTools;
  - amplitude square calculation uses C++ to provide a user-friendly interface.
Reduction of spinor structures

spinor structures (standard matrix element or SME) → oriented graphs:
- nodes → spinor, gamma matrices, projectors, polarizations, ...
- links → contraction of indices and direction.

\[ \gamma^\mu \gamma_\mu \]

\[ \gamma^\mu \gamma^\alpha \gamma^\mu \]

\[ \gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\mu \]
oriented graphs $\rightarrow$ stored as relations and graph operations that are automatically implemented over the entire structure of a diagram at once,

$$\gamma^\mu \gamma^\alpha \gamma^\beta P_+ \times \gamma_\mu = \gamma^\mu P_+ \times (\gamma_\mu \gamma^\beta \gamma^\alpha P_+ + \gamma^\alpha \gamma^\beta \gamma_\mu P_-)$$

- algebraic relations (based on $d=4$ identities) translate into graph operations (e.g. shrinking of edges, exchange or addition of nodes, ...) and result into disconnected elementary graphs;
- number of final SME much smaller (from thousands to a few hundreds);
- coefficients of single elementary graphs collected via systematic labeling.
Reduction of tensor-integral coefficients

- reduce to standard pattern of momenta and masses;
- create list of dependences that are then reused every time the same pattern appears, including subdiagrams;
- choose evaluation order.

- if presence of numerical instabilities detected, switch to different reduction method.
$Z + 1b$ jet vs $Z + 2b$ jets in a nutshell:

LO processes, depend on choice of 4FNS vs 5FNS:

\[
\begin{array}{ccc}
q & \rightarrow & Z \\
\bar{q} & \rightarrow & \bar{Q} \\
g & \rightarrow & Q \\
g & \rightarrow & \bar{Q} \\
\end{array}
\]

\[\quad + \quad O(\alpha_s) \text{ corrections}\]

\[
\begin{array}{ccc}
Q & \rightarrow & Z \\
\quad + \quad Q & \rightarrow & Z \\
g & \rightarrow & Q \\
g & \rightarrow & \bar{Q} \\
\end{array}
\]

\[\quad + \quad O(\alpha_s) \text{ corrections}\]

Correspondently, at NLO:

1. $bg \rightarrow Zb$ at tree level and one loop (with $m_b = 0$);
2. $bg \rightarrow Zb + g$, $bq \rightarrow Zb + q$ (with $m_b = 0$);
3. $q\bar{q}, gg \rightarrow Zb\bar{b}$ at tree level and one loop (with $m_b \neq 0$);
4. $q\bar{q}, gg \rightarrow Zb\bar{b} + g$ and $gq(g\bar{q}) \rightarrow Zb\bar{b} + q(\bar{q})$ (with $m_b \neq 0$).

avoiding double counting in (3) and (4).

▷ $Z + 2b$ jets: processes 3 + 4

▷ $Z + 2$ jets with at least one $b$ jet: processes 1 + $\cdots$ + 4 (?).
Observe that,

\[
gg + \cdots \rightarrow gQ QZ gQ + \cdots
\]

are one loop corrections to \( q\bar{q}, gg \rightarrow b\bar{b}Z \) to be interpreted as a piece of the NNLO 5FNS calculation, comparable to two-loop corrections to (and double parton emission from) \( bg \rightarrow Zb \)?

overall better agreement with data.

\[
\text{MCFM} \rightarrow 1 + 2 + (3 + 4)_{LO}
\]
Conclusions and Outlook

• Heavy quark production ($Q\bar{Q}$) and associated heavy quark production ($Q\bar{Q} + H$, $Q\bar{Q} + W/Z$) play a fundamental role in the physics scenario of the LHC:

  ▶ precision studies ($m_t$ and parton luminosity from $Q\bar{Q}$);
  ▶ signal of new physics: $t\bar{t}H$, $b\bar{b}H$;
  ▶ background to new physics signals: $b\bar{b}W$, $b\bar{b}Z$.
  ▶ test ground of QCD ($2 \rightarrow 2$ at NNLO, $2 \rightarrow 3$ at NLO).

• Prepare to use $t\bar{t}H$, $b\bar{b}H$ to test couplings of the 125 GeV resonance:

  ▶ interface with parton shower MC available, for both signal and background;
  ▶ refine $H + 1b$ calculation.

• Improve $W/Z + b$-jets comparison with data.