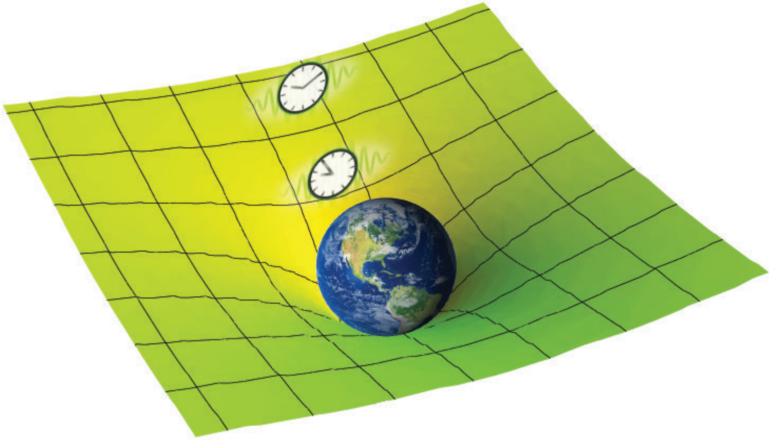
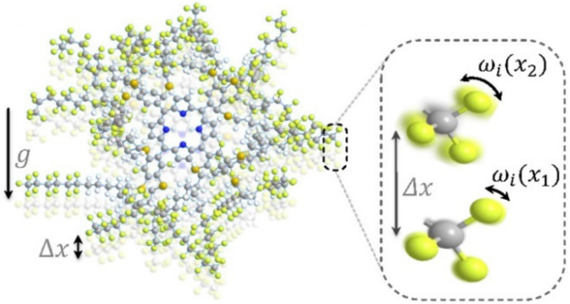
Macroscopic quantum systems and gravitational phenomena

Igor Pikovski





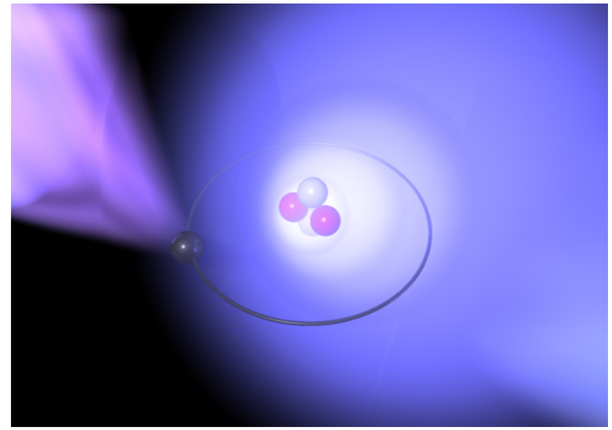
Experimental Search for Quantum Gravity

SISSA/ISAS, Trieste, Italy

01.09.2014

Testing the interplay between quantum theory and gravity

High-energy scattering experiments



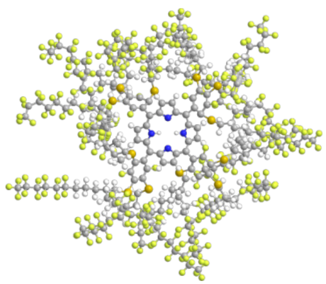
High-precision quantum metrology

Astrophysics and cosmology

Novel systems, that allow for precision measurement in a „quantum gravtitational“ paramter regime?

Outline

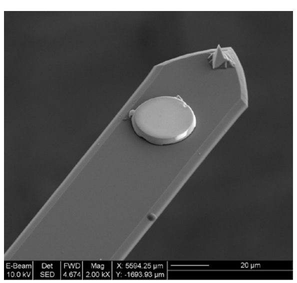
Quantum mechanics on fixed background space-time



- Measurable effects of *classical* gravity in quantum mechanics
- Time dilation in quantum mechanics
- Universal decoherence due to gravitational time dilation

  I. Pikovski, M. Zych, F. Costa, Č. Brukner.
  Universal Decpherence due to Gravitational Time Dilation.
  arXiv:1311.1095 (2013).

