Lessons from holography: probing universality

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Insight into strong coupling

Many faces of holography:

- Top-down studies (string/M-theory based) focused on probing features of quantum gravity

- Bottom-up approaches
  \[ \rightarrow \text{'pheno' applications to QCD-like and condensed matter systems} \]

valuable tool for probing
thermal and hydrodynamic properties
of strongly coupled field theories

few theoretical tools available
Microscopic description of such systems is challenging

- often we want their macroscopic behavior, at large distances and long time scales

System typically exhibits universal features

(independent of details of underlying micro description)

Given ‘exotic’ nature of AdS/CFT constructions, crucial to find universal properties:

- gain intuition about real systems in same universality class
- input into realistic simulations
Focus on a particular universal quantity, \( \eta/s \)

**Relativistic Hydrodynamics:**

\[
T_{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg_{\mu\nu} - \sigma_{\mu\nu}
\]

\[
\sigma_{ij} = \eta \left( \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u^k \right) + \zeta \delta_{ij} \partial_k u^k
\]

\[
G_{xy,xy}^R(\omega, 0) = \int dt \ d\mathbf{x} \ e^{i\omega t} \theta(t) \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle = -i\eta \omega + O(\omega^2)
\]

\[
\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \text{Im} \ G_{xy,xy}^R(\omega, 0)
\]

**Effective description of dynamics of system at large wavelengths and long time scales**
Many lessons and still some challenges...

Why interest in $\eta/s$?

Lessons we learned from:

- String theory/sugra constraints
- Consistency of the theory as a relativistic QFT
- ‘toy models’ not realized in string theory

Interesting behavior in systems with very different physics in IR vs UV $\rightarrow$ open issues

Work with:
- J. Liu, K. Hanaki, P. Szepietowski (Michigan)
- A. Buchel (PI)

arXiv:0812.3572, 0903.3244, 0910.5159
arXiv:1007.2963
arXiv:1109.xxxx
$\rightarrow$ arXiv:1108.0677 (Review)
Why interest in $\eta/s$?
I) universality of $\eta/s$

- For $\mathcal{N}=4$ SU(N) SYM plasma [PSS hep-th/0104066]
  (planar limit, infinite 't Hooft coupling)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- universal in all gauge theories with
  Einstein gravity duals [Buchel, Liu th/0311175]
  regardless of
  - matter content
  - amount of SUSY
  - conformality
  - with or without chemical potential
II) elliptic flow measurements at RHIC

- RHIC DATA $\rightarrow$ very small $\eta/s$ for QGP comparable to $1/4\pi$

  well described by hydrodynamics with a very small shear viscosity/entropy density ratio -- “perfect fluid”

DATA FAVORS $4\pi \eta/s \leq 2.5$ (e.g. Song et al. 1101.2783)

- Contrast to weak coupling calculations in thermal gauge theories:

  $\frac{\eta}{s} \sim \frac{1}{\lambda^4 \log 1/\lambda^2} \gg 1$
Order of magnitude agreement with data

large effort to use ads/cft to probe transport properties of sQGP
Can we use CFTs to probe QCD?

For $T \sim T_c - 3T_c$

- Both strongly coupled
- QGP is nearly conformal
  (small bulk viscosity away from $T_c$)

Some properties may be universal

Karsch, hep-lat/0106019

RHIC ~ 200 GeV
LHC ~ 2.7 TeV

generic relations provide INPUT into realistic simulations of sQGP
Shear Viscosity Bound

- $\frac{\eta}{s} = \frac{1}{4\pi}$ agrees well with naïve dilute gas approximation suggesting QM bound:

$$\frac{\eta}{s} \sim p l_{mfp} \Rightarrow \frac{\eta}{s} \gtrsim \mathcal{O}(\hbar)$$

Conjecture that any fluid in nature would obey a bound:

[Kovtun, Son, Starinets th-0309213]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$
Natural Next Step: Role of Higher Derivatives?

Motivations?

