Astroparticle Physics Phd Admission 2023

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1 Gravity

1. The order of magnitude of the metric perturbation produced by a binary systems with masses m_i and positions x_i at distance D is

   A. $h \sim \frac{G}{c^3 D} \frac{\mathrm{d}}{\mathrm{d}t} (m_1 x_1 + m_2 x_2)$

   B. $h \sim \frac{G}{c^4 D} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left[m_1 (x_1)^2 + m_2 (x_2)^2\right]$

   C. $h \sim \frac{G}{c^3 D^2} \frac{\mathrm{d}}{\mathrm{d}t} \left[m_1 (x_1)^2 + m_2 (x_2)^2\right]$

   D. $h \sim \frac{G}{c^5 D} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left[m_1 (x_1)^2 + m_2 (x_2)^2\right]$

   E. $h \sim \frac{G}{c^5 D^2} \frac{\mathrm{d}}{\mathrm{d}t} \left[m_1 (x_1)^2 + m_2 (x_2)^2\right]$

   [Note that all indices have been dropped.]

2. In a theory with n large spatial extra-dimensions, the Newtonian tidal force is

   A. $\propto 1/r^{n+1}$

   B. $\propto 1/r^3$

   C. $\propto 1/r^{n+3}$

   D. $\propto 1/r^n$

   E. $\propto 1/r^{n-1}$

3. On a planet with twice the mass of the Earth and twice its radius by how much your mass would change?

   A. It would be 2 times the one on Earth

   B. It would be one fourth of the one on Earth

   C. It would be twice the one on Earth

   D. It would not change

   E. It would be one 1/2 of the one on Earth

4. A spaceship moves at 99% of the speed of light. If the spaceship is sent to a solar system 1 light year away, how much the trip lasts for the ship crew?

   A. 1.2 years

   B. 7 months

   C. 51 days

   D. 2 days

   E. 2 hours

5. Newtonian gravity predicts that the angular advancement $\Delta\phi$ of a planet's perihelion (i.e. the orbital point closest to the Sun) over a period is

   A. $\Delta\phi = 0$
   B. $\Delta\phi \sim GM_\odot/(Rc^2)$
   C. $\Delta\phi \sim Gm/(Rc^2)$
   D. $\Delta\phi \sim G^2 m M_\odot/(R^2 c^4)$
   E. $\Delta\phi = \pi$

   where $M_\odot$ is the mass of the Sun (assumed to be perfectly spherical), m is the planet's mass, R its heliocentric distance and the orbital eccentricity is assumed to be small.

2 Cosmology

1. In flat Universe with only one fluid, with an equation of state $p = w\,\rho$ and no cosmological constant, which is the value of w which will result in the largest acceleration of the scale factor at the present time?

   A. $w = -2$
   B. $w = -1/3$
   C. $w = 0$
   D. $w = 1/3$
   E. $w = 1$

2. Consider a spatially flat FLRW universe that contains radiation and a negative cosmological constant, which is initially expanding. What happens at late times?

   A. The universe expands forever, with a decelerated expansion
   B. The universe expands forever, with an accelerated expansion
   C. The universe reaches a maximum expansion and then recollapse
   D. The universe may recollapse or expand forever depending on the quantity of radiation and cosmological constant
   E. The universe may recollapse or expand forever depending on the initial velocity

3. The Hubble expansion rate of the present universe is $H_0 \approx (1.5 \times 10^{10}\,\text{yrs})^{-1}$ in inverse units of years. Supposing that the universe has always been dominated by radiation, what is the age of the universe at the present $z = 0$ time?

   A. $t_{\text{age}} \approx 0.50 \times 10^{10}\,\text{yrs}$
   B. $t_{\text{age}} \approx 0.75 \times 10^{10}\,\text{yrs}$
   C. $t_{\text{age}} \approx 1.0 \times 10^{10}\,\text{yrs}$
   D. $t_{\text{age}} \approx 1.4 \times 10^{10}\,\text{yrs}$
   E. $t_{\text{age}} \approx 3.0 \times 10^{10}\,\text{yrs}$

4. In a flat ΛCDM universe, the redshift of equality between matter and radiation is $z_{eq} = 10^5$. If $\Omega_\Lambda = 0.9$ at $z = 0$, what is the present contribution of the radiation, relative to the critical density $\Omega_{r,0}$?

