coarse-graining, BH thermodynamics and quantum gravitation

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ESF Exploratory Workshop
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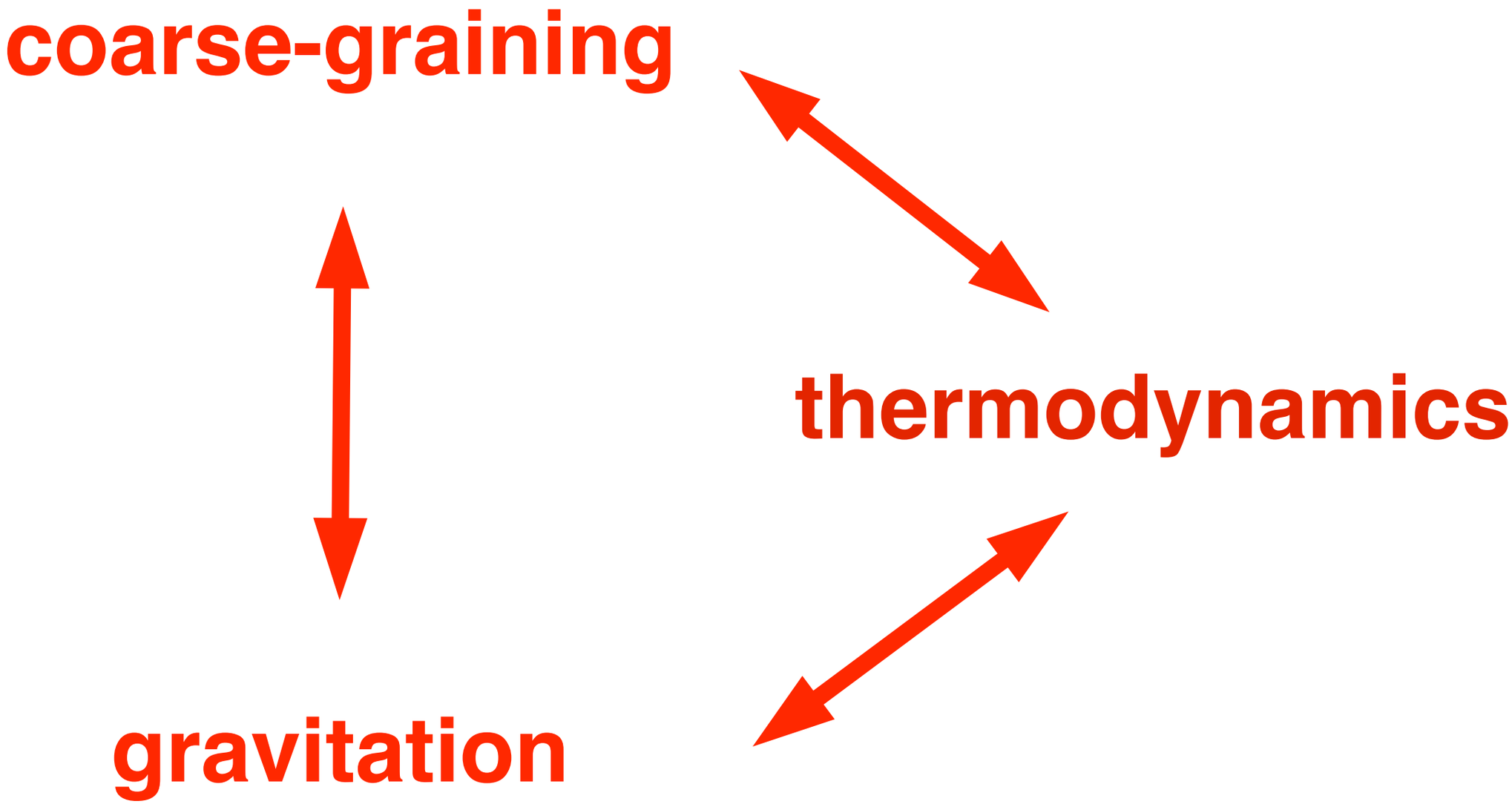
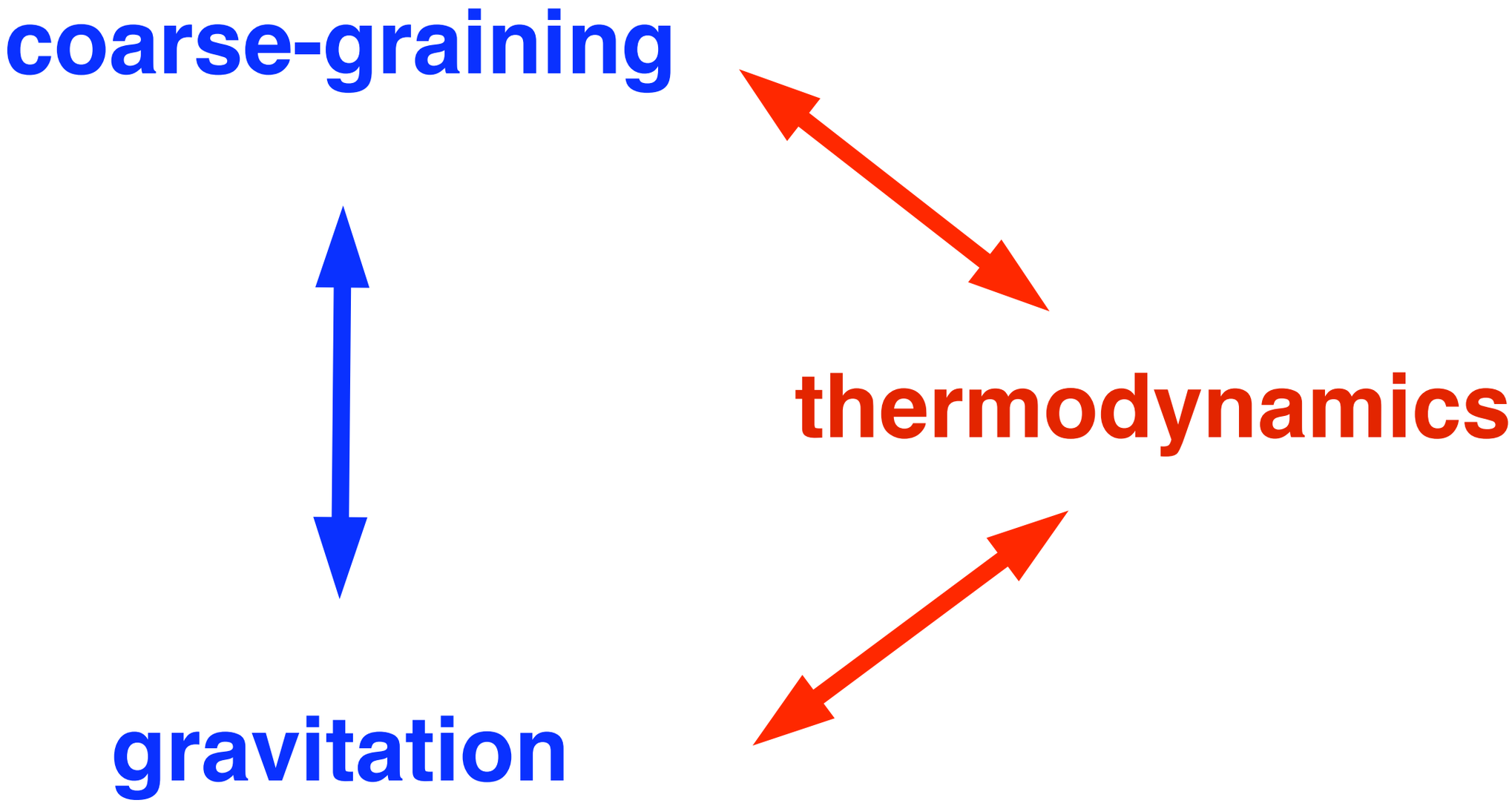
SISSA/ISAS Trieste – Italy


