SISSA
Entrance
Examination

Astroparticle Physics Curriculum

SOLVE two out of the four problems. Concisely answer two out of the five questions.

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2009 spring session

Problem 1.

Consider the non-relativistic quantum mechanical problem of a particle of mass m placed in the two-dimensional potential:

$$V(x,y) = \begin{cases} 0 & \text{for} \quad 0 \le x \le L_x \text{ and } 0 \le y \le L_y \\ +\infty & \text{for} \quad x < 0,\ x > L_x \text{ and } y < 0,\ y > L_y \end{cases} , \qquad (1)$$

i.e. an infinitely deep rectangular well of sizes L_x and L_y.

1. Compute the solution to the Schrödinger equation for such particle.

2. Write the energy spectrum for arbitrary L_x and L_y; discuss its properties when $L_x = L_y$. What happens instead in the limit when $L_y \ll L_x$?

3. Consider again the case when $L_x = L_y$. Discuss how the second energy level changes when a weak perturbation is introduced by adding to the potential an extra term V_ϵ, in the following two cases:

$$\begin{aligned} (a) \qquad & V_\epsilon = \epsilon \left(x^4 + y^4\right) \\ (b) \qquad & V_\epsilon = \epsilon \left(x^4 - y^4\right) , \end{aligned} \qquad (2)$$

   where ϵ is some small constant.

Problem 2.

There are different options to define cosmological distance measures in an homogeneous and isotropic expanding Universe. One is the angular size distance D_A, defined by:

$$D_A = \frac{R}{\theta}\,, \tag{1}$$

where R is the proper size of a given source and θ the angular size (in radians) that it subtends on the sky. A second distance indicator is the luminosity distance D_L, defined in terms of the absolute luminosity L of a given source (the energy per time produced by the source in its rest frame) and its measured flux F (the energy per unit time per area measured by an observer) through the relation:

$$F = \frac{L}{4\,\pi\,D_L^2}\,. \tag{2}$$

1. Consider the source emitting a black body radiation characterized by a temperature T_e at emission, and T_o at observation. Assume that the radiation propagates freely in space. Use the relation between T_e and T_o in an expanding Universe to derive the relation between D_L and D_A as a function of the redshift z at emission; you can assume, for simplicity, that emitter and observer have negligible proper velocities.

2. Assume now that the observer has a non-negligible proper velocity. Show that this velocity cannot be determined from the spectral shape of a distant black body emitter in case the temperature of the emitter is not known.

3. Assume that the cosmic microwave background (CMB) has a perfect black body spectrum. Estimate the earth velocity in the CMB reference frame from the fact that the COBE satellite has measured a dipole asymmetry (namely a forward to backward asymmetry with respect to the direction of the motion) in the CMB temperature of about 1 part in 1000.

Problem 3.

Consider a flat Friedmann-Robertson-Walker (FRW) metric described by:

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \ , \qquad (1)$$

where $a(t)$ is the cosmological scale factor. It obeys the Friedmann equation:

$$H = \sqrt{\frac{8\pi G}{3}\rho_0 \left(\frac{a}{a_0}\right)^n} \ , \qquad (2)$$

where G is the Newton constant, ρ_0 and a_0 are the energy density and scale factor at the present time, respectively, $H = \frac{1}{a}\frac{da}{dt}$ the Hubble expansion rate, which has the value $H_0^{-1} = 2200$ Mpc.

1. Assume perfect matter domination, $n = -3$. Determine the distance r_{1000} traveled by a photon from the epoch of decoupling between matter and radiation, $1 + z = 1000$, to the present, $z = 0$, where $1 + z = a_0/a$ is the cosmological redshift.

2. Determine now the distance r_∞ from $z = 0$ to $z = +\infty$, assuming perfect matter domination up to the equivalence (eq) between matter and radiation, occurring at $1 + z = 3 \cdot 10^4$, and perfect radiation domination at earlier times, when the energy density is given by $\rho = \rho_{eq}(a/a_{eq})^n$ and $n = -4$. Show that $r_\infty - r_{1000} \ll r_{1000}$.

3. Suppose that, in addition to the energy density of matter or radiation having the same values as above, the Friedmann equation contains a constant Λ:

   $$H = \sqrt{\frac{8\pi G}{3}\rho_0 \left(\frac{a}{a_0}\right)^n + \frac{\Lambda}{3}} \ . \qquad (3)$$

   Are the distances calculated above shortened or elongated?

Problem 4.

Consider the Schwartzschild metric:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\ . \quad (1)$$

1. Using the normalization of the 4-velocity u^μ, prove that the coordinate r of an observer behind the horizon, $r \leq 2GM$, must decrease with a rate
$$\left|\frac{dr}{d\tau}\right| \geq \left(\frac{2GM}{r} - 1\right)^{1/2}\ , \quad (2)$$
where τ is the proper time of the observer.

2. Show that this implies that the maximum proper lifetime of an observer behind the horizon before hitting the singularity at $r = 0$, is $\tau_{\max} = \pi GM$. [Hint: It may be useful to make the substitution $x^2 = 1 - r/(2GM)$.]

3. For a geodesic motion on the plane $\theta = \pi/2$ in the Schwartzschild metric, the two quantities:
$$E \equiv \left(1 - \frac{2GM}{r}\right)\frac{dt}{d\tau} \qquad \text{and} \qquad L \equiv r^2\frac{d\phi}{d\tau} \quad (3)$$
are constant. Prove that the maximum lifetime found above is approached for a geodesic motion in the limit $E \to 0$ and $L \to 0$.

Questions:

1. Give an example of astrophysical system in which a synchrotron emission mechanism plays a dominant role.

2. What is the qualitative difference between the spontaneous breaking of a global and a local (gauge) symmetry?

3. Observations indicate that the TeV γ ray radiation from distance sources is highly absorbed. Can you guess what is the main mechanism in quantum electrodynamic responsible for the absorption?

4. Dark energy, dark matter, dark baryons. Discuss one of these cosmological issues in light of recent observations.

5. Discuss parity non-conservation in the context of the Standard Model of Particle Physics.