   A. $\Omega_{r,0} \approx 1 \times 10^6$
   B. $\Omega_{r,0} \approx 9 \times 10^{-2}$

C. $\Omega_{r,0} \approx 9 \times 10^{-4}$
D. $\Omega_{r,0} \approx 1 \times 10^{-5}$
E. $\Omega_{r,0} \approx 1 \times 10^{-6}$

5. The Hubble law, that is the proportionality between distance to a galaxy and its recession velocity, is, under a theoretical point of view, a consequence of

   A. Inflation
   B. Gravitational redshift
   C. Homogeneity and isotropy
   D. The flatness of the Universe
   E. None of the above

3 Particles in the early universe and astrophysics

1. During Big Bang Nucleosynthesis, deuterium is produced through $\mathrm{n}+\mathrm{p} \to \mathrm{D}+\gamma$. However this process is delayed because of (a) the small binding energy of the deuteron $E_{\mathrm{D}} \sim 2\,\mathrm{MeV}$, and (b) the large number of photons, some of which destroy the deuterium through the inverse process. In a universe with the photon number being 10^5 times larger than the baryon number, what is the cosmic temperature when deuterium starts to get abundantly produced? [Hint: Use Wien's law for the number of photons with an energy E_γ larger than the temperature; $n_\gamma \propto \exp(-E_\gamma/T)$.]

   A. $T \sim 10\,\mathrm{MeV}$
   B. $T \sim 10^{-1}\,\mathrm{MeV}$
   C. $T \sim 10^{-3}\,\mathrm{MeV}$
   D. $T \sim 10^{-5}\,\mathrm{MeV}$
   E. $T \sim 10^{-7}\,\mathrm{MeV}$

2. Consider a hypothetical scalar particle ϕ with mass $m = 10\,\mathrm{eV}$, which decays into two photons through a dimension-five operator $(\phi/f)F_{\mu\nu}F^{\mu\nu}$. For this particle's lifetime to be longer than the age of the universe: $\tau \sim 10^{10}$ years $\sim 10^{33}/\mathrm{eV}$, what is the condition that the cutoff f needs to satisfy?

   A. $f \gtrsim 10^{13}\,\mathrm{GeV}$
   B. $f \lesssim 10^{13}\,\mathrm{GeV}$
   C. $f \gtrsim 10^{8}\,\mathrm{GeV}$
   D. $f \lesssim 10^{8}\,\mathrm{GeV}$
   E. $f \gtrsim 10^{3}\,\mathrm{GeV}$

3. The mass density of dark matter in our Milky Way Galaxy is about $0.4\,\mathrm{GeV\,cm^{-3}}$. Supposing that dark matter particles have mass $10^9\,\mathrm{GeV}$ and are moving with velocity $300\,\mathrm{km\,sec^{-1}}$ in the Galaxy, what is the order of magnitude of the number of dark matter particles crossing a detector with surface area $1\,\mathrm{m}^2$ per second?

   A. 10^{-2}
   B. 1
   C. 10^{2}
   D. 10^{4}
   E. 10^{6}

4. An astrophysical source emits a mono-energetic beam of electrons (energy of the electrons E_e). The beam transverses a volume filled with a population of mono-energetic low energy photons (energy of photons $E_\gamma \ll E_e$). In the elastic scattering $e(E_e) + \gamma(E_\gamma) \to e(E_e^f) + \gamma(E_\gamma^f)$ what would be the approximate value for the maximum energy of the outgoing photon? (in the following m_e is the electron mass). [Hint: consider the scattering in the center of mass frame and transform back to laboratory frame.]

   A. $E_\gamma^f \sim E_\gamma$
   B. $E_\gamma^f \sim E_e$
   C. $E_\gamma^f \sim E_\gamma \cdot (E_e/m_e)$
   D. $E_\gamma^f \sim E_\gamma \cdot (E_e/m_e)^2$
   E. $E_\gamma^f \sim E_\gamma \cdot (E_e/m_e)^4$

5. Soon before electrons and protons recombine to form neutral hydrogen atoms, at a temperature $T \ll m_e \sim m_p/2000$ (where m_e and m_p are the electron and proton masses), sketch the early Universe as an environment populated by photons, electrons and protons. When evaluating the photon coupling in such a medium, it is a good approximation to include photon-electron elastic scatterings and neglect photon-proton elastic scatterings because:

   A. the number density of protons is suppressed compared to the number density of electrons by the factor m_e/m_p
   B. the cross section with protons is suppressed compared to the cross section with electrons by the factor m_e/m_p
   C. the cross section with protons is suppressed compared to the cross section with electrons by the factor $(m_e/m_p)^2$
   D. the number density of protons is suppressed compared to the number density of electrons by the factor $(m_e/m_p)^3$
   E. the cross section with protons is suppressed compared to the cross section with electrons by the factor $(m_e/m_p)^4$

4 Particle physics and field theory

1. Consider the following Lagrangian,

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\Phi\,(D^\mu\Phi)^* - \lambda\left(|\Phi|^2 - \frac{f^2}{2}\right)^2 ,$$

   where Φ is a complex scalar field, A_μ is a U(1) gauge field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $D_\mu = \partial_\mu - ieA_\mu$. Consider the masses of the scalar boson $m_{\rm H}$ and gauge boson $m_{\rm V}$ after spontaneous symmetry breaking, in a gauge where the Goldstone boson is removed. How does the mass ratio change if e, λ, and f are all increased by a factor of two?