coarse-graining
thermodynamics
gravitation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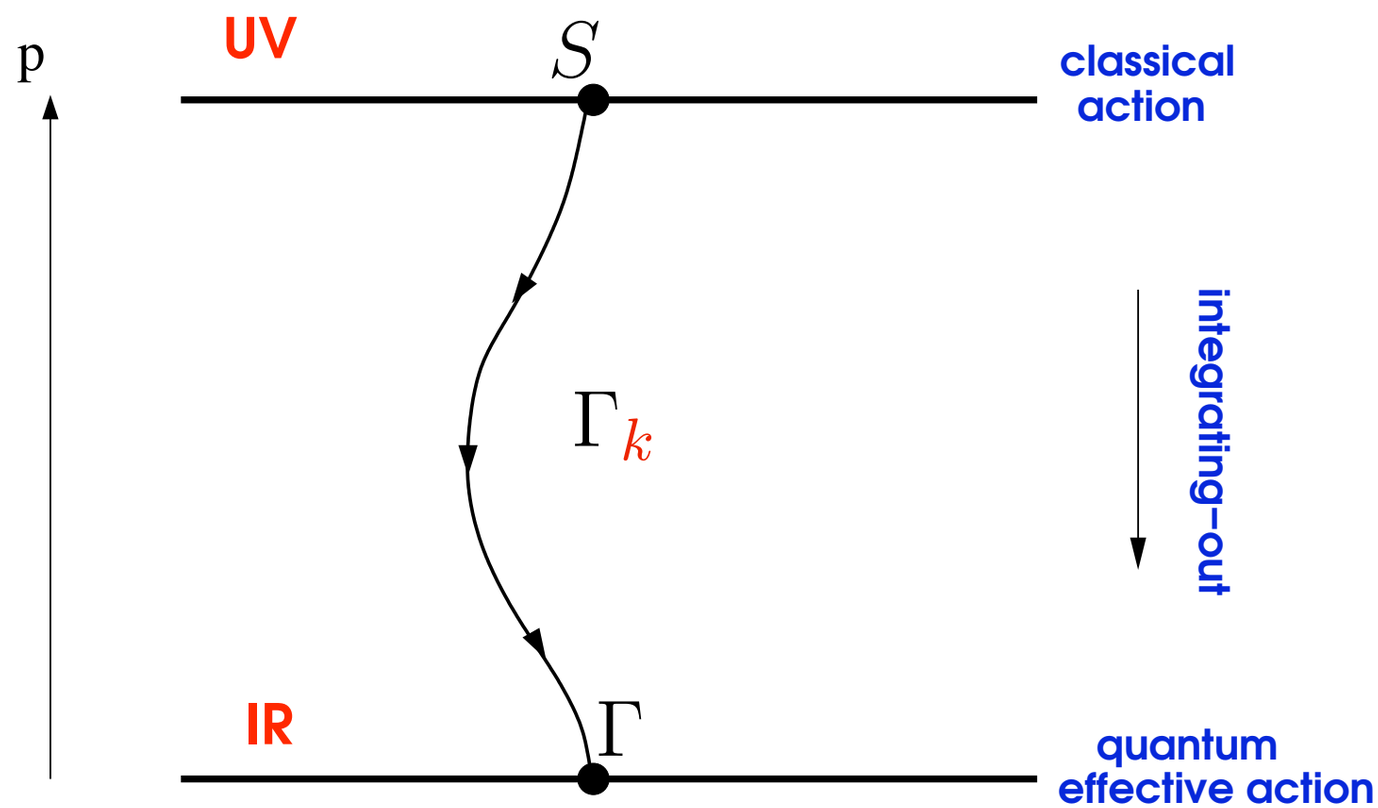
coarse-graining

integrating-out momentum degrees of freedom: "top-down" (Wilson '71)



`coarse-graining' of quantum fields

quantum field theory

running couplings

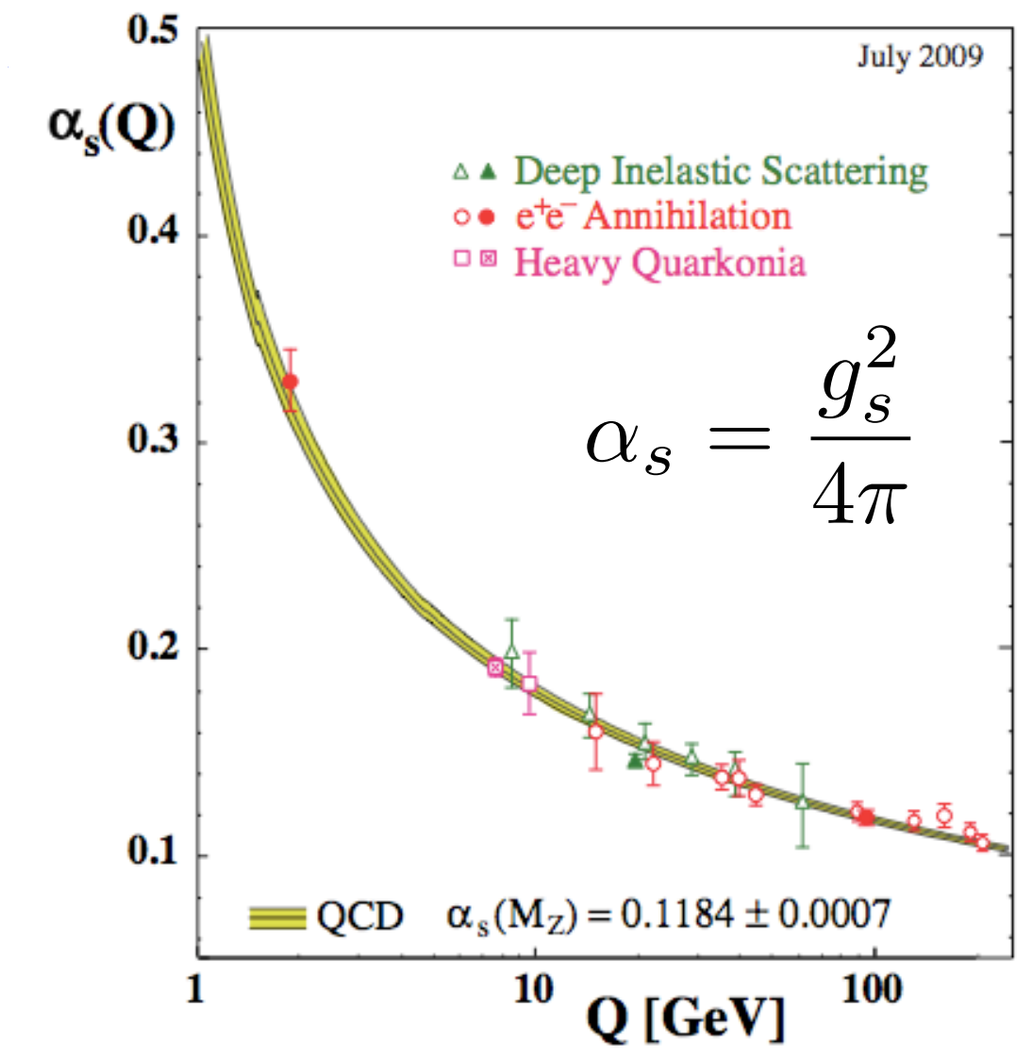
quantum fluctuations modify interactions