Quantum gravity phenomenology



- Pulsed quantum opto-mechanics
- Opto-mechanical scheme to experimentally test possible quantum gravitational deformations of the center-of-mass canonical commutator

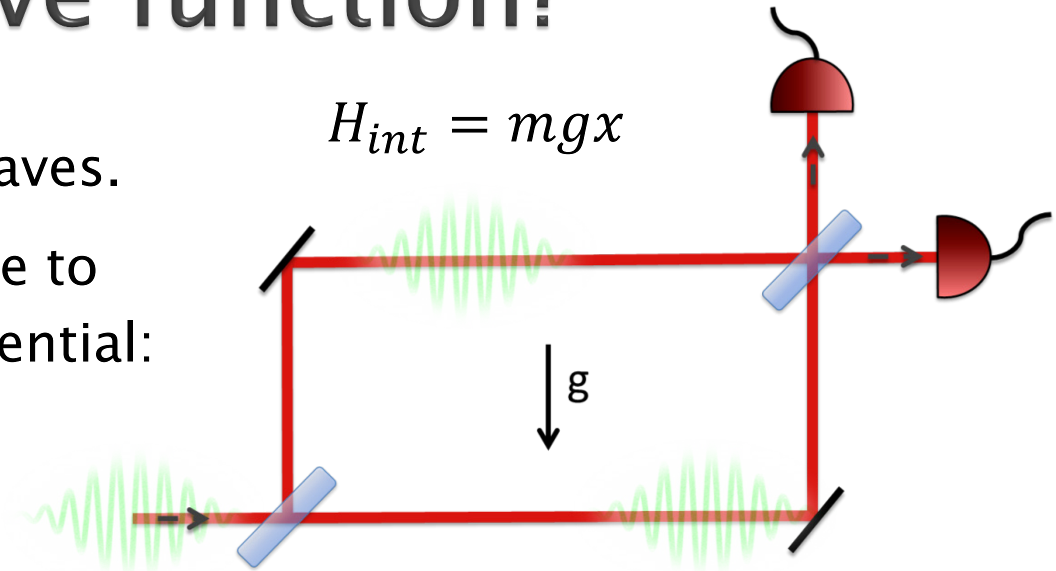
  I. Pikovski, M. Vanner, M. Aspelmeyer, M. S. Kim, Č. Brukner.
  Probing Planck-Scale Physics with Quantum Optics.
  Nature Physics 8, 393 (2012); arxiv:1111.1979.

Can one see the influence of gravity on a quantum wave function?

Yes!
Earth's gravity affects matter waves.

Aharonov-Bohm-type phase due to the Newtonian gravitational potential:

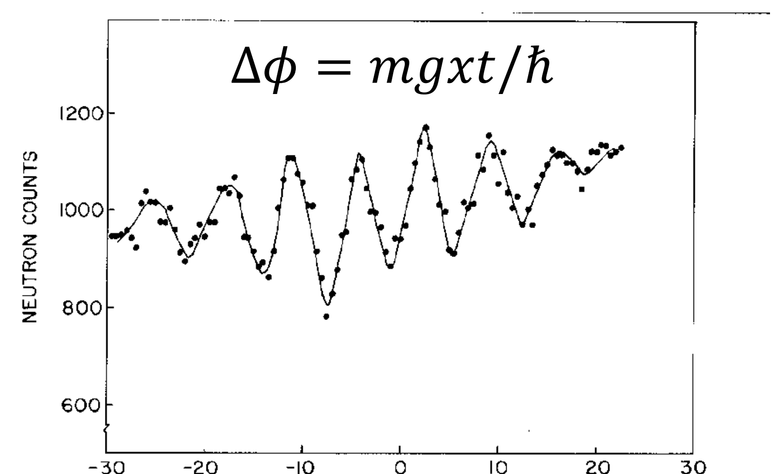
$$\Delta\phi = mgxt/\hbar$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_{down}\rangle + e^{-i\Delta\phi}\,|\psi_{up}\rangle\right)$$

Tested with:

- Neutron interferometry
  e.g. *R. Colella, A. W. Overhauser, S. A. Werner, PRL 34, 1472-1474 (1975)*
- Atomic fountains
  e.g. H. Müller, A. Peters, S. A. Chu, Nature 463, 926-929 (2010)



Our work: Incorporate time dilation into description of QM systems

Beyond the Newtonian limit: Dynamics of particles on background space–time

$$p_\mu p^\mu = -\frac{H_{rest}^2}{c^2}, \qquad cp_0 = H = H(p^i, g_{\mu\nu}, H_{rest}) = \sqrt{-g_{00}}\sqrt{H_{rest}^2 + g_{ij}c^2p^ip^j}$$

- low–energy limit:

$$i\hbar\frac{\partial}{\partial\tau}|\psi\rangle = H_{rest}|\psi\rangle$$

- composite systems: $H_{rest} = H_0 + mc^2$

internal dynamics *remaining static part*

Hamiltonian in the weak–field limit $O(c^{-2})$:

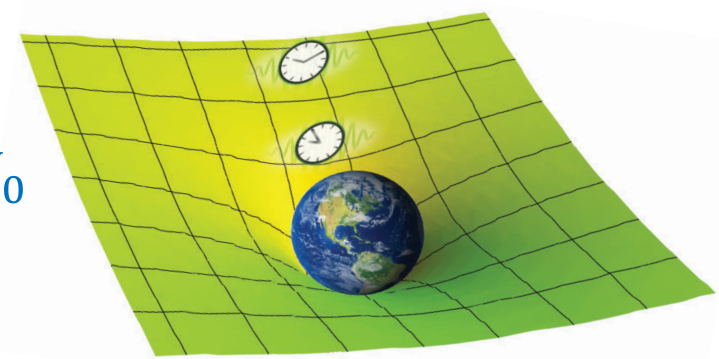
$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \left(H_0 + mc^2 + \frac{p^2}{2m} + m\Phi(x) + \frac{m\Phi^2(x)}{2c^2} - \frac{p^4}{8m^3c^2} + \left[\frac{\Phi(x)}{c^2} - \frac{p^2}{2m^2c^2}\right]H_0\right)|\psi\rangle$$

Coupling between *internal* and *external* d.o.f.

$\boxed{\text{QM on fixed (classical) background space–time with time dilation}}$

Gravitational part of interaction with $\Phi(x) = gx$:

$$H_{int} = \frac{gx}{c^2}H_0$$

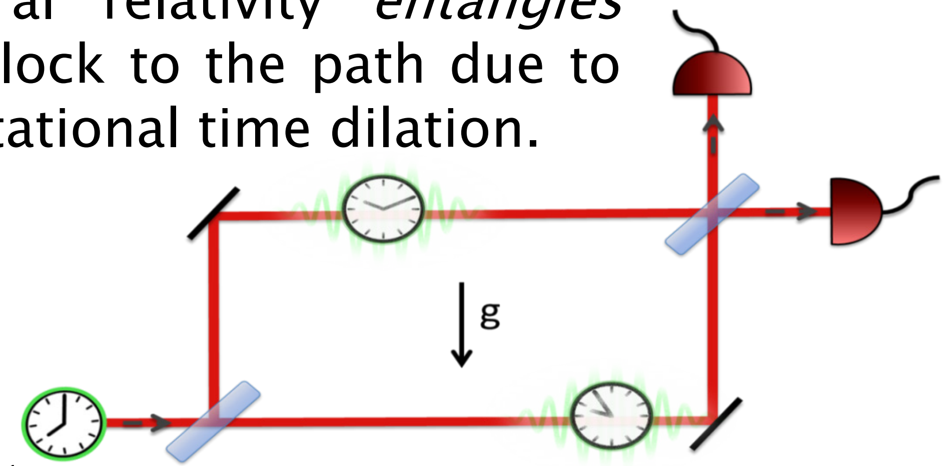


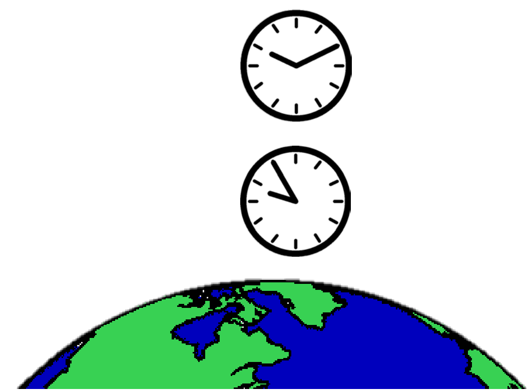
Time dilation in QM

Consequences of time dilation in quantum mechanics

Quantum mechanically: $$H \approx mgx + H_0 + \frac{gx}{c^2} H_0$$ *Classically:*

General relativity *entangles* any clock to the path due to gravitational time dilation.





$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_{down}\rangle|C_{down}\rangle + e^{-i\Delta\phi}\,|\psi_{up}\rangle|C_{up}\rangle\right)$$

Time dilation: $|\langle C_{down}|C_{up}\rangle| < 1$

Drop in interference visibility!

Experimental implications:

- Matter wave interferometry with additional internal clock-states $|C\rangle$ (e.g. $|C\rangle = |g\rangle + |e\rangle$) *(M. Zych, F. Costa, I. Pikovski, Č. Brukner. Nature comm. 2, 505 (2011))*
- Shapiro delay: Photons slowed down by gravity *(M. Zych, F. Costa, I. Pikovski, T.C. Ralph, Č. Brukner. Class. Quant. Grav. 29, 224010 (2012))*

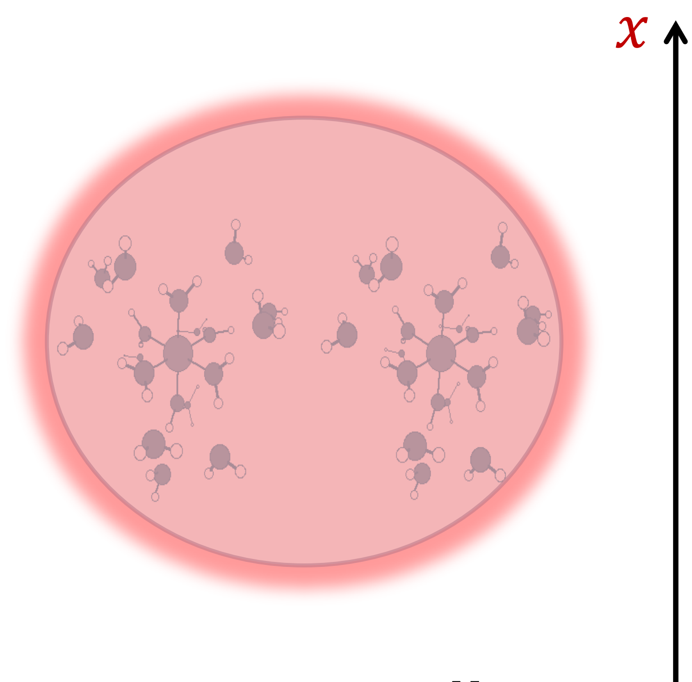
Time dilation decoherence

Composite quantum particle under general relativistic time dilation

Arbitrary composite system in Earth's gravitational field.

Simple model: Particle has N internal harmonic oscillators:

$$H_0 = \sum_{i=1}^{N} n_i \hbar\omega_i$$

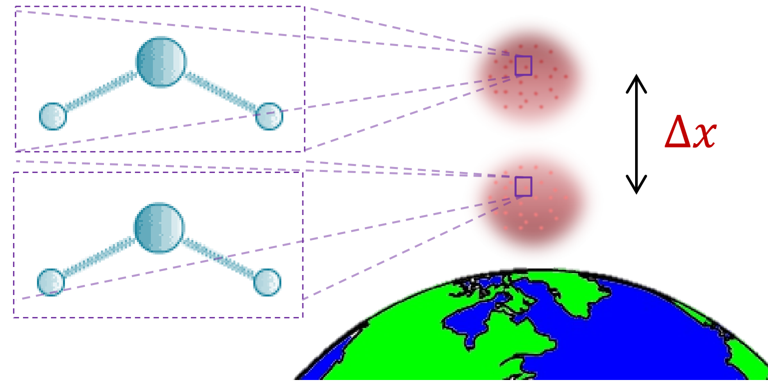


Each constituent in equilibrium at temperature T: $\bar{n}_i = (e^{\hbar\omega_i/k_B T} - 1)^{-1}$

$$\rho_i = \frac{1}{\pi\bar{n}_i}\int d^2\alpha_i \, e^{-|\alpha_i|^2/\bar{n}_i} \, |\alpha_i\rangle\langle\alpha_i|$$

GR time dilation induces interaction with center-of-mass position x : $H_{int} = mgx + \frac{\hbar g x}{c^2}\sum_{i=1}^{N} n_i\omega_i$

Time-dilation decoherence



Spatial superposition, internal temperature T : $|\psi_{cm}\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$,

$$\rho(0) = |\psi_{cm}\rangle\langle\psi_{cm}| \otimes \prod_{i=1}^{N} \rho_i$$

Evovles under $H = mgx + (1 + \frac{gx}{c^2}) \sum_{i=1}^{N} \hbar n_i \omega_i$

Quantum coherence of center-of-mass reduces due to time-dilation:

$$V(t) \approx \left(1 + \left(\frac{k_B T g \Delta x\, t}{\hbar c^2}\right)\right)^{-N/2} \approx e^{-\left(\frac{t}{\tau_{dec}}\right)^2}$$

$$\boxed{\tau_{dec} = \sqrt{2/N}\,\frac{\hbar c^2}{k_B T g \Delta x}}$$

- Universal for all composite systems
- Gaussian decay of quantum coherence (for t $\ll \sqrt{N}\tau_{dec}$)
- Decoherence mediated by time dilation, depends on internal composition
- Relativistic, thermodynamic and quantum mechanical effect
- Regular quantum theory and general relativity

Master-equation

Include full dynamics of center-of mass in Born approximation:

$$\dot{\rho}_{cm}(t) = -\frac{i}{\hbar}\left[H_{cm} + \frac{\langle H_0 \rangle}{c^2}\Gamma(x,p), \rho_{cm}(t)\right] - \left(\frac{\Delta H_0}{\hbar c^2}\right)^2 \int_0^t ds \left[\Gamma(x,p), [\Gamma(x,p), \rho_{cm}(t-s)]\Big|_s\right]$$

where: $\Gamma(x,p) = gx - \frac{p^2}{2m^2}$ and $[\Gamma,\rho]\Big|_s = e^{-isH_{cm}/\hbar}[\Gamma,\rho]e^{isH_{cm}/\hbar}$

Master equation with only gravitational interaction after build-up of superosition:

$$\dot{\rho}_{cm}(t) = -\frac{i}{\hbar}\Big[H_{cm} + \underbrace{\left(m + \frac{Nk_BT}{c^2}\right)gx}_{\text{Unitary part. „A piece of iron weighs more when red-hot than when cool“}}, \rho_{cm}(t)\Big] \underbrace{- Nt\left(\frac{k_BTg}{\hbar c^2}\right)^2 [x,[x,\rho_{cm}(t)]]}_{\text{Decoherence into position basis}}$$

Off-diagonal elements supressed: $\langle x_1|\rho_{cm}(t)|x_2\rangle \sim \rho_{cm}(0)e^{-\left(\frac{t}{\tau_{dec}}\right)^2}$

- No dissipation
- Gaussian decay

- No „external“ environment
- Position pointer-basis
  - No hidden assumptions, relies only on time dilation

$$\boxed{\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_BTg\Delta x}}$$

Time dilation decoherence

Strength of decoherence

$$\boxed{\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_B T g \Delta x}}$$

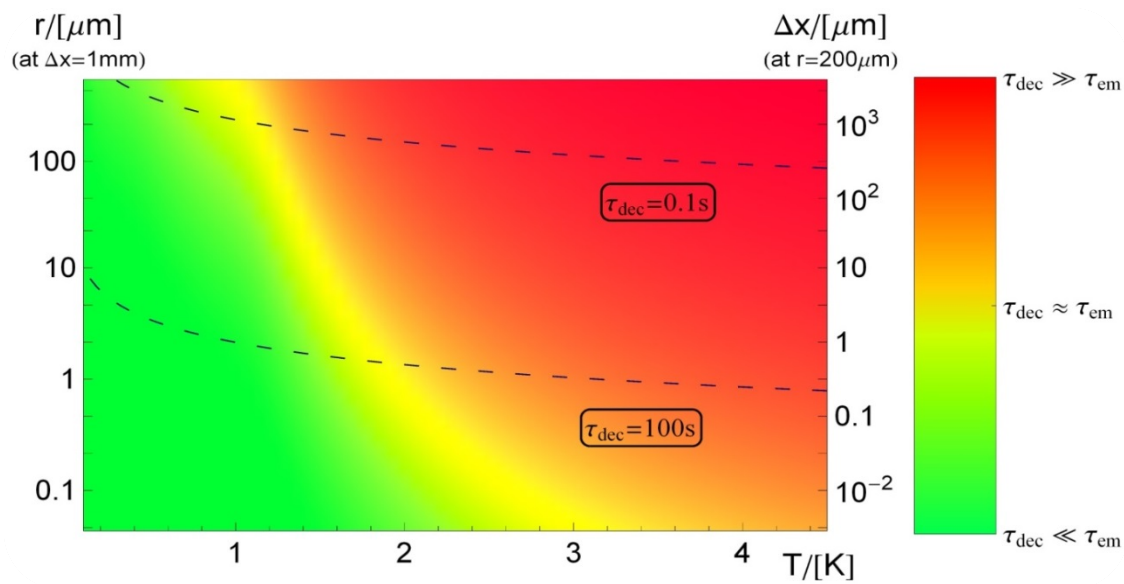
μm-scale object on Earth at room temperature, $\Delta x = 10^{-6}$m: $\boldsymbol{\tau_{dec} = 10^{-3}s}$

- Despite small redshift, decoherence is substantial.
- Fundamental limit for spatial superpositions on Earth.
- Decoherence universally present on curved space-time.
- For strong gravitational fields / high accelerations: Very strong decoherence

Other decoherence sources also present.

Main competing mechanism black-body radiation:

$$\tau_{em} \sim Im\left[\frac{\varepsilon+2}{\varepsilon-1}\right]\left(\frac{\hbar c}{k_B T}\right)^6 \frac{10^{-3}}{c r^3 \Delta x^2}$$



Green region: decoherence due to time dilation dominates over BB-emission.

Time dilation in QM Summary

- Gravitational effect in quantum theory
- General relativistic time dilation leads to entanglement between position and internal degrees-of-freedom
- Quantum Hamiltonian can be probed with matter waves with clock-states or with photons via Shapiro delay
- Time dilation leads to decoherence of all composite particles, timescale: $\tau_{dec} = \sqrt{\frac{2}{N}} \frac{\hbar c^2}{k_B T g \Delta x}$
- No breakdown of quantum mechanics, as opposed to collapse-theories
- Time dilation on Earth decoheres mesoscopic systems
- Could be verified in future experiments with molecules or trapped nanospheres

I. Pikovski, M. Zych, F. Costa, Č. Brukner. **Universal Decpherence due to Gravitational Time Dilation.** *ArXiv:1311.1095 (2013).*

Quantum gravity phenomenology: Modified uncertainty relations

Can quantum gravity have signatures in low-energy quantum mechanics?

Novel effects seem inevitable at some scale.

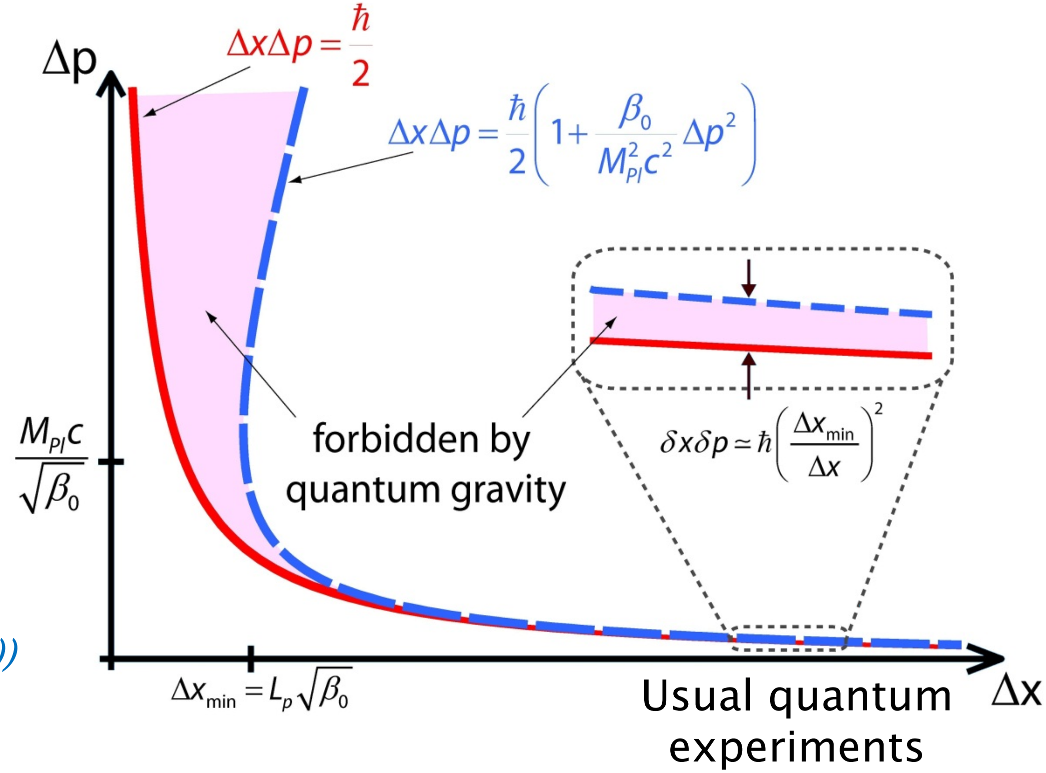
Modification of Heisenberg uncertainty relation, common to many approaches to QGR:

(L. Garay, Int. J. Mod. Phys. A10, 145 (1995))

$$\Delta x\, \Delta p \;\geq \frac{\hbar}{2}(1+\beta_0 \frac{\Delta p^2}{M_{Pl}^2 c^2})$$

standard QM

response of the space-time,
$\mathrm{M_{Pl}} \approx 22\mu g$ Planck-mass,
β_0 dimensionless parameter



Current experimental bound from quantum systems : $\beta_0 < 10^{33}$

(S. Das & E. C. Vagenas, PRL 101, 221301 (2008))

Probing Planck-scale physics

Possible commutator modifications

A. Kempf, M. Maggiore: $\Delta x\, \Delta p \geq \frac{\hbar}{2}(1+\beta\Delta p^2+\cdots)$ implies a modified commutator. E.g.:

- $\left[\hat{X},\hat{P}\right]_\beta = i\left(1+\beta_0\frac{p_0^2\hat{P}^2}{M_{Pl}^2c^2}+\cdots\right)$ *(A. Kempf, G. Mangano and R. Mann, PRD, 52, 2 (1995))*
- $\left[\hat{X},\hat{P}\right]_\mu = i\sqrt{1+2\mu_0\frac{(p_0\hat{P}/c)^2+m^2}{M_{Pl}^2}+\cdots}$ *(M. Maggiore, Phys. Lett. B, 319 (1993))*
- $\left[\hat{X},\hat{P}\right]_\gamma = i\left(1-\gamma_0\frac{p_0\hat{P}}{M_{Pl}c}+\gamma_0{}^2\frac{p_0^2\hat{P}^2}{M_{Pl}^2c^2}+\cdots\right)$ *(A. F. Ali, S. Das and E. C. Vagenas, Phys. Lett. B, 678 (2009))*

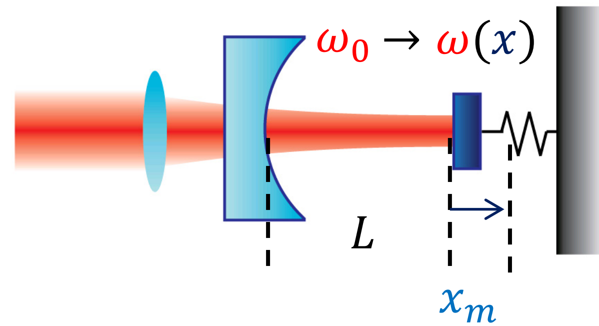
Examples:

Ions in harmonic trap: $\begin{matrix}\omega/2\pi = 10\ \mathrm{kHz},\\ m = 10^{-27}\mathrm{kg}\end{matrix} \rightarrow \frac{p_0^2}{M_{Pl}^2c^2} \sim 10^{-60}$ very small

Optomechanics: $\begin{matrix}\omega/2\pi = 100\ \mathrm{MHz},\\ m = 10^{-12}\mathrm{kg}\end{matrix} \rightarrow \frac{p_0^2}{M_{Pl}^2c^2} \sim 10^{-40}$

Optomechanics

Light in a cavity displaces a small mirror by radiation pressure: Massive mechanical oscillator interacts with light.



Opto-mechanical interaction:

i. Free cavity: $\hat{H} = \hbar\omega_0 \hat{n}_L$, $\qquad \omega_0 = \frac{4\pi c}{n\,L}$

ii. Radiation pressure changes $L \rightarrow L + x$: $\qquad \omega_0 \rightarrow \omega_0\,(1 - \frac{x}{L})$

iii. Quantize $x \rightarrow \hat{x}$

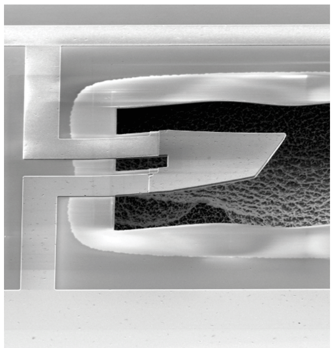
$$\hat{H} = \hbar\omega_m \hat{n}_m + \hbar\omega_L \hat{n}_L - \hbar g_0 \hat{n}_L \hat{X}_m, \quad g_0 = \frac{\omega_L}{L}\sqrt{\frac{\hbar}{m\omega_m}}$$ coupling rate

Can add laser drive with detuning $\Delta = \omega_{in} - \omega_L$: Photons will scatter depending on the detuning:

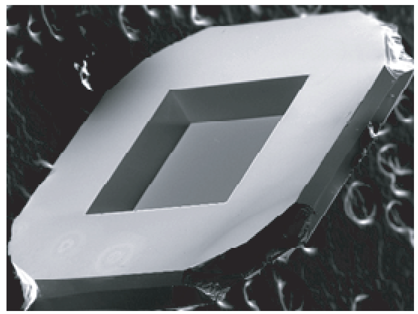
anti-Stokes (red sideband, $\Delta = -\omega_m$) → cooling, state swap

Stokes (blue sideband, $\Delta = \omega_m$) → heating, squeezing

