APP admission test

15 March 2021

1. **COSMOLOGY**

   Consider a flat Universe containing only matter and a cosmological constant. If their mean densities at the present time, in units of the critical density, are, respectively, $\Omega_{\rm m}^0 = 1/9$ and $\Omega_\Lambda^0 = 8/9$, at which redshift equality between the densities in matter and cosmological constant is reached?

   A. $z = 2$
   B. $z = 1$
   C. $z = 0.5$
   D. $z = 0.25$
   E. $z = 0.125$

2. **COSMOLOGY**

   The comoving particle horizon is defined as $\Xi(t) = c\int_0^t \frac{dt'}{a(t')}$, with c the speed of light and a the scale factor, assuming $a = 1$ at the present time. We have two *flat* Universes: Universe A, filled with a matter fluid component $p = 0$, and Universe B, filled with a generic fluid component $p = w\rho$, with $w > -1/3$. Both Universes do not have any other fluid component at any stage in their evolution and have the same energy densities at the present time. Which conditions on w results in a comoving particle horizon $\Xi_A(a = 1) = 2\,\Xi_B(a = 1)$?

   A. $w = 1/3$
   B. $w = 2/3$
   C. $w = 1$
   D. $w = 4/3$
   E. No solution is possible

3. **COSMOLOGY**

   Assuming a static and closed Universe having as energy density components only radiation and the cosmological constant, what is the relation, in natural units ($c = 1$), between the cosmological constant Λ and the (constant) scale factor a_0?

   A. $\Lambda = \frac{1}{a_0^2}$
   B. $\Lambda = \frac{3}{2}a_0^2$
   C. $\Lambda = \frac{2}{3}\frac{1}{a_0^2}$
   D. $\Lambda = a_0^2$
   E. $\Lambda = \frac{3}{2}\frac{1}{a_0^2}$

4. **GR**

Which of the following is a necessary and sufficient condition to ensure that a metric is flat?

A. The Ricci tensor is zero, $R_{\mu\nu} = 0$

B. The Riemann tensor contracted twice with the timelike Killing vector(s) χ^ρ is zero, $R_{\mu\nu\alpha\beta}\chi^\mu\chi^\nu = 0$

C. Kretschmann invariant is zero, $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = 0$

D. The Gauss-Bonnet invariant $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is zero

E. None of the above

5. **GR**

A satellite orbits around the Earth with speed $v \ll 1$ ($c = 1$) at an altitude h over the ground, and is endowed with an atomic clock. An identical clock is placed on Earth. Denoting as $\Delta t'$ and Δt the time intervals measured respectively by the clocks on the satellite and on Earth, compute $\Delta t' - \Delta t$: [The mass of the Earth is denoted by M, its radius is R ($R \gg h$) and the Earth's exterior spacetime is well described by the Schwarzschild geometry]

A. $\approx \Delta t(GhM/R^2 - v^2/2)$

B. $\approx \Delta t(-GhM/R - v^2/2)$

C. $\approx \Delta t(-GhM/R^2 - v^2/2)$

D. $\approx \Delta t(GhM/R - v^2/2)$

E. $\approx \Delta t(-GhM/R^3 - v^2/2)$

6. **GR**

Two test masses falling radially into a (non-spinning) black hole have a radial separation of 30 cm. The relative acceleration of the two masses when they cross the event horizon is about 0.003 m/s^2. How many solar masses is the mass of the black hole? [Use the fact that the Schwarschild radius of a black hole of one solar mass is about 3 km.]

A. 10^{-10} $M_\odot$

B. 10^{-2} $M_\odot$

C. 10 $M_\odot$

D. 1000 $M_\odot$

E. 10^6 $M_\odot$

7. **SPECIAL RELATIVITY**

A supernova explosion, occurring at distance of about $2 \cdot 10^{11}$ Light-seconds, emitted neutrinos in a short burst. Suppose that a detector on Earth has detected a neutrino of energy $E_2 = 1$ MeV about 10 seconds later than a neutrino of energy $E_1 = 50$ MeV; what value would you deduce for the neutrino mass m_ν under the assumption that the neutrino burst is instantaneous? [Assume 1 neutrino flavour and no oscillations.]

A. $m_\nu = 10^{-6}$ MeV

B. $m_\nu = 1/\sqrt{2} \cdot 10^{-5}$ MeV

C. $m_\nu = 10^{-5}$ MeV

D. $m_\nu = \sqrt{2} \cdot 10^{-5}$ MeV

E. $m_\nu = 10^{-4}$ MeV

8. **SPECIAL RELATIVITY**

A particle P, shot on its antiparticle $\bar{P}$, annihilates generating two photons:

$$P + \bar{P} \to \gamma + \gamma \, .$$

Assuming that the particle $\bar{P}$ is at rest, and that the energy and mass of P are, respectively, 5 GeV and 3 GeV, determine the angle θ of emission for the two photons with respect to the direction of the incident particle P.