couplings depend on eg. energy or distance

asymptotic freedom of the strong force

(taken from PDG)

$$S_{\mathrm{YM}} = \frac{1}{4g_s^2}\int F^2$$



$$\alpha_s = \frac{g_s^2}{4\pi}$$

quantum field theory

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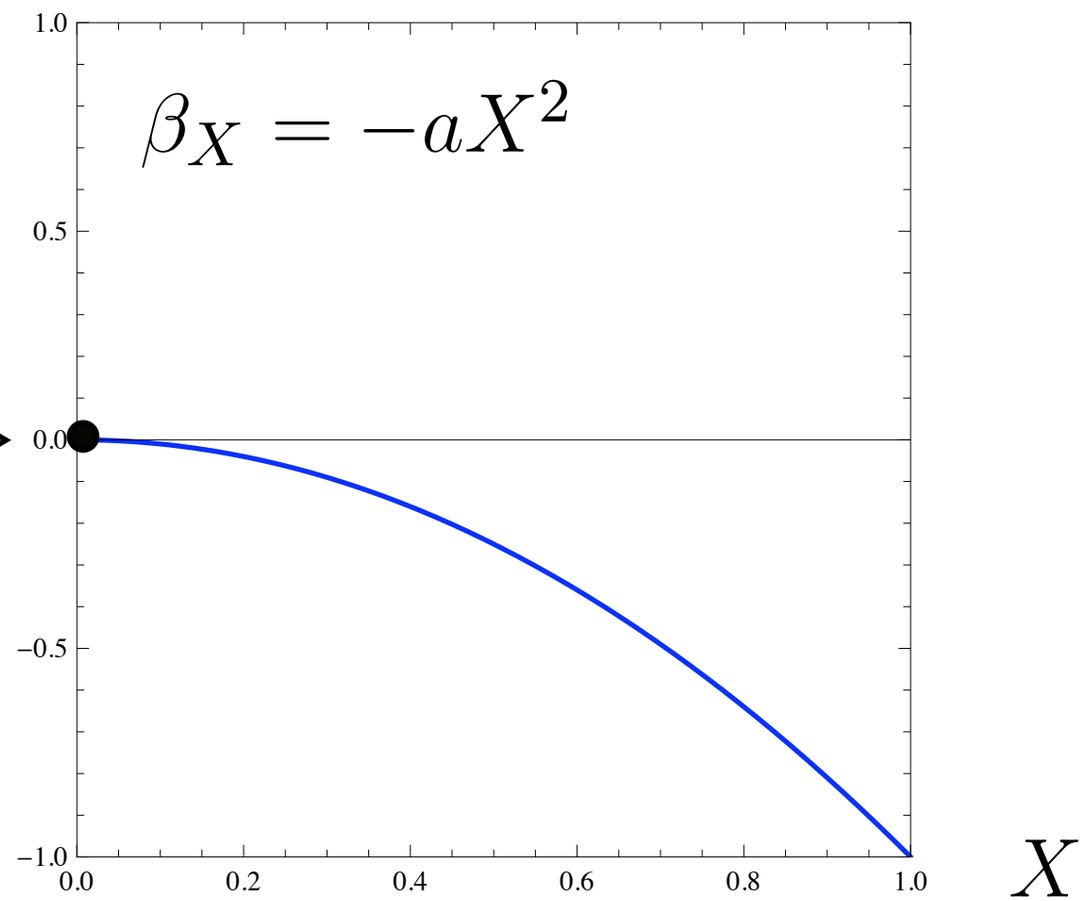
asymptotic freedom of the strong force

coupling $X = g_s^2/(4\pi)$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

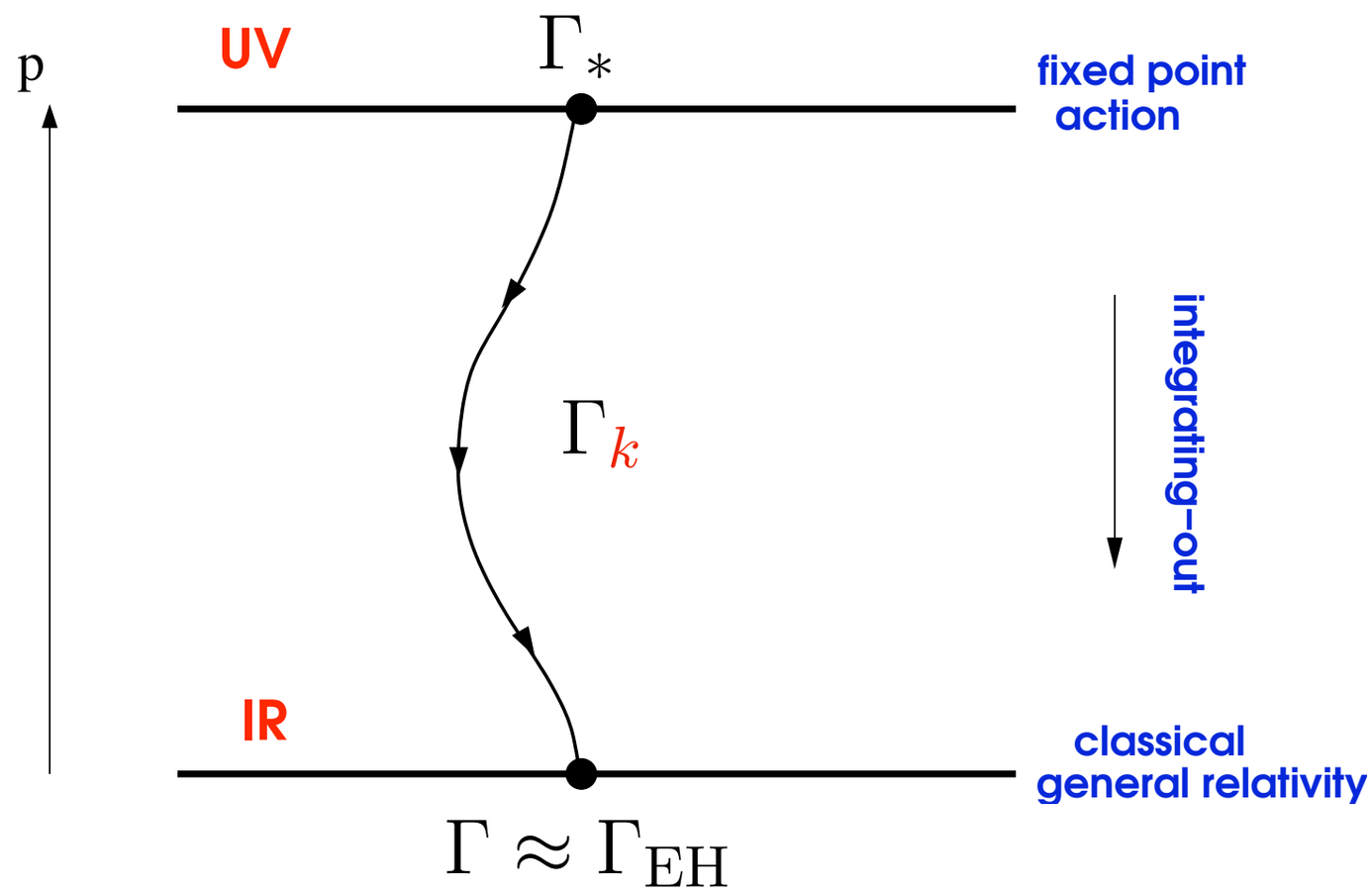
trivial UV fixed point

$$X_* = 0$$



coarse-graining

for quantum gravity: "bottom-up" (Reuter '96, Percacci et al '97, DL '03, Niedermaier '06)



