coarse-graining, BH thermodynamics and quantum gravitation

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coarse-graining

gravitation

thermodynamics
coarse-graining

gravitation

thermodynamics
coarse-graining

integrating-out momentum degrees of freedom: “top-down” (Wilson '71)

\[ \Gamma_k \rightarrow \Gamma \]

\text{query:} \text{`coarse-graining’ of quantum fields}
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

asymptotic freedom of the strong force

\[ S_{\text{YM}} = \frac{1}{4g_s^2} \int F^2 \]

\[ \alpha_s(Q) = \frac{g_s^2}{4\pi} \]

Saturday, 10 September 2011
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

asymptotic freedom of the strong force

coupling \( X = \frac{g_s^2}{4\pi} \)

\[ \beta_X \equiv \frac{dX}{d\ln \mu} \]

trivial UV fixed point

\( X_* = 0 \)
coarse-graining

for quantum gravity: “bottom-up” (Reuter '96, Percacci et al '97, DL '03, Niedermaier '06)

\[ \Gamma_k \approx \Gamma_{EH} \]

`coarse-graining' of quantum fields
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

gravitation

coupling \( X = G_N \mu^2 \)

\( \beta_X \equiv \frac{dX}{d\ln \mu} \)

trivial IR fixed point

classical general relativity
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

gavitation

replace

\[ G_N \rightarrow G(\mu) \]

\[ g_{\text{eff}} = G_N \mu^2 \rightarrow g(\mu) \equiv G(\mu) \mu^2 \]
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

gravitation

coupling \( X = G(\mu) \mu^2 \)

\[ \beta_X \equiv \frac{dX}{d\ln \mu} \]

non-trivial UV fixed point

Weinberg ('79)
Reuter ('96), Percacci et. al. ('97)
DL ('03), Niedermaier ('06)
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

gravitation

coupling \( X = G(\mu) \mu^2 \)
\[
\beta_X \equiv \frac{dX}{d\ln \mu}
\]

non-trivial UV fixed point

asymptotic safety (S Weinberg '79)
quantum field theory

running couplings

quantum fluctuations modify interactions
couplings depend on eg. energy or distance

gravitation

coupling \( X = G(\mu) \mu^2 \)

\[ \beta_X \equiv \frac{dX}{d\ln \mu} \]

non-trivial UV fixed point

asymptotic safety \( (S \text{ Weinberg '79}) \)

UV fixed point implies weakly coupled gravity at high energies

\[ \mu \to \infty : \quad G(\mu) \to g_* \mu^{2-D} \ll G_N \]
asymptotic safety

gravitation

Einstein-Hilbert  (Souma '99, Reuter, Lauscher '01, DL '03)
f(R), polynomials in R  (Codello, Percacci, Rahme  '08, Machado, Saueressig '09)
higher-derivative gravity  (Codello, Percacci '05)
  (Benedetti, Saueressig, Machado '09, Niedermaier '09)
higher dimensions  (DL '03, Fischer, DL '05)
conformally reduced gravity  (Reuter, Weyer '09, Machado, Percacci '10)
Holst action + Immirzi parameter  (Daum, Reuter '10, Benedetti, Speciale '11)

gravitation + matter

matter  (Percacci '05, Perini, Percacci '05)
  (Narain, Percacci '09, Narain, Rahmede '09, Codello '11)
Yang-Mills gravity
  1-loop:  (Robinson, Wilzcek '05, Pietrokowski, '06, Toms '07, Ebett, Plefka, Rodigast '08)
  beyond:  (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawlowski, 11, Harst, Reuter '11)

gravitation + lattice

consistency with
  simplicial gravity / Regge calculus  (Hamber '00, Hamber, Williams '04)
  causal and euclidean DT  (Amjorn, Jurkiewicz, Loll '04, Laiho, Coumbe '11)
coarse-graining

gravitation

thermodynamics
black hole thermodynamics

black holes in general relativity

solutions to Einstein’s classical equations
event horizon with area $A$

most general static BH solution for long-ranged forces:
Kerr Newman black holes

uniqueness theorem:

$$A = A(M, J, q)$$

singularities / break-down of predictivity
cosmic censorship?
black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking ’73)

infinitesimal amount of matter crossing the horizon

\[ \delta M = \frac{\kappa}{8\pi G_N} \delta A + \Omega \delta J + \Phi \delta q \]

first law of thermodynamics

\[ \delta U = \delta Q + \sum_i \mu_i \delta N_i \]

with

\[ U \leftrightarrow M \]

\[ \mu_i \leftrightarrow \{ \Omega, \Phi \} \]

\[ \delta Q \leftrightarrow \frac{\kappa}{8\pi G_N} \delta A \]

\[ N_i \leftrightarrow \{ J, q \} \]
black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking ’73)

infinitesimal amount of matter crossing the horizon

\[ \delta M = \frac{\kappa}{8\pi G_N} \delta A + \Omega \delta J + \Phi \delta q \]

reversible thermodynamical process at temperature \( T \)

\[ \frac{\delta Q}{T} = \delta S \]

second law of thermodynamics

\( S \propto A \)

(Bekenstein ’73)

\( T = \frac{\kappa}{2\pi} \)

(Hawking ’75)
black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

\[ \delta M = \frac{\kappa}{8\pi G_N} \delta A + \Omega \delta J + \Phi \delta q \]

identify the heat flow at temperature \( T \) as

\[ \delta Q = \delta M - \Omega \delta J - \Phi \delta q \]

to find that the first law of thermodynamics leads to

\[ S_{BH} = \frac{A}{4 G_N} \]

derivation from euclidean path integral statistical entropy

(Gibbons, Hawking '77)
coarse-graining

quantum gravitation

challenge for any UV completion of gravity:
identify the underlying coarse-grained degrees of freedom

``flowing” effective action

\[
\Gamma_k = \int d^4 x \sqrt{-g} \left[ \frac{1}{8\pi G_k} R + \frac{1}{4\alpha_k} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_m \right].
\]

IR limit \( G_0 \approx 6.674 \times 10^{-11} \text{ N (m/kg)}^2 \quad \alpha_0 \approx 1/137 \)

family of Kerr-Newman BH solutions

\[
A = A(M, J, q; k) \quad S = \frac{A}{4G_k}
\]
coarse-graining

(choice of scale)

“not too coarse- and not too fine-grained”

$\delta Q = \delta A / 4G_k$

$M \rightarrow M + \delta M \quad J \rightarrow J + \delta J$

$q \rightarrow q + \delta q \quad k_{\text{opt}} \rightarrow k_{\text{opt}} + \delta k_{\text{opt}}$

(BH settles in a new state)

$\delta A = \frac{2\pi}{\kappa} T \delta A + \frac{\partial A(M, J, q; k)}{\partial k} \bigg|_{k=k_{\text{opt}}} \delta k_{\text{opt}}$

(RG thermodynamics)

infinitesimal amount of matter crossing the horizon, with heat flow
coarse-graining

(Falls, Litim ’11)

choice of scale

“not too coarse- and not too fine-grained”

results

optimal scale

\[ k = k_{\text{opt}}(M, J, q) \]

\[ k_{\text{opt}}^2(M, J, q) \equiv k_{\text{opt}}^2(A) = \frac{4\pi \xi^2}{A} \]

mass function

\[ M^2 \equiv \frac{4\pi}{A} \left[ \left( \frac{A + 4\pi G(A)e^2(A)q^2}{8\pi G(A)} \right)^2 + J^2 \right] \]

temperature

\[ T = 4G(A) \frac{\partial M}{\partial A} \]

entropy

\[ S = \frac{A}{4G_k} \quad \text{with} \quad k = k_{\text{opt}} \]
asymptotically safe black holes

(RG thermodynamics)

asymptotic safety:

\[
\frac{1}{G_k} = \frac{1}{G_N} + \frac{k^2}{g_*}
\]

outer/inner horizon

\[
A_{\pm} = 4\pi G_N \left[ 2G_NM^2 - \frac{1}{g_*\xi^2} \pm 2\sqrt{G_N^2M^4 - J^2 - \frac{G_NM^2}{g_*\xi^2}} \right]
\]
RG improved metrics

basic idea

replace $G_N \rightarrow G(r, \cdots)$ using $k \rightarrow k(r, \cdots)$ (Bonanno, Reuter '01)

Kerr space-time

$$ds^2 = - \left(1 - \frac{2G(r)Mr}{\rho^2}\right) dt^2 - \frac{G(r)Mar s^2}{\rho^2} dtd\phi$$

$$+ \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{s^2}{\rho^2} \left[(r^2 + a^2)^2 - a^2 \Delta s^2\right] d\phi^2$$

with

$$\Delta(r) = r^2 - 2G(r)Mr + a^2$$

$$\rho^2 = r^2 + a^2 \cos^2 \theta$$

horizons

solve $\Delta(r_h) = 0$
RG improved metrics

thermodynamical equivalence

provided if

\[ k^2(r) = \frac{\xi^2}{r^2 + (J/M)^2} \]

statistical entropy

“off-shell” conical singularity method

Schwarzschild black hole, free energy

improved metric

\[ F = \frac{r+}{2G_N} - \frac{A}{4G_N} T \]

here

\[ F = M - S T \]
conclusion

**RG gravitation**
increasing evidence for UV fixed point
metric field remains fundamental carrier of the dynamics

**gravitation and thermodynamics**
RG puts black holes under a microscope
BH thermodynamics consistent and meaningful
beyond the semi-classical approximation

**predictions, extensions, challenges**
thermodynamical consistency + asymptotic safety implies
smallest black holes + maximum temperature
high-energy degrees of freedom?
cosmological constant, higher curvature invariants
non-equilibrium thermodynamics
some extra material

course-graining

gravitation

thermodynamics
further directions

higher derivative gravity

1-loop  
Codello, Percacci (’05) Niedermaier (’09)
1-loop and beyond  
Benedetti, Machado, Saueressig (’09)

conformal symmetry

Weyl coupling

\[
\frac{1}{\sigma} \int d^4x \sqrt{g} C_{\mu\nu\rho\tau} C^{\mu\nu\rho\tau}
\]

asymptotically `free' fixed point

\[
\sigma_\ast = 0
\]

entails

\[
g_\ast > 0 \quad \lambda_\ast \neq 0
\]

QCD versus quantum gravitation:  ```as close as it gets'’
renormalisation group

**functional RG** (Wetterich '93)

\[ k \frac{d\Gamma_k}{dk} = \frac{1}{2} \text{Tr} \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R_k \right)^{-1} k \frac{dR_k}{dk} \right] = \frac{1}{2} \]

`all-in-one`  
`exact`  
finite  
systematic  
Callan-Symanzik type  

`optimised` choices of \( R_k \)

\[ \text{stability, analyticity, convergence} \]

(DL '01,'02)
renormalisation group

**functional RG**  (Wetterich '93)

\[
k \frac{d \Gamma_k}{dk} = \frac{1}{2} \text{Tr} \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R_k \right)^{-1} k \frac{d R_k}{dk} \right] = \frac{1}{2}
\]

**QCD**

signatures of confinement  (Pawlowski, DL, Nedelko, Smekal '03)

**Ising-type universality**

phase transitions, high precision exponents  (DL '02, Bervillier, Juettner, DL '07)

quality control, systematic uncertainties  (DL, Zappala '10)

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(DL, Zappala '10)