   A. $m_{\rm H}/m_{\rm V}$ increases by a factor of 2
   B. $m_{\rm H}/m_{\rm V}$ increases by a factor of $\sqrt{2}$
   C. $m_{\rm H}/m_{\rm V}$ does not change
   D. $m_{\rm H}/m_{\rm V}$ decreases by a factor of $1/\sqrt{2}$
   E. $m_{\rm H}/m_{\rm V}$ decreases by a factor of $1/2$

2. Consider a scalar field ϕ in 2+1 dimensions whose dynamics is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{g}{4!}\phi^4 .$$

   What is the mass dimension of g in natural units $\hbar = c = 1$?

A. 1/2

B. 1

C. 0

D. -1

E. -2

3. Consider a free scalar described by the action

$$\mathcal{S} = -\int d^4x \frac{1}{2}(\partial\phi)^2 \ .$$

Consider the transformation that shifts the scalar by a constant c: $\phi \to \phi + c$. Which of the following statements is correct?

A. This is not a symmetry of the action

B. It is a symmetry but the Noether current does not exist

C. It is not a symmetry of the action, but only of the Lagrangian

D. It is a symmetry, but only in the vacuum

E. It is a symmetry, but it is always spontaneously broken

4. Consider (in QED) the scattering $\gamma\gamma \to \gamma\gamma$. The cross section σ depends on the electron charge e as

A. It does not depend on e

B. e^0

C. e^2

D. e^8

E. e^4

5. Let $\sigma_A(E_{CM})$ be the cross section for the annihilation of a spin-1/2 fermion A with its antiparticle $\bar{A}$ into two photons at the center of mass energy E_{CM}. What would be the cross section for the annihilation of the spin-1/2 fermion B with its antiparticle $\bar{B}$ into two photons, at the same energy, knowing that the mass of B is equal to the mass of A and the electric charge of B is equal to -2 times the electric charge of A?

A. $\sigma_B(E_{CM}) = -16 \cdot \sigma_A(E_{CM})$

B. $\sigma_B(E_{CM}) = 4 \cdot \sigma_A(E_{CM})$

C. $\sigma_B(E_{CM}) = 16 \cdot \sigma_A(E_{CM})$

D. $\sigma_B(E_{CM}) = 1/4 \cdot \sigma_A(E_{CM})$

E. $\sigma_B(E_{CM}) = -1/4 \cdot \sigma_A(E_{CM})$

5 Miscellanea (e.g. statistics, astro)

1. In Italy, 80% of the population watches TV. Cardiopathic disease has an incidence of 20% in the whole population, and of 22% among TV watchers. What fraction of the cardiopathic population watches TV?

A. 88%

B. 80%

C. 72.7%

D. 82%

E. 78.6%

2. A population of merging black hole binaries has orbital frequency f evolving according to $\dot{f} \propto f^{11/3}$ under gravitational wave emission during the inspiral. Assuming the population properties are time independent (i.e. the merger rate dn/dt is constant), the number of black hole binaries per unit frequency dn/df is

   A. $\propto f^{11/3}$

   B. constant

   C. $\propto f^{-22/3}$

   D. $\propto f^{-10/3}$

   E. none of the above

3. According to Hawking, a black hole has a temperature that is inversely proportional to the black hole mass, $T = \alpha M^{-1}$. The proportionality coefficient α can be estimated on dimensional grounds by using $c \sim 10^8\,\mathrm{m\,s^{-1}}$, $\hbar \sim 10^{-34}\,\mathrm{kg\,m^2\,s^{-1}}$, $G \sim 10^{-10}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, and $k_B \sim 10^{-23}\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$. What is the temperature of a black hole with a solar mass $M \sim 10^{30}\,\mathrm{kg}$?

   A. $T \sim 10^{-17}\,\mathrm{K}$

   B. $T \sim 10^{-7}\,\mathrm{K}$

   C. $T \sim 10^{3}\,\mathrm{K}$

   D. $T \sim 10^{13}\,\mathrm{K}$

   E. $T \sim 10^{23}\,\mathrm{K}$

4. Consider a particle in a potential

$$V(x) = \infty \quad \text{for} \quad x < 0\,, \qquad V(x) = \frac{1}{2} m\omega^2 x^2 \quad \text{for} \quad x > 0$$

   The energy of the ground state is

   A. $E = \frac{1}{4}\hbar\omega$

   B. $E = \frac{1}{2}\hbar\omega$

   C. $E = \hbar\omega$

   D. $E = \frac{3}{2}\hbar\omega$

   E. $E = 2\hbar\omega$

5. A gravitational wave signal has narrow width Δf in the frequency domain. T is the observation time. What does it correspond to in the time domain?

   A. A damped quasi-monochromatic signal with decay time $\sim 1/\Delta f$

   B. A damped quasi-monochromatic signal with decay time $\sim T$

   C. A monochromatic signal with frequency Δf

   D. A monochromatic signal with frequency $1/T$

   E. A damped quasi-monochromatic signal with decay time $\sim \sqrt{T/\Delta f}$