`coarse-graining' of quantum fields

quantum field theory

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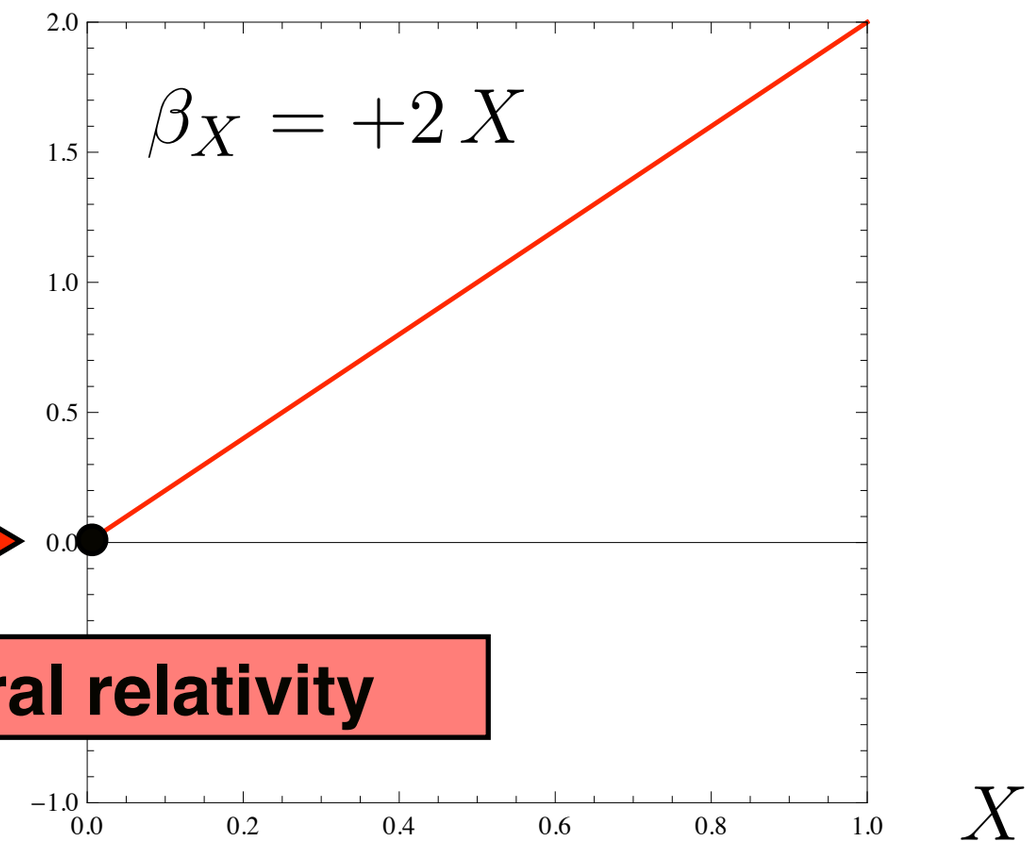
gravitation

coupling $X = G_N \mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

trivial IR fixed point



classical general relativity

quantum field theory

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

gravitation

replace

$$G_N \to G(\mu)$$

$$g_{\rm eff} = G_N\,\mu^2 \to g(\mu) \equiv G(\mu)\,\mu^2$$

quantum field theory

running couplings

quantum fluctuations modify interactions

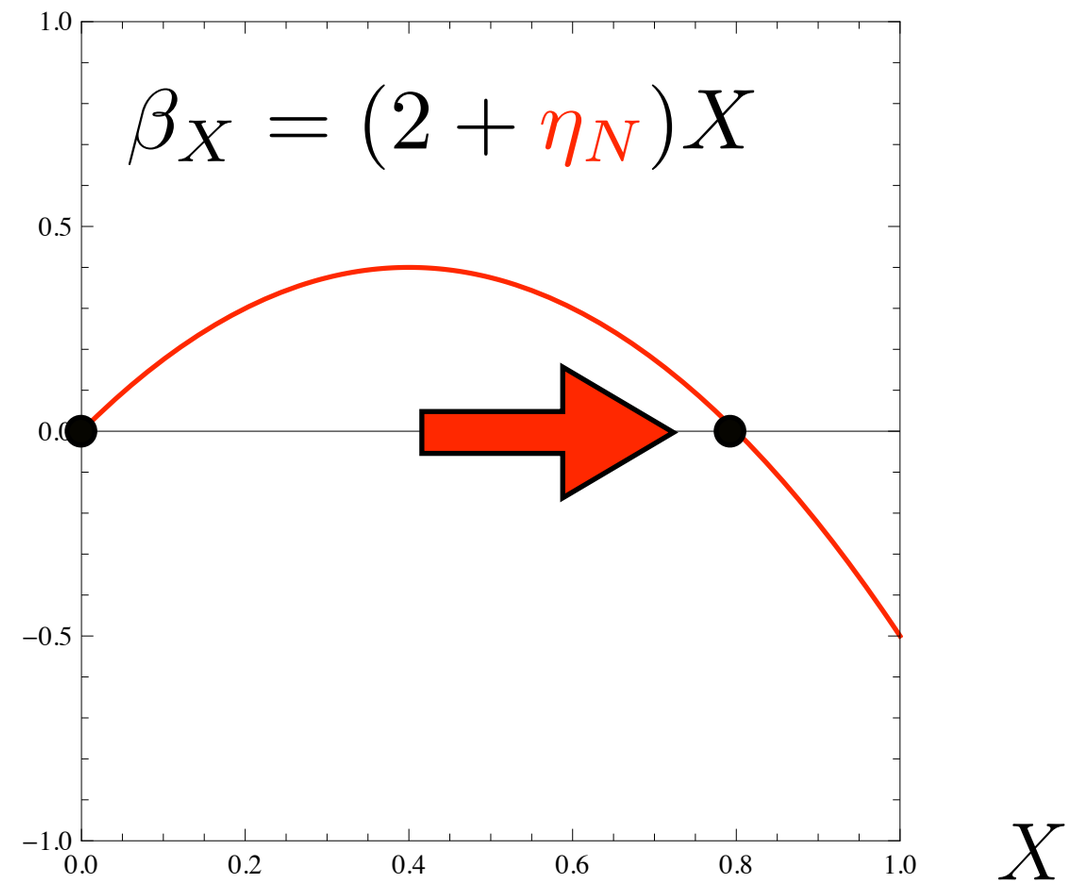
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



