SISSA
Entrance
Examination

Astroparticle Physics Curriculum

SOLVE at least two out of the four problems – the best two solutions will count towards the evaluation of the written exam. Concisely answer two out of the six questions (no more than one page per question).

July 18, 2011

Problem 1

A particle with mass m and electric charge q can move in one dimension subject to a harmonic force and a homogeneous electrostatic field, which is turned on at the time $t = 0$. The potential is:

$$V = \frac{1}{2} m\, \omega^2\, x^2 - q\, E\, x\, \theta(t)\,, \tag{1}$$

with the Heaviside function $\theta(t)$ being 0 for $t < 0$ and 1 for $t > 0$.

1. Find the energy levels for this system.

2. Suppose that for $t < 0$ the system is in the ground state; calculate the probability of finding the system in the ground state at the time $t > 0$. [**Hint**: you may need to use the operator identity: $e^{O_1+O_2} = e^{O_1} e^{O_2} e^{-[O_1,O_2]/2} = e^{O_2} e^{O_1} e^{-[O_2,O_1]/2}$]

3. Still under the hypothesis that for $t < 0$ the system is in the ground state, calculate at $t > 0$ the expectation value for the particle electric dipole moment $d_e \equiv q\, x$.

Problem 2

Consider N real scalar fields with Lagrangian

$$L = -\frac{1}{2}\sum_{a=1}^{N}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\left(\phi^2\right)^2 \quad (1)$$

(where $\phi^2 = \sum_a \phi^a\phi^a$, and the signature of the metric is $-,+,+,+$).

1. What is the internal symmetry group of this Lagrangian? Find the corresponding Noether current and show that it is conserved.

2. Assuming that the parameter m^2 is negative, find the classical vacuum of the theory. What is the symmetry group of the vacuum? Write the energy-momentum tensor of the vacuum. What would be its effect, if the theory was coupled to gravity?

3. Determine the masses of the linearized quantum fluctuations around the vacuum. What states of this theory can decay, and what do they decay into?

Problem 3

Consider a homogeneous isotropic universe with scale factor $a(t)$, dominated by two non-relativistic cosmological fluids, denoted by 1 and 2 and characterized by background densities ρ_1 and ρ_2. Suppose that the speed of sound of the fluid 1 is equal to c_S, with $c_S^2 = \frac{k_B T_0}{m_1 a}$, T_0 the present temperature of the universe, m_1 the mass of the particles. The Poisson equation for the Fourier amplitude of the density contrast δ_1 is:

$$\ddot{\delta}_1 + 2H\dot{\delta}_1 + \frac{k^2 c_s^2}{a^2}\delta_1 = 4\pi G\rho_1\delta_1 + 4\pi G\rho_2\delta_2 \ , \qquad (1)$$

where $H = \dot{a}/a$ is the Hubble expansion rate. The universe is matter-dominated, so $a(t) = (t/t_0)^{2/3}$.

1. Suppose the fluid 2 dominates the right hand side and its density contrast scales as $\delta_2 \propto a$; determine the particular solution to (1) which is a power law in time and discuss the relation between δ_1 and δ_2 as a function of the scale k.

2. Determine the scale $\lambda_p = 2\pi/k_p$ at which the pressure term dominates over gravity for fluid 1; calculate the related characteristic mass M_p and discuss its scaling with ρ_1.

3. For $k \gg k_p$ and neglecting the right hand side, determine the solution of (1) and discuss its behavior with time.

Problem 4

In general relativity, the spacetime around an isolated, spherically symmetric star is described by the Schwarzschild line element

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \qquad (1)$$

with the surface of the star lying at some radius larger than $2M$. Consider the coordinate redefinition

$$r = \bar{r}\left(1 + \frac{M}{2\bar{r}}\right)^2 . \qquad (2)$$

1. Show that the new coordinate provides a good cover for all of the exterior spacetime ($r > 2M$) and write down the metric in the new coordinates.

2. As $\bar{r} \to \infty$, give the metric to leading order in $1/\bar{r}$. Argue how the procedure you just followed justifies identifying M with the mass of the central object in units where $G/c^2 = 1$. Explain why this procedure is necessary and we should not instead take the limit $r \to \infty$ in the line element of equation (1) in order to identify the mass. (Hint: recall the derivation of the Poisson equation as the newtonian limit of Einstein's equations.)

3. The Schwarzschild solution can also describe a spherically symmetric black hole, in which case the part of the spacetime for which $r < 2M$ becomes relevant. In this region g_{rr} in the line element of equation (1) becomes negative and, thus, r becomes a timelike coordinate. Does this mean that the metric signature changes? (Please justify your answer.) What are the implications regarding the endpoint of a free-falling particle's trajectory, once the particle enters this region?

Questions:

1. Briefly discuss one of the questions regarding physics beyond the standard model of particle physics being currently addressed by the Large Hadron Collider (LHC) which may have an impact on cosmology as well.
2. Discuss the notion of chirality in the standard model of particle physics and the origin of fermion masses.
3. Discuss the observational evidence which suggests that the Universe is spatially flat, and the consequences of this fact for early and late cosmology.
4. Describe why, in the standard cosmological model, most of the mass is thought to be non-baryonic.
5. Discuss the open problems in inflationary cosmology in light of recent cosmological observations.
6. Discuss the definition of the shortest line and straightest line between two points in flat space (in general coordinates) and the extension of these definitions to curved space. Explain why these two curves coincide both in flat space and in general relativity.