APP admission test

21 March 2022

1. **COSMOLOGY**

A flat Universe contains only matter and a positive cosmological constant, whose energy densities are $\rho_{\rm M}$ and $\rho_{\rm V}$. Which of the conditions below marks the transition to an epoch of cosmic accelerated expansion?

A. $\rho_{\rm M} = \rho_{\rm V}/2$
B. $\rho_{\rm M} = \rho_{\rm V}$
C. $\rho_{\rm M} = 2\,\rho_{\rm V}$
D. $\rho_{\rm M} = 3\,\rho_{\rm V}$
E. $\rho_{\rm M} = 4\,\rho_{\rm V}$

2. **COSMOLOGY**

According to the current cosmological model, why is the night sky not bright with starlight?

A. The Universe is expanding now faster than the speed of light
B. The Universe is flat and the speed of light is finite
C. The expansion of the Universe is accelerating
D. The age of the Universe and its size are finite
E. The age of the Universe and the speed of light are finite

3. **COSMOLOGY**

The Hubble radius today is about $10^{29}\,$cm. Estimate the time when the redshift of the universe was $z = 10^{28}$. Here, assume that the Universe has always been dominated by radiation, and the speed of light is $c \sim 10^{10}\,$cm/sec.

A. $10^{-22}\,$sec after the Big Bang
B. $10^{-27}\,$sec after the Big Bang
C. $10^{-32}\,$sec after the Big Bang
D. $10^{-37}\,$sec after the Big Bang
E. $10^{-42}\,$sec after the Big Bang

4. **SPECIAL RELATIVITY**

At what percent of the speed of light does a car need to move so that approaching a traffic light the red light is perceived as green? (Wavelengths: green: 540nm, red 650nm, c being the speed of light)

A. $v \approx 39\%$ c

B. $v \approx 18\%$ c

C. $v \approx 6\%$ c

D. $v \approx 0.2\%$ c

E. $v \approx 56\%$ c

5. **RELATIVISTIC KINEMATICS**

Let X be a particle of mass m_X, and $\bar{X}$ its antiparticle. Consider the inelastic scattering $X + X \to X + X + X + \bar{X}$; what is the minimum energy E_{th} of one of the X particles in the initial state, in the rest frame of the other, for the process to be kinematically allowed?

A. $E_{th} = m_X$

B. $E_{th} = 3\, m_X$

C. $E_{th} = 5\, m_X$

D. $E_{th} = 7\, m_X$

E. $E_{th} = 10\, m_X$

6. **GENERAL RELATIVITY**

What is the value of the Ricci scalar for a FRW Universe filled only with electromagnetic radiation? (a being the scale factor)

A. $\propto 1/a^2$

B. $\propto 1/a^3$

C. 0

D. $\propto 1/a^4$

E. None of the above

7. **GENERAL RELATIVITY**

If a spaceship is at rest above a non rotating black hole at $r = 4M$ for 1yr (as measured by the pilot), how much time has meanwhile elapsed for an observer at infinity?

A. 1.4 yr

B. 100 yr

C. 10^4 yr

D. 0.5 yr

E. 0.1 yr

8. **QUANTUM MECHANICS**

Strong electric fields can create electron-positron pairs out of the vacuum. This can also be viewed as a quantum tunneling in a background electric field, from a vacuum state to a state with an electron-positron pair. Based on this picture, what is the minimum distance between electron and positron upon creation? Here m is the electron mass, e is the electron charge, E is the electric field strength, and we set $c = \hbar = 1$.

A. $e^3E/(2m)^2$

B. $eE/(2m)^3$

C. $(2m)^2/eE$

D. $1/2m$

E. $2m/eE$

9. **QUANTUM MECHANICS**

Three identical spin 1/2 fermions are subject to the potential of a tridimensional isotropic harmonic oscillator with angular frequency ω. Neglecting mutual interactions, what is the energy of the ground state E_g and its degeneracy d_g?

A. $E_g = 3/2\hbar\omega$ and $d_g = 1$

B. $E_g = 3/2\hbar\omega$ and $d_g = 3$

C. $E_g = 5/2\hbar\omega$ and $d_g = 3$

D. $E_g = 5/2\hbar\omega$ and $d_g = 6$

E. $E_g = 11/2\hbar\omega$ and $d_g = 6$

10. **QUANTUM MECHANICS**

A system with two spins ($s_1 = 1/2$ and $s_2 = 1/2$) is described by the Hamiltonian: $H = \lambda\, \mathbf{s_1} \cdot \mathbf{s_2}$. What are the eigenvalues of H?