Weinberg ('79)
Reuter ('96), Percacci et. al. ('97)
DL ('03), Niedermaier ('06)

quantum field theory

running couplings

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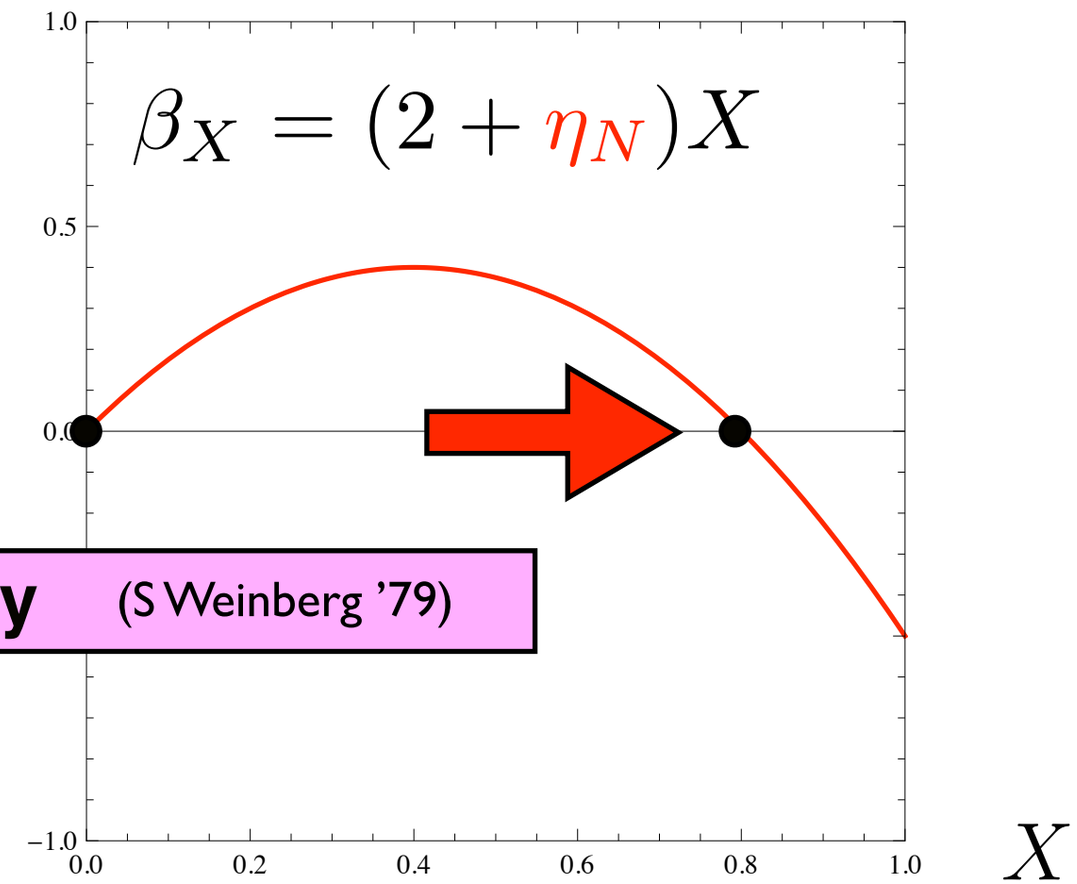

asymptotic safety (S Weinberg '79)

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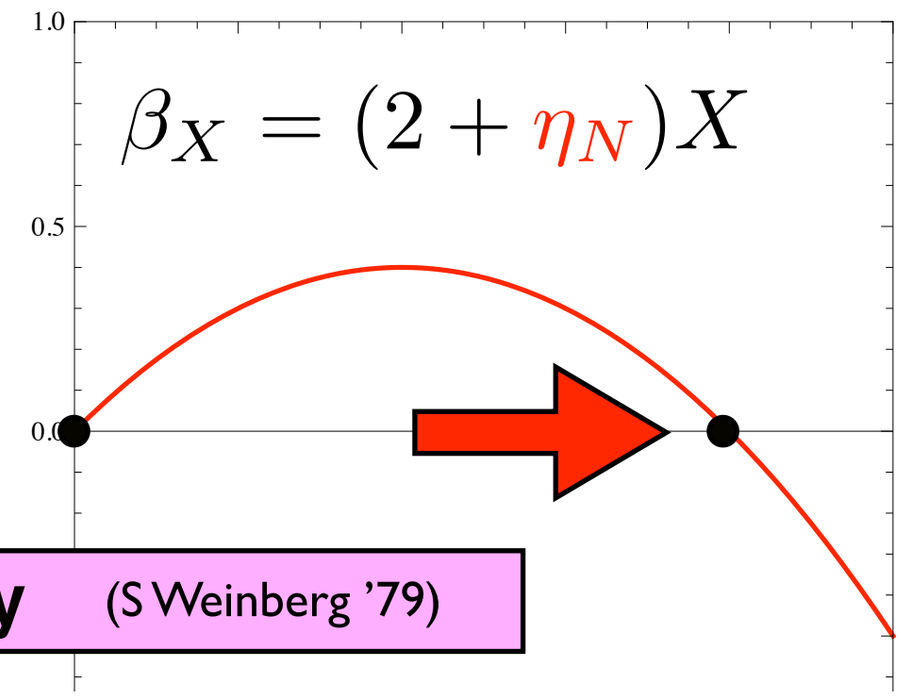


asymptotic safety (S Weinberg '79)