- Testing validity of bound

More than that:

- role of string constraints on hydrodynamics (if any)

- Dependence of \( \eta/s \) on physical parameters of the theory (various charges, chemical potential...) \( \Rightarrow \) valuable for pheno applications

Important point: leading sugra approximation hides any interesting sub-structure (\( \eta/s \) is universal)
Leading $\alpha'$ correction on $\text{AdS}_5 \times S^5$ ($N = 4$ SYM) increased the ratio [Buchel, Liu, Starinets th/0406264]

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 + 15\zeta(3)\lambda^{-3/2} + \ldots \right]
\]

finite $\lambda$ correction to $N = 4$ SYM

$\alpha'^3 R^4$ in Type IIB
- Type IIB on $\text{AdS}_5 \times S^5/\mathbb{Z}_2$
  (decoupling limit of $N$ D3’s with collection of D7’s and 07)

$$S = \int d^Dx \sqrt{-g} \left( \frac{R}{2\kappa} - \Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right)$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - 4(D - 4)(D - 1) \frac{c_3}{L^2/\kappa} \right]$$

$c_3$ from effective action on world-volume of D7/07 system
  (determined by fundamental matter content of theory)
By now many examples of corrections violating the bound
what have we learned?

Questions you can ask:

- What parametrizes bound violation on gauge theory side?
- Role of SUSY/stringy constraints?
- Any remnant of universality with higher derivatives?
- Dependence on various physical parameters?

will answer some of these questions by looking at a specific example (string theory construction)
Corrections to $\eta/s$ at finite chemical potential

[SC, K. Hanaki, J. Liu, P. Szepietowski, 0812.3572, 0903.3244, 0910.5159]

Here we are interested in:

- **Electrically charged black holes** (chemical potential)

  \[
  \mathcal{L}_0 = -R - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{12\sqrt{3}} \epsilon^{\mu\nu\rho\lambda\sigma} F_{\mu\nu} F_{\rho\lambda} A_{\sigma} + 12 g^2
  \]

  - **R-charge** $Q$

- **Curvature terms constrained by supersymmetry**

  higher derivative corrections start at $R^2$
  $\Rightarrow$ their SUSY completion is known
  
  [off-shell, Hanaki, Ohashi, Tachikawa, th/0611329]
With curvature corrections...

\[ \mathcal{L} = -R - \frac{1}{4} F^2 + \frac{1}{12\sqrt{3}} \left( 1 - \frac{1}{6} c_2 g^2 \right) \epsilon_{\mu \nu \rho \lambda \sigma} A_\mu F_{\nu \rho} F_{\lambda \sigma} + 12 g^2 \]

\[ + \frac{c_2}{24} \left[ \frac{1}{48} R F^2 + \frac{1}{576} (F^2)^2 \right] + \mathcal{L}_1^{\text{unrgauged}}, \]

\[ \mathcal{L}_1^{\text{unrgauged}} = \frac{c_2}{24} \left[ \frac{1}{16\sqrt{3}} \epsilon_{\mu \nu \rho \lambda \sigma} A_\mu R^{\nu \rho \delta \gamma} R^{\lambda \sigma \delta \gamma} + \frac{1}{8} C^2_{\mu \nu \rho \sigma} \right] + \frac{1}{16} C_{\mu \nu \rho \lambda} F^{\mu \nu} F^{\rho \lambda} - \frac{1}{3} F^{\mu \rho} F_{\rho \nu} R_\mu^{\nu} \]

\[ - \frac{1}{24} R F^2 + \frac{1}{2} F_{\mu \nu} \nabla^\nu \nabla^\rho F^{\mu \rho} + \frac{1}{4} \nabla^\mu F^{\nu \rho} \nabla_\mu F_{\nu \rho} + \frac{1}{4} \nabla^\mu F^{\nu \rho} \nabla_\nu F_{\rho \mu} \]

\[ + \frac{1}{32\sqrt{3}} \epsilon_{\mu \nu \rho \lambda \sigma} F_{\mu \nu} (3 F^{\rho \lambda} \nabla_\delta F^{\sigma \delta} + 4 F^{\rho \delta} \nabla_\delta F^{\lambda \sigma} + 6 F^{\rho \delta} \nabla^\lambda F^{\sigma \delta}) \]

\[ + \frac{5}{64} F_{\mu \nu} F^{\nu \rho} F_{\rho \lambda} F^{\lambda \mu} - \frac{5}{256} (F^2)^2 \].