A. $-3/4\lambda\hbar^2$ and $1/4\lambda\hbar^2$

B. $-3/4\lambda\hbar^2$, $-1/4\lambda\hbar^2$ and $1/4\lambda\hbar^2$

C. $-3/4\lambda\hbar^2$ and $-1/4\lambda\hbar^2$

D. $-1/4\lambda\hbar^2$, $1/4\lambda\hbar^2$ and $3/4\lambda\hbar^2$

E. None of the others.

11. **QUANTUM FIELD THEORY**

In the Standard Model (SM) for particle physics $SU(3) \times SU(2) \times U(1)$ it is not possible to write a Majorana mass term for the neutrino because:

A. In the SM there is no right-handed neutrino component.

B. The term would not be Lorentz invariant.

C. The term would violate a globally conserved charge.

D. The term would violate the SM gauge invariance.

E. The term would violate the CPT symmetry.

12. **QUANTUM FIELD THEORY**

Consider an effective field theory where a real scalar ϕ interacts with another light real scalar σ through a term $g\phi^2\sigma^2$. If one demands that the one-loop correction to the mass of ϕ is smaller than the tree-level mass m_ϕ, which of the following bounds is obtained for the cutoff energy Λ of the effective theory? Here we ignore numerical factors.

A. $\Lambda \lesssim g^{-1}\, m_\phi^2$

B. $\Lambda \lesssim g^{-1/2}\, m_\phi$

C. $\Lambda \gtrsim g^{-1/2}\, m_\phi$

D. $\Lambda \lesssim g^{1/2}\, m_\phi$

E. $\Lambda \gtrsim g\, m_\phi$

13. QUANTUM FIELD THEORY

A massive real scalar Φ, a massless real scalar ϕ and a massless fermion ψ are described by the Lagrangian density:

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \Phi)^2 - \tfrac{1}{2} M^2 \Phi^2 + \tfrac{1}{2}(\partial_\mu \phi)^2 + i\bar{\psi}\partial\!\!\!/\,\psi - g_\phi \phi^2 \Phi - g_\psi \bar{\psi}\psi\Phi\,.$$

Consider the ratio of partial decay widths $r \equiv \Gamma(\Phi \to \phi\phi)/\Gamma(\Phi \to \psi\bar{\psi})$. Use dimensional analysis to estimate r' in the specular model in which you replace $M \to M' = 2\,M$, $g_\phi \to g'_\phi = 2\,g_\phi$ and $g_\psi \to g'_\psi = 2\,g_\psi$.

A. $r' = r/4$

B. $r' = r/2$

C. $r' = r$

D. $r' = 2\,r$

E. $r' = 4\,r$

14. THERMAL HISTORY

Let A be a hypothetical fermion with mass m_A and electric charges Q_A. Assume that A is kept in kinetic equilibrium in the early Universe by elastic scatterings with photons:

$$A + \gamma \to A + \gamma\,.$$

Considering the stage at which A is non-relativistic, and knowing that in this regime it takes about m_A/T collisions for transferring to A an energy of the order of the temperature T, estimate the scaling with Q_A, m_A and T, of the timescale for kinetic equilibrium τ_{ke} (in all the answers proposed below c is some dimensionless numerical factor).

A. $\tau_{ke} = c\, Q_A^{-2}\, m_A^2\, T^{-4}$

B. $\tau_{ke} = c\, Q_A^{-2}\, m_A^{3/2} T^{-5/2} \exp(m_A/T)$

C. $\tau_{ke} = c\, Q_A^{-4}\, m_A^3\, T^{-4}$

D. $\tau_{ke} = c\, Q_A^{-4}\, m_A^{3/2} T^{-5/2} \exp(-m_A/T)$

E. $\tau_{ke} = c\, Q_A^{-4}\, m_A\, T^{-3}$

15. ASTROPHYSICS

A stellar system has mass of $10^3 M_\odot$ and radius of 100 pc. What is the order of magnitude of the velocity dispersion of the stars? [1 pc $= 3.08 \times 10^{16}$ m, $M_\odot = 2 \times 10^{30}$ kg and $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}]

A. $O(10^5)$ m/s

B. $O(10^4)$ m/s

C. $O(10^2)$ m/s

D. $O(1)$ m/s

E. $O(10^{-1})$ m/s

16. **STATISTICS**

A galaxy survey counts 10^6 galaxies in an region (a) of the sky of 1 square degree. The same survey counts 2×10^6 (b), 1.1×10^6 (c), 0.9×10^5 (d), 0.95×10^6 (e) and 0.99×10^6 (f) galaxies in different sky regions, again of 1 square degree. Which of these counts are statistically different from that of region (a) at 3 σ level?

A. (b)

B. (b,c,d)

C. (b,c,d,e)

D. None

E. All

17. **PROBABILITY**

Today is sunny. On average, 60% of days are sunny, on 40% of days it is cold, and 20% of cold days are sunny. What is the probability that today is cold?

A. 66.6%

B. 13.3%

C. 4.8%

D. 30.0%

E. 8.0%

18. **THERMODYNAMICS**

According to Hawking, a Schwarzschild black hole with mass M radiates as a blackbody with temperature $T \propto M^{-1}$, and eventually evaporates. Supposing that this is true, how does the lifetime of a black hole scale in terms of its mass? Note that the radius of a Schwarzschild black hole scales as $r \propto M$.

A. $\tau \propto M^3$

B. $\tau \propto M^2$

C. $\tau \propto M$

D. $\tau \propto M^{-1}$

E. The lifetime is independent of mass.