UV fixed point implies **weakly coupled gravity** at high energies

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

asymptotic safety

overviews: DL **0810.3675** and **1102.4624**

gravitation

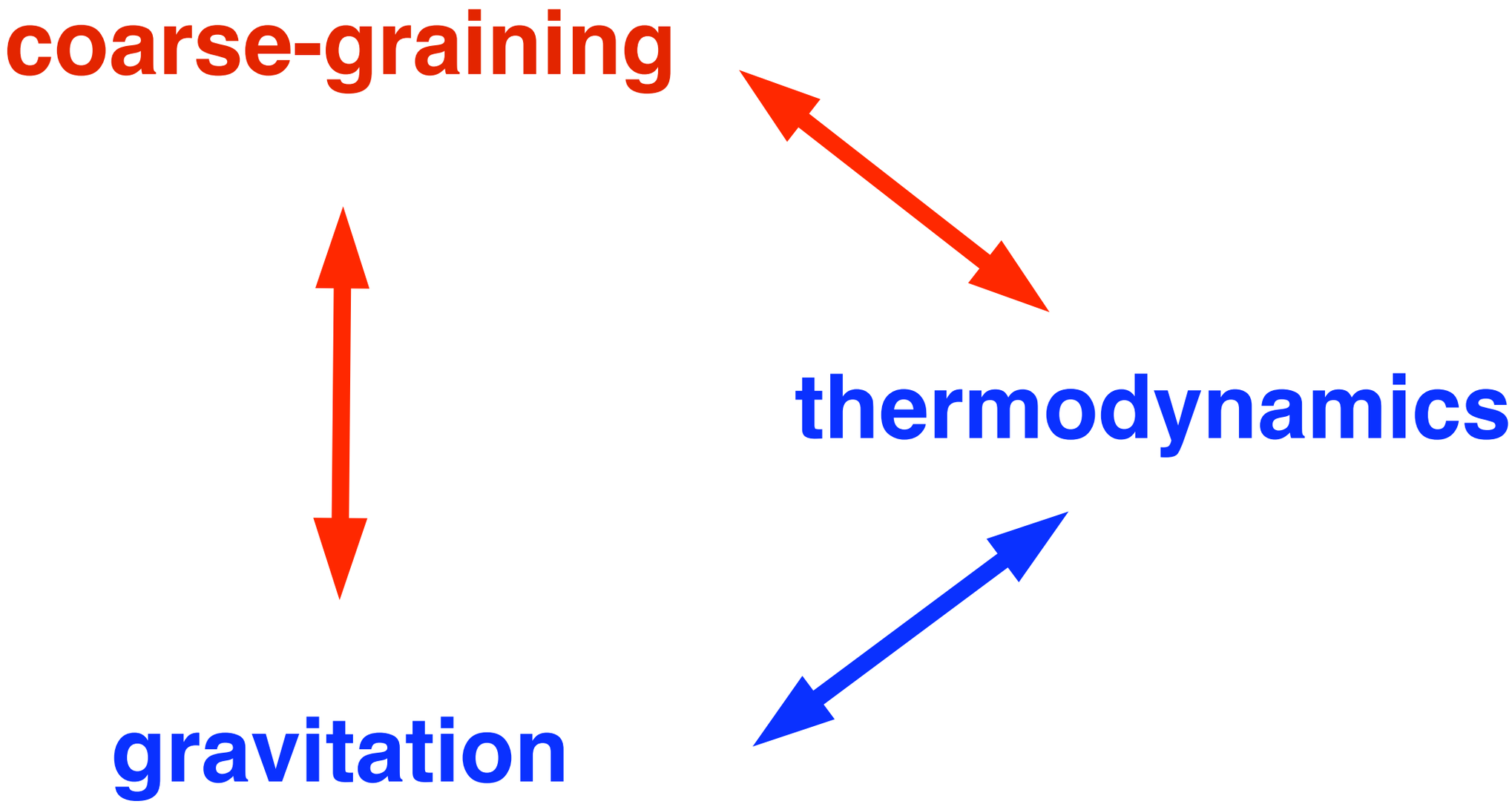
- Einstein-Hilbert (Souma '99, Reuter, Lauscher '01, DL '03)
- f(R), polynomials in R (Codello, Percacci, Rahmede '08, Machado, Saueressig '09)
- higher-derivative gravity (Codello, Percacci '05)
  (Benedetti, Saueressig, Machado '09, Niedermaier '09)
- higher dimensions (DL '03, Fischer, DL '05)
- conformally reduced gravity (Reuter, Weyer '09, Machado, Percacci '10)
- Holst action + Immirzi parameter (Daum, Reuter '10, Benedetti, Speciale '11)

gravitation + matter

- matter (Percacci '05, Perini, Percacci '05)
  (Narain, Percacci '09, Narain, Rahmede '09, Codello '11)
- Yang-Mills gravity
  - 1-loop: (Robinson, Wilzcek '05, Pietrokowski, '06, Toms '07, Ebett, Plefka, Rodigast '08)
  - beyond: (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawlowski, 11, Harst, Reuter '11)

gravitation + lattice

- consistency with
  - simplicial gravity / Regge calculus (Hamber '00, Hamber, Williams '04)
  - causal and euclidean DT (Amjorn, Jurkiewicz, Loll '04, Laiho, Coumbe '11)


coarse-graining
thermodynamics
gravitation

black hole thermodynamics

black holes in general relativity

solutions to Einstein's classical equations
event horizon with area **A**

most general static BH solution for long-ranged forces:
Kerr Newman black holes

uniqueness theorem:

$$A = A(M, J, q)$$

singularities / break-down of predictivity
cosmic censorship ?

black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

$$\delta M = \frac{\kappa}{8\pi G_N}\,\delta A + \Omega\,\delta J + \Phi\,\delta q$$

first law of thermodynamics

$$\delta U = \delta Q + \sum_i \mu_i\,\delta N_i$$

with $U \leftrightarrow M$ $\qquad$ $\mu_i \leftrightarrow \{\Omega, \Phi\}$

$$\delta Q \leftrightarrow \frac{\kappa}{8\pi G_N}\,\delta A \qquad N_i \leftrightarrow \{J, q\}$$

black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

$$\delta M = \frac{\kappa}{8\pi G_N}\,\delta A + \Omega\,\delta J + \Phi\,\delta q$$

reversible thermodynamical process at temperature **T**

$$\frac{\delta Q}{T} = \delta S$$

second law of thermodynamics

$$S \propto A$$

(Bekenstein '73)

$$T = \frac{\kappa}{2\pi}$$

(Hawking '75)

black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

$$\delta M = \frac{\kappa}{8\pi G_N}\,\delta A + \Omega\,\delta J + \Phi\,\delta q$$

identify the heat flow at temperature **T** as

$$\delta Q = \delta M - \Omega\,\delta J - \Phi\,\delta q$$

to find that the **first law of thermodynamics** leads to

$$S_{\rm BH} = \frac{A}{4\,G_N}$$

derivation from euclidean path integral

(Gibbons, Hawking '77)

statistical entropy

coarse-graining

(Falls, Litim '11)

quantum gravitation

challenge for any UV completion of gravity:
identify the underlying coarse-grained degrees of freedom

``flowing'' effective action

$$\Gamma_k = \int d^4x\sqrt{-g}\left[\frac{1}{8\pi G_k}R + \frac{1}{4\alpha_k}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{\rm m}\right].$$

IR limit $\quad G_0 \approx 6.674 \times 10^{-11}\ \mathrm{N\,(m/kg)^2} \qquad \alpha_0 \approx 1/137$

family of Kerr-Newman BH solutions

$$A = A(M, J, q; k) \qquad\qquad S = \frac{A}{4G_k}$$

coarse-graining

(Falls, Litim '11)

choice of scale

"not too coarse- and not too fine-grained"

$$k = k_{\rm opt}(M, J, q)$$

RG thermodynamics

infinitesimal amount of matter crossing the horizon, with heat flow

$$\frac{\delta Q}{T} = \frac{\delta A}{4G_k}$$

BH settles in a new state

$$M \to M + \delta M \quad J \to J + \delta J$$

$$q \to q + \delta q \quad k_{\rm opt} \to k_{\rm opt} + \delta k_{\rm opt}$$

total change of horizon area

$$\delta A = \frac{2\pi}{\kappa} T \, \delta A + \left. \frac{\partial A(M, J, q; k)}{\partial k} \right|_{k = k_{\rm opt}} \delta k_{\rm opt}$$

coarse-graining

(Falls, Litim '11)

choice of scale

"not too coarse- and not too fine-grained" $k = k_{\rm opt}(M, J, q)$

results

optimal scale $$k^2_{\rm opt}(M, J, q) \equiv k^2_{\rm opt}(A) = \frac{4\pi\xi^2}{A}$$

mass function $$M^2 \equiv \frac{4\pi}{A}\left[\left(\frac{A + 4\pi G(A)e^2(A)q^2}{8\pi G(A)}\right)^2 + J^2\right]$$

