SISSA
Entrance
Examination

Astroparticle Physics Curriculum

SOLVE two out of the five exercises given below and write one short essay (no more than three pages) choosing among the proposed titles.

Trieste, 26 September, 2005

Problem 1.

Consider a very high energy astrophysical neutrino scattering off a relic neutrino and producing N particles in the final state.

1. Determine the largest value of the neutrino mass such that (a) the scattering process proceeds through the resonant (on shell) exchange of a Z boson ($M_Z \approx 91\,\mathrm{GeV}$) and (b) at least one of the decay products has an energy larger than the GZK cutoff $E_{\mathrm{GZK}} \sim 5 \cdot 10^{19}\,\mathrm{eV}$. Assume that all neutrino types have the same mass. Neglect the thermal energy of relic neutrinos.

2. Consider the case of $N = 2$. Determine the angular distribution of the decay products as a function of the incoming neutrino energy assuming that the scattering process is isotropic in the center of mass reference frame. Estimate the opening angle of the cone containing most of the decay products. Assume that all the particles have the same mass.

Problem 2.

Consider a high energy photon γ propagating in the interstellar medium. Since the Galaxy is filled with background photons, there is a finite probability for γ to be absorbed in the electromagnetic process producing an electron/positron pair:

$$\gamma + \gamma_b \rightarrow e^- + e^+ \tag{1}$$

with γ_b a background photon.

1. Sketch the process with the Feynmann diagram technique. Using dimensional arguments and taking into account the couplings, try to guess an approximate expression for the cross section in the limit of small center-of-mass velocity for the outgoing electron and positron, and in the limit in which they are ultra-relativistic.

2. Find the minimum energy for the photon γ at which the process becomes kinematically allowed, as a function of the energy of the background photon E_γ^b and of the angle between the direction of the incident photon and that of the background photon. What is the threshold energy in case of absorption on a local background starlight photon (say, with an energy $E_\gamma^b \sim 3$ eV) and a local background infrared photon (say, with an energy of $E_\gamma^b \sim 10^{-2}$ eV)?

3. Suppose that at the center of a far away globular cluster (redshift z) there is a gamma-ray source emitting photons with continuum energy spectrum and up to energies of about 1 TeV. Suppose the cluster is filled with an intense starlight field, such that photons emitted with energy larger than the threshold energy for the process in Eq. 1 are absorbed. Let n_γ^b be mean starlight background density in the cluster and R_c the cluster radius; find the lower limit on the product $n_\gamma^b \cdot R_c$ such that the interaction length for absorption is smaller than R_c. Suppose that the source is bright enough to be observable at Earth over the full energy range of its emission spectrum; at what energy would we find a cut-off in the observed spectrum?

Problem 3.

One can define a geodesics as: *i)* the curve of minimal length connecting two points in spacetime (as a straight line on a plane or an arc of a great circle on a sphere); or *ii)* the curve whose tangent vector is parallel propagated along itself, i.e., if $u^\mu = \dot{x}^\mu$ and the derivative is defined with respect to an affine parameter, then $u^\mu \nabla_\mu u^\nu = 0$.

1. Using the definition *i)*, derive the geodesic equation

$$\frac{\mathrm{d}^2 x^\tau}{\mathrm{d}s^2} + \Gamma^\tau{}_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}s} \frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0$$

   where

$$\Gamma^\alpha{}_{\beta\gamma} \equiv g^{\sigma\alpha} \Gamma_{\sigma\beta\gamma} \equiv g^{\sigma\alpha} \frac{1}{2} (g_{\sigma\beta,\gamma} + g_{\sigma\gamma,\beta} - g_{\beta\gamma,\sigma})$$

2. Using the definition *ii)*, prove that if a geodesic is timelike at some point P, then it will remain timelike at any other point along its length.

   Hint: Use the fact that to determine if a curve is timelike (or spacelike or null) at some point P it is sufficient to look at the sign of $u \cdot u$ where u is the tangent vector to the curve.

PROBLEM 4.

THE relativistic law of motion of a system which uniformly accelerates (constant acceleration in its reference frame) is described in a frame at rest by

$$v(t) = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}}\,, \tag{1}$$

where a is the constant acceleration and t is the time as measured in the rest frame.

1. Determine the relation between the time t and the proper time τ (as measured by an observer that uniformly accelerates with acceleration a).

2. Try to give an order of magnitude of a possible effect associated to the difference between t and τ (twins' paradox). Consider a system which accelerates with $a = 1m/sec^2$, until $v = c/\sqrt{2}$, at which point it starts to decelerate with deceleration $-a$ until $v = 0$ and then it comes back, again by accelerating until $v = c/\sqrt{2}$ and decelerating until $v = 0$ (Assume that the transition between the accelerating and decelerating phases are instantaneous and neglect all possible effects related to the transition).

PROBLEM 5.

THE cosmological expansion may be described as a stretching of all physical lengths λ, according to the law

$$\lambda(t) = \lambda_0 \cdot a(t) \ , \tag{1}$$

where $a(t)$ is an increasing function of time with $a(t_0) = 1$ and t_0 corresponds to the present.

0.0.1 question

1. Consider the occupation number of a blackbody radiation as a function of frequency, with temperature T_0 at t_0. Imposing the conservation of the occupation number during the expansion given by (1), find the temperature T of the blackbody radiation at $t \neq t_0$ as a function of $a(t)$.

2. Using the Stephan-Boltzmann law, relating energy density of the blackbody radiation and temperature with constant $a = 4\sigma/c^3$ where $\sigma = 5.7 \cdot 10^{-5}$ erg/cm^2/K^4/s, calculate the energy density of the blackbody radiation at $T_0 = 2.7$ Kelvin, corresponding to the cosmic microwave background (CMB) at present, and compare it with the present cosmological density of non-relativistic matter, $\rho_m \simeq 5.0 \cdot 10^{-30}$ g/cm^3. Using the Stephan-Boltzmann law, and the fact that non-relativistic energy density scales as the inverse of the volume, calculate the temperature of the CMB at the epoch of equivalence between matter and radiation.

Short Essays

1. The largest contributions to the energy density of the Universe today come from dark energy and dark matter. Describe evidences and/or theoretical aspects of one or both of these components.

2. Describe the mechanism of spontaneous symmetry breaking in field theory.

3. Discuss one fundamental open issue in particle physics, astrophysics and cosmology.