A. $\theta = 0$

B. $\theta = \pi/6$

C. $\theta = \pi/4$

D. $\theta = \pi/3$

E. None of the above

9. **QM**

A point mass in 3 spatial dimensions is subject to a central potential proportional to the square of the distance from the center. What is the degeneracy of the first excited state?

A. 1

B. 2

C. 3

D. 4

E. 6

10. **QM**

For a particle in a one-dimensional infinite square well, which state will have the fastest variation in time for the position probability density? (The state ψ_n corresponds to an energy eigenstate of energy E_n)

A. ψ_1

B. ψ_4

C. $\frac{1}{\sqrt{2}}(\psi_2 + \psi_3)$

D. $\frac{1}{\sqrt{2}}(\psi_1 - \psi_3)$

E. All these states have time-independent position probability density

11. **QM**

Given a quantum system with Hamiltonian H and components of the angular momentum L_x, L_y, L_z, which of the following statements is correct?

A. $[H, L_z] = 0$ and $[H, L_x] = 0$ guarantee the full rotational invariance of the Hamiltonian.

B. H cannot commute simultaneously with all components of the angular momentum.

C. $[[L_x, L_y], L_x] = 0$

D. One can always find a complete set of states that are eigenstates both of H and L_x.

E. A rotationally invariant Hamiltonian may not commute with L^2.

12. **QFT**

The following Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + im\bar{\psi}\gamma_5\psi \,,$$

describes the propagation of Dirac particles with mass:

A. m
B. $\sqrt{2}m$
C. $2m$
D. $m \pm m$ depending on chirality
E. the masses are not real

13. **QFT**

By approximately what factor the mass of the proton would change if the fine structure constant of the strong interactions computed at 100 GeV ($\alpha_s(100\text{ GeV}) = g_s^2(100\text{ GeV})/(4\pi)$) is doubled?

A. $1/100$
B. $1/10$
C. $1/\sqrt{10}$
D. $\sqrt{10}$
E. 10

14. **QFT**

In the U(1) Higgs model described by the Lagrangian

$$\mathcal{L} = |D_\mu\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4 - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} \,,$$

the ratio of the Higgs mass squared over the vector mass squared m_h^2/m_V^2 is proportional to which ratio of couplings:

A. $\frac{\lambda}{g}$
B. $\frac{g^2}{\lambda}$
C. $\frac{\lambda^2}{g^2}$
D. $\frac{g}{\lambda^2}$
E. $\frac{\lambda}{g^2}$

15. **THERMAL HISTORY**

Consider the existence of a hypothetical fermion F, with mass m_F and electric charge Q_F in units of the electron charge, in the context of the Early Universe thermal plasma; assume that m_F is much larger than the electroweak scale (namely much larger than the W boson mass).Use dimensional analysis to estimate an approximate condition on Q_F so that F is not in thermal equilibrium at any temperature, even extrapolating the Universe temperature as high as the Planck scale M_{Pl}.

A. $Q_F \lesssim 10^{-4}\,(M_F/M_{Pl})^{1/4}$
B. $Q_F \lesssim 10\,(M_F/M_{Pl})^{1/4}$
C. $Q_F \lesssim 10^{-2}\,\sqrt{M_F/M_{Pl}}$
D. $Q_F \lesssim 100\,\sqrt{M_F/M_{Pl}}$
E. $Q_F \lesssim 10^{-2}\,(M_F/M_{Pl})$

16. **RADIATIVE PROCESSES**

An electron with energy $E_e = 4$ MeV, a muon with energy $E_\mu = 160$ MeV and a proton with $E_p = 6400$ MeV travel through a medium with refractive index $n = \sqrt{5/4}$. Which of the three particles emit Cherenkov radiation in the medium?

A. None of them.

B. The proton only.

C. The electron and the proton only.

D. The muon and the proton only.

E. All of them.

17. **EARLY UNIVERSE**

Having defined $\epsilon = -\dot{H}/H^2$, with H the Hubble parameter, which of the conditions below is the most general condition that will give rise to inflation (accelerated expansion of the Universe)?

A. $\epsilon > 1$

B. $\epsilon < 1$

C. $0 < \epsilon < 1$

D. $\epsilon < 0$

E. $\epsilon = 0.001$

18. **HIGH ENERGY ASTROPHYSICS**

A nearby source is detected by a gamma-ray telescope to have, between 1 GeV and 100 GeV, an energy spectrum $\propto E_\gamma^{-1.7}$. Among the following particle emissions at the source and subsequent interactions close to the source, which is compatible with the detected flux?

A. electrons with spectrum $\propto E_e^{-2.7}$ radiating on a 10 μG magnetic field.

B. electrons with spectrum $\propto E_e^{-1.7}$ interacting with monochromatic photons.

C. electrons with spectrum $\propto E_e^{-2.7}$ interacting with gas.

D. protons with spectrum $\propto E_p^{-1.7}$ interacting with gas.

E. protons with spectrum $\propto E_p^{-2.7}$ interacting with gas.