SISSA
Entrance
Examination

Astroparticle Physics Curriculum

SOLVE two out of four problems. If you hand in more than two, then state clearly which two problems you wish to be evaluated.
Concisely answer two out of the six questions (no more than one page per question).

July 16, 2012

Problem 1

Consider the decay of an unknown particle X, of spin J, to two photons

$$X \to \gamma\gamma\,. \tag{1}$$

Recall that a photon is a spin-1 massless particle. Work in the center-of-mass frame with the z-axis along the direction of one of the photons.

1. What are the possible values of the total spin S of the two-photon final state? Find the possible spin configurations of the two photons (it will be easier to work with combinations which are eigenstates of the parity operator P).

2. For each value of S, what values of the total orbital angular momentum L are allowed?

3. Consider $J = 0, 1$. Which of the configurations found in part 1 are possible? What are the transformation properties of such states under $R_z(\alpha)$ and $R_x(\pi)$ (rotation of an angle α around $\hat{z}$ and rotation of π around $\hat{x}$, respectively). Use these results to conclude that the decay (1) is forbidden for $J = 1$ (this is the so-called "Landau-Yang" theorem).

4. Does the Landau-Yang theorem apply for $X \to ZZ$ (where Z is the neutral massive gauge boson of the Electroweak theory)? And for $X \to gg$, where g is the gluon (massless gauge boson associated to $SU(3)_c$ gauge symmetry)? Why?

5. Consider X to be scalar or a pseudo-scalar, $J^P = 0^+, 0^-$ respectively, and assume the decay (1) proceeds via parity-conserving interactions. What is the selection rule on the L of the final states, in the two cases? What are the corresponding values of S? Are there examples in Nature of a scalar or a pseudo-scalar particle decaying to two photons?

Problem 2

1. Derive the geodesic equation by using the definition of a geodesic as the “straightest line”, instead of the “shortest line” between two points. Explain why the two definition yield the same result in General Relativity.

2. Show that a parameterization of the curve exist for which the geodesic equation takes the form

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\nu\sigma}\frac{dx^\nu}{d\lambda}\frac{x^\sigma}{d\lambda} = 0. \qquad (1)$$

3. Show that starting from the equations $\nabla_\nu T^{\mu\nu} = 0$, and taking the stress energy tensor to be that of dust (pressure-less perfect fluid) can also yield the geodesic equation. What does this imply for the particles constituting the dust?

Problem 3

Consider a system of two real scalar fields $\phi_1(x)$, $\phi_2(x)$ with potential

$$V(\phi_1, \phi_2) = \frac{\lambda}{2}(\phi_1^2 - \phi_2^2)^2 + \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 - m_{12}^2\phi_1\phi_2, \tag{1}$$

where $\lambda > 0$, m_1^2, m_2^2 are real parameters, possibly negative, and $m_{12}^2 > 0$.

1. Under which conditions on m_1^2, m_2^2, m_{12}^2:

   a. the potential is bounded from below;

   b. the quadratic part of the potential $(m_1^2/2)\phi_1^2 + (m_2^2/2)\phi_2^2 - m_{12}^2\phi_1\phi_2$ is bounded from below;

   c. the origin $\phi_1 = \phi_2 = 0$ is a local minimum of the potential;

   d. the origin $\phi_1 = \phi_2 = 0$ is a global minimum of the potential.

2. Suppose now that the system is coupled to two additional real scalar fields ϕ_3, ϕ_4 though the quartic term:

$$\begin{aligned} V(\phi_1, \phi_2, \phi_3, \phi_4) &= \frac{\lambda}{2}(\phi_1^2 - \phi_2^2 + \phi_3^2 - \phi_4^2)^2 + \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 - \\ &\quad m_{12}^2\phi_1\phi_2 + V_{34}(\phi_3, \phi_4). \end{aligned}$$

   Consider the potential as a function of ϕ_1, ϕ_2 and answer the questions a) b) c) d) for any fixed values of ϕ_3 and ϕ_4.

3. Now, consider again the theory described by the potential in (1), with $m_2^2 = 0, m_{12}^2 = 0$. Consider the $2 \to 2$ scatterings of two ϕ_1 particles into two other particles. What are the possible final states? Draw the tree-level Feynman diagrams for these processes. Give a parametric estimate of the cross section.

Problem 4

Our Universe is accelerating, i.e. it is expanding faster today than it was in the recent past. This is based on the assumption of a linear Hubble law: that, at any given time, the speed v with which any two observers are separating from one another (because of the expansion) is linearly proportional to their separation d, i.e. $v = Hd$, where H is called the Hubble constant. Today, H is approximately 70 (km/s)/Mpc (a Mpc is approximately 3 million light years).

1. At any given time, knowledge of the Hubble constant H yields a crude estimate of the age of the Universe. What is the estimated age (in years) for the value of H given above?

2. What is the physical basis for assuming that, at any given time, v must be linearly proportional to d?

3. In practice, v and d are measurements of speeds and distances between our galaxy and other objects. The speed v is estimated from the spectra of the other objects, similarly to how one measures Doppler shifts, and is sometimes referred to as the redshift z. Distinguish between the Newtonian and Special Relativistic Doppler shifts, the General Relativistic gravitational redshift, and the cosmological redshift associated with the expansion of the Universe, providing a formula for each effect.

4. The actual measurements of the relationship between v and d show a curvature. Why? One way of expressing this curvature is to show how d differs from (cz/H), where z is the cosmological redshift. Derive the expected relation between d and z for a spatially-flat matter-dominated Universe (sometimes called an Einstein-de Sitter Universe).

Questions

1. How can you tell whether a given metric is equivalent to the Minkowski metric of flat space?

2. Origin and relevance of the Lepton number for particle physics and cosmology.

3. Dark energy and dark matter are two key ingredients of the standard cosmological model. In the context of this model, what is the evidence that most of the mass density in the Universe is in the form of non-baryonic dark matter?

4. Discuss an argument in favor of dark matter to be not far from the electroweak scale.

5. How many degrees of freedom does the metric have? How many of these degrees of freedom can be removed by arbitrary transformations of the spacetime coordinates, and therefore how many physical degrees of freedom are there in spacetime?

6. Discuss briefly the meaning and the consequences of spontaneous breaking of global and local symmetries in particle physics.