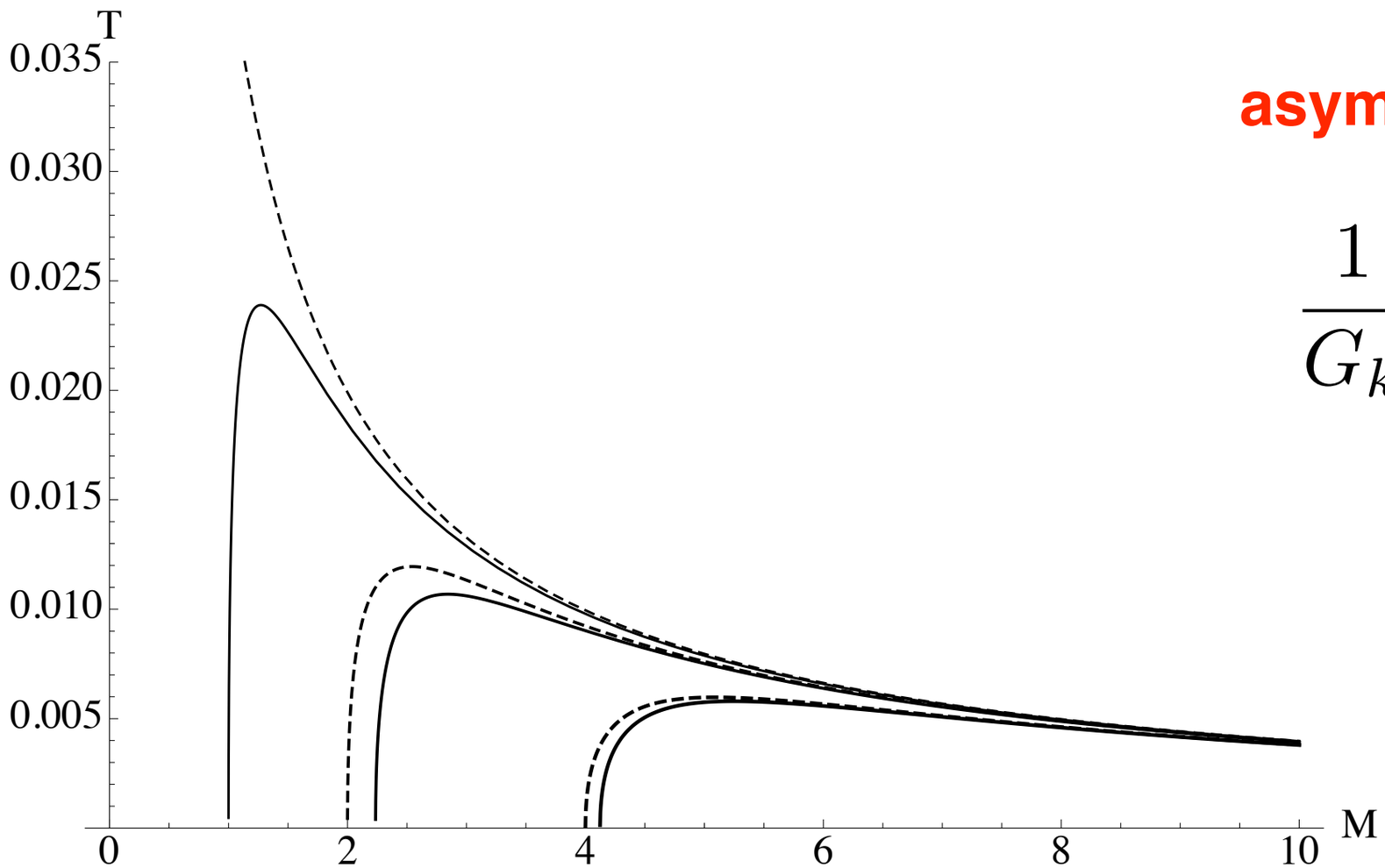
temperature $$T = 4G(A)\,\frac{\partial M}{\partial A}$$

entropy $$S = \frac{A}{4G_k}$$ with $$k = k_{\rm opt}$$

asymptotically safe black holes

(Falls, Litim '11)

RG thermodynamics



asymptotic safety:

$$\frac{1}{G_k} = \frac{1}{G_N} + \frac{k^2}{g_*}$$

outer/inner horizon

$$A_\pm = 4\pi G_N \left[2G_N M^2 - \frac{1}{g_* \xi^2} \pm 2\sqrt{G_N^2 M^4 - J^2 - \frac{G_N M^2}{g_* \xi^2}} \right]$$

RG improved metrics

basic idea

replace $G_N \to G(r, \cdots)$ using $k \to k(r, \cdots)$ (Bonanno, Reuter '01)

(Reuter, Tiuran '10)

Kerr space-time

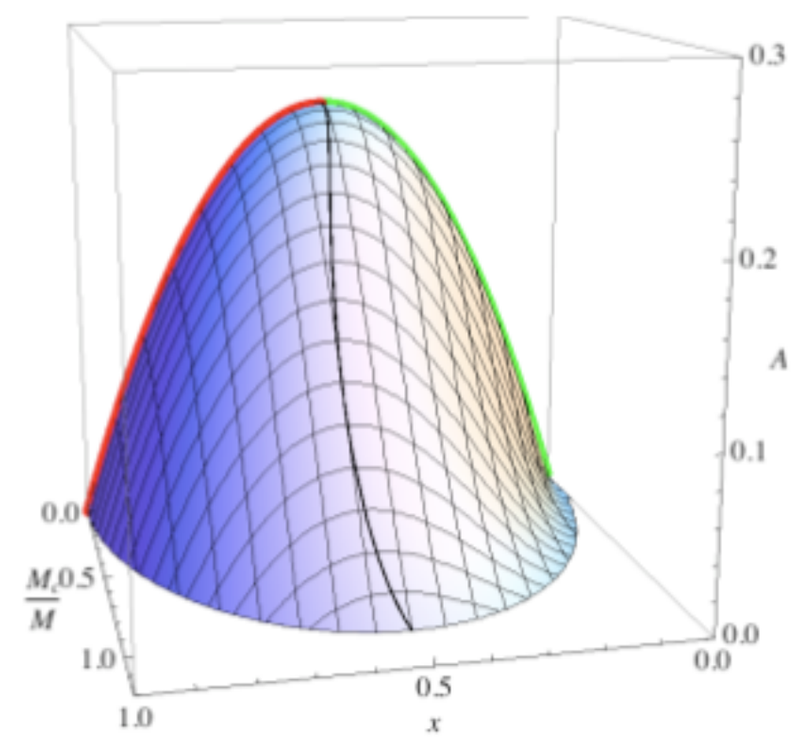
(DL, Nikolakopoulos, ERG '10)

$$ds^2 = -\left(1 - \frac{2G(r)Mr}{\rho^2}\right) dt^2 - \frac{G(r)Mar\, s^2}{\rho^2} dt d\phi$$
$$+ \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{s^2}{\rho^2}\left[(r^2+a^2)^2 - a^2 \Delta\, s^2\right] d\phi^2$$

with
$$\Delta(r) = r^2 - 2G(r)Mr + a^2$$
$$\rho^2 = r^2 + a^2 \cos^2\theta$$

horizons

solve $\Delta(r_h) = 0$



RG improved metrics

thermodynamical equivalence

(Falls, DL '11)

provided if

$$k^2(r) = \frac{\xi^2}{r^2 + (J/M)^2}$$

statistical entropy

"off-shell" conical singularity method (Soludkhin '96)

Schwarzschild black hole, free energy

improved metric

$$F = \frac{r_+}{2G_N} - \frac{A}{4G_N}T$$

(Bonanno, Reuter '00)

here

$$F = M - ST$$

conclusion

RG gravitation

increasing evidence for UV fixed point
metric field remains fundamental carrier of the dynamics

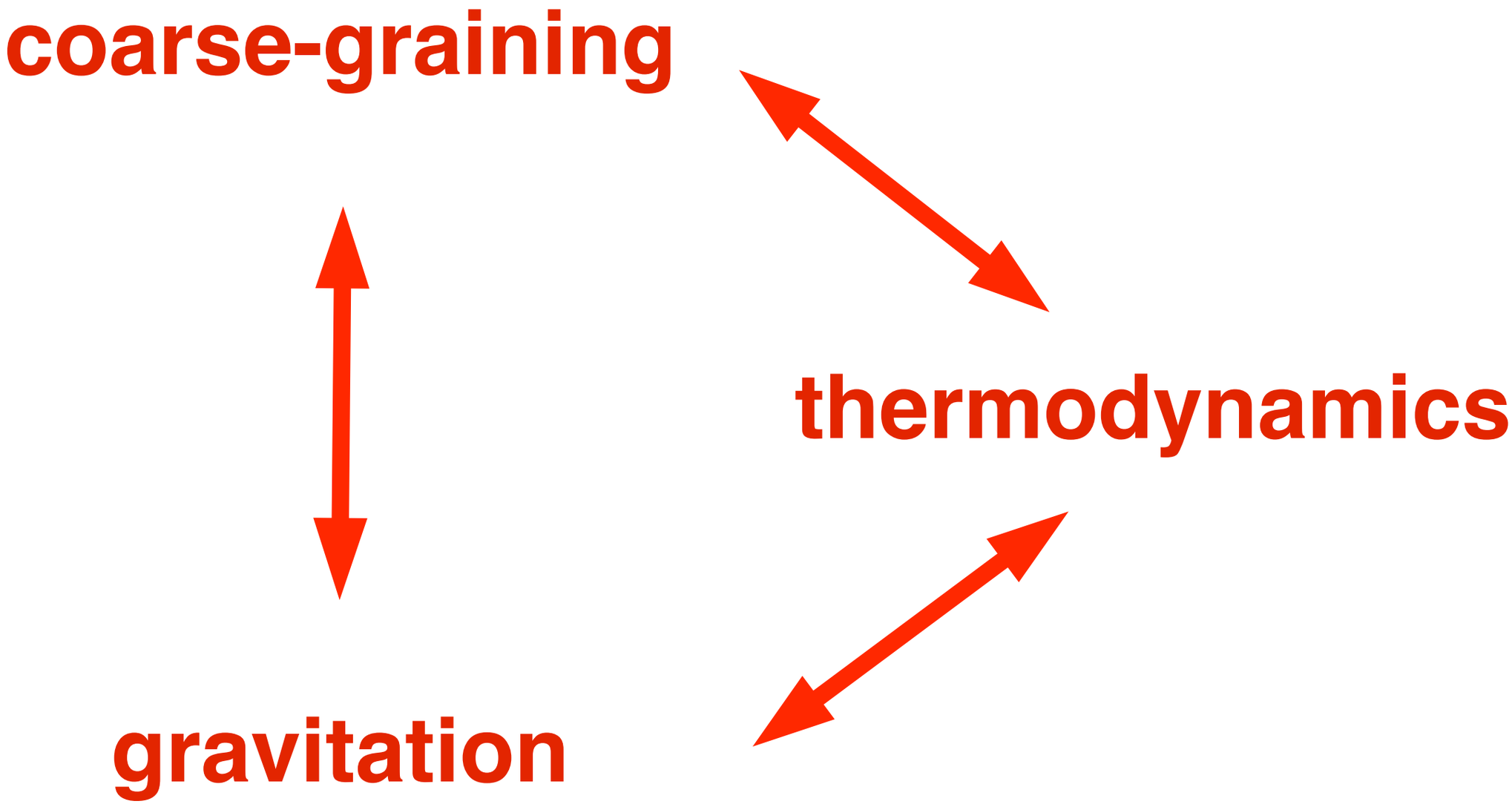
gravitation and thermodynamics

RG puts black holes under a microscope
BH thermodynamics consistent and meaningful
beyond the semi-classical approximation

predictions, extensions, challenges

thermodynamical consistency + asymptotic safety implies
→ smallest black holes + maximum temperature
high-energy degrees of freedom?
cosmological constant, higher curvature invariants
non-equilibrium thermodynamics

some extra material



further directions

higher derivative gravity

1-loop — Codello, Percacci ('05) Niedermaier ('09)

1-loop and beyond — Benedetti, Machado, Saueressig ('09)

conformal symmetry

Weyl coupling

$$\frac{1}{\sigma}\int d^4x\sqrt{g}\, C_{\mu\nu\rho\tau}\, C^{\mu\nu\rho\tau}$$

asymptotically `free' fixed point $\sigma_* = 0$ — DL, Rahmede ('11)

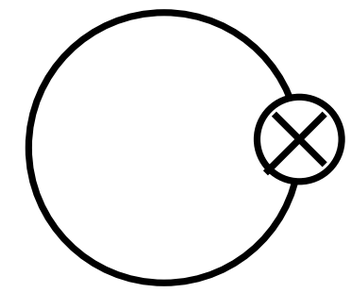
entails $g_* > 0 \quad \lambda_* \neq 0$

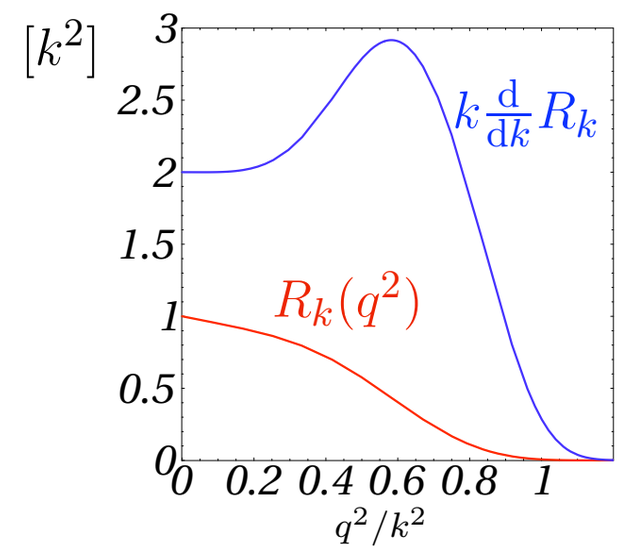
QCD versus quantum gravitation: ``as close as it gets''

renormalisation group

functional RG (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$





`all-in-one'
`exact'
finite
systematic
Callan-Symanzik type

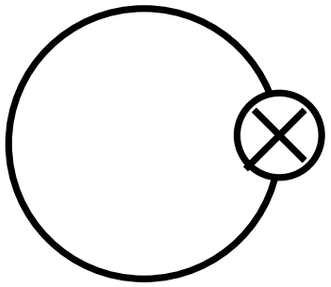
`optimised' choices of R_k

→ stability, analyticity, convergence (DL '01,'02)

renormalisation group

functional RG (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$



QCD

signatures of confinement (Pawlowski, DL, Nedelko, Smekal '03)

Ising-type universality

phase transitions, high precision exponents (DL '02, Bervillier, Juettner, DL '07)

quality control, systematic uncertainties

	η	ν	ω
resummed PT	**0.0335(25)**	**0.6304(13)**	**0.799(11)**
ϵ-expansion	**0.0360(50)**	**0.6290(25)**	**0.814(18)**
world average	**0.0364(5)**	**0.6301(4)**	**0.84(4)**
Monte Carlo	**0.03627(10)**	**0.63002(10)**	**0.832(6)**
functional RGs	**0.034(5)**	**0.630(5)**	**0.82(4)**

(DL, Zappala '10)