\( c_2 \) can be related to the central charges of dual UV CFT via:

- Holographic trace anomaly
- \( R \)-current anomaly

Interpretation on dual gauge theory side?
The Link to the Central Charges

Dual theory: 4D CFT with N=1 SUSY

- 4D CFT central charges \( a, c \) defined in terms of trace anomaly:
  (CFT coupled to external metric)

\[
\langle T_{\mu \nu} \rangle_{CFT} = \frac{c}{16\pi^2} (\text{Weyl})^2 - \frac{a}{16\pi^2} (\text{Euler})
\]

Prescription for extracting trace anomaly for higher derivative GR:

\[
\langle T_{\mu} \rangle = \frac{1}{16\pi^2} \left[ \left( \frac{c}{3} - a \right) R^2 + (4a - 2c) R_{\mu \nu}^2 + (c - a) R_{\mu \nu \rho \sigma}^2 \right]
\]

\[
\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu \nu}^2 + \alpha_3 R_{\mu \nu \rho \sigma}^2 + \ldots
\]

For us: \( c_2 = \frac{24}{g^2} \frac{c - a}{a} \)
**Finite N effect**

To recap: \( \mathcal{L} = R + \alpha_3 R_{\mu\nu\rho\sigma}^2 + \ldots \)

\[ \alpha_3 \sim \frac{c - a}{a} \]

For \( \mathcal{N} = 4 \) SYM \( a = c \) (no \( R^2 \) corrections)

In general \( a = c = \mathcal{O}(N^2) \) only, and \( \frac{c - a}{a} \sim \frac{1}{N} \)

- \( R^2 \) terms will correspond to a 1/N correction
  \( \rightarrow \) non-trivial physics parametrized by \((c-a)\)

- Contrast to \( \alpha' \ 3 \ R^4 \rightarrow \text{finite} \ \lambda \) (IIB on AdS\(_5 \times S^5\))
Features

- **Bound violated** for $c-a > 0$ (finite N effect)

- R-charge (chemical potential) makes violation worse (surprisingly simple dependence on $Q$: universality?)

- Eta/s affected only by terms with explicit Riemann tensor:
  \[
  R_{\mu\nu\rho\sigma}^2, \ R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}
  \]
  reminiscent of Wald’s entropy formula

\[\Rightarrow\] **SUSY completion did not play any role** (except for $a,c$)

Partially justifies looking at ‘effective models’ and scanning through CFTs
Features...

- Correlation between sign of higher derivative terms required by weak gravity conjecture and bound violation? [see arXiv:0910.5159]

- What happens to universality (appeal behind eta/s)? Seemingly lost with higher derivatives...

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 + \frac{(\ldots)}{\lambda^{3/2}} + \frac{(\ldots)}{N} + \ldots \right]
\]

But we see `sub-structure’ which was masked by universality $\Rightarrow$ useful for pheno applications
Violation of the bound can be traced to inequality of central charges of dual CFT:

\[ c - a > 0 \]

generic in superconformal gauge theories with unequal central charges

[Buchel et al. 0812.2521]
In holographic models realized in string theory, violation of the KSS bound is necessarily perturbative → always small (curvature corrections small)
Original KSS bound is clearly violated. Is there a bound at all? How low can eta/s get?

Any finite (large) violation of the bound will entail working in a model of gauge/gravity duality (not a ST realization)
Gauss-Bonnet as a toy model [Brigante et al, 0712.0805, 0802.3318]

Black brane solutions known for finite GB coupling

\[ I = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g} \left[ R - 2\Lambda + \frac{\lambda_{GB}}{2} L^2 (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) \right] \]

Finite \( \lambda_{GB} \) leads to natural question: arbitrary violation of the bound?

No! Must look at the consistency of the dual QFT:

- once the coupling becomes too large, one finds modes that propagate faster than light

**microcausality violation** \( \lambda_{GB} > \frac{9}{100} \)

\[ \frac{\eta}{s} > \frac{1}{4\pi} \frac{16}{25} \]

same bound by requiring positivity of energy measured by a detector in the plasma (Hofman 0907.1625)
Causality Violation and the Link to $\eta/s$

- In this model (and many generalizations), consistency of the GB plasma as a relativistic QFT ensures small violation of the bound (and gives new lower bound)

We considered a slight modification of the GB model, realized in a theory with a superfluid phase transition.
Idea is generic:

While *transport* properties are determined by the IR features of the theory, *causality* is determined by the propagation of UV modes (whose dynamics is not that of hydro)
IR vs. UV Physics

- **shear viscosity**: coupling of effective hydro description at low momentum and frequency

- **microcausality**: determined by propagation of modes in UV

  \[
  \omega \ll \min(T, \mu, \cdots), \quad |\vec{k}| \ll \min(T, \mu, \cdots) \quad \text{UV}
  \]

  \[
  \omega \gg \max(T, \mu, \cdots), \quad |\vec{k}| \gg \max(T, \mu, \cdots) \quad \text{IR}
  \]

Link only if same phase of the theory extends over all energy scales (e.g. no phase transitions decoupling UV from IR)
Features of our Toy Model [S.C., A. Buchel arXiv:1007.2963]

Based on: holographic model of superfluidity proposed by GHPT 0907.3510 (consistent truncation of Type IIB)

\[ \mathcal{L} = R - \frac{L^2}{3} F_{\mu \nu} F^{\mu \nu} + \left( \frac{2L}{3} \right)^3 \frac{1}{4} \varepsilon^{\lambda \mu \nu \sigma \rho} F_{\lambda \mu} F_{\nu \sigma} A_\rho + \mathcal{L}_{\text{scalar}} \]

\[ \mathcal{L}_{\text{scalar}} = -\frac{1}{2} \left( (\partial_\mu \phi)^2 + 4\phi^2 A_\mu A^\mu \right) + \frac{12}{L^2} \phi^2 + \frac{3}{2L^2} \phi^2 \]

\[ \mathcal{L}_{\text{GB}} = \beta \phi^4 L^2 \left( R^2 - 4R_{\mu \nu} R^{\mu \nu} + R_{\mu \nu \rho \lambda} R^{\mu \nu \rho \lambda} \right) \]

dual operator develops a VEV below \( T_c \)

\[ \langle \mathcal{O}_c \rangle \begin{cases} = 0, & T > T_c \\ \neq 0, & T < T_c \end{cases} \]

\[ \lambda_{\text{GB}} \bigg|_{\text{effective}} \begin{cases} = 0, & \text{UV} \\ \neq 0, & \text{IR.} \end{cases} \]
unbroken phase
- electrically charged bh
- Einstein GR

broken symmetry phase
- bh develops scalar hair
- GB higher derivative corrections
The shear viscosity bound \([\text{arXiv:1007.2963}]\)

\[
\frac{\eta}{s} = \frac{1}{4\pi}
\]

expected from universality

when \(\lambda_{\text{GB}}\) is non-zero \(\eta/s\) gets corrected:
- \(\eta/s\) goes well below pure GB bound (finite \(\lambda_{\text{GB}}\))
- no causality violation (down to low \(T\), scalar channel)

Will not set lower bound on \(\eta/s\)
Phase transition has decoupled UV from IR physics

→ example suggests that link between a lower bound on \( \eta/s \) and causality violation is not fundamental
Radial vs. Temperature Flow

Well known that eta/s doesn’t run in any Wilsonian sense (membrane paradigm):

\[ \partial_r \Pi = 0 + \mathcal{O}(\omega^2) \]

trivial radial flow between horizon and boundary even with higher derivatives

\[
\frac{\eta}{s} = \frac{1}{4\pi}
\]

In our superfluid GB model, eta/s has a function of temperature jump in Eta/s and temperature flow
Temperature dependence relevant for QGP

Any other ways to get interesting behavior/flow (or “UV/IR decoupling”) for $\eta/s$ ?
Non-Trivial Scalar Profile?

Charged dilatonic branes with Lifshitz solutions:

\[
S = \frac{1}{16\pi G_{d+2}} \int d^{d+2}x \sqrt{-g} \left( R - 2\Lambda - 2(\nabla \phi)^2 - e^{2\alpha \phi} g^2 \right)
\]

\[
ds^2 = L^2 \left( r^{2z} dt^2 + r^2 dx^i dx^j \delta_{ij} + \frac{dr^2}{r^2} \right)
\]

Finite T solutions interpolate smoothly between the two (no phase transition)
IR behavior (Lifshitz) is different enough from UV behavior (AdS) that we expect interesting \( \eta/s \) behavior without need for phase transition.
In Conclusion...

- We are learning to use gravity to model interesting field theory systems – how much more mileage can we get?

- Important to identify universal relations – but also understand more systematically how they are modified

- Role of string constraints on hydrodynamics, if any?

- KSS bound is violated. But is there a new lower bound on $\eta/s$? What is the physics underlying it?

Transport properties: IR feature of theory

$\Rightarrow$ micro constraints (although important for consistency of theory) should not set lower bound on $\eta/s$
Although $\eta/s$ does not flow in any Wilsonian sense, it still has a different behavior in the UV than in the IR. Can we understand non-trivial temperature flow of $\eta/s$ in a more systematic way? (valuable for QGP applications)

More broadly, generate interesting IR physics by adding relevant deformations to CFT.

Recent attempts to refine the Wilsonian approach to gauge gravity duality + apply to fully fledged viscous hydro:

1006.1902 (Bredberg, Keeler, Lysov, Strominger)
1009.3094 (Nickel and Son)
1010.1264 (Heemskerk and Polchinski)
1010.4036 (Faulkner, Liu, Rangamani)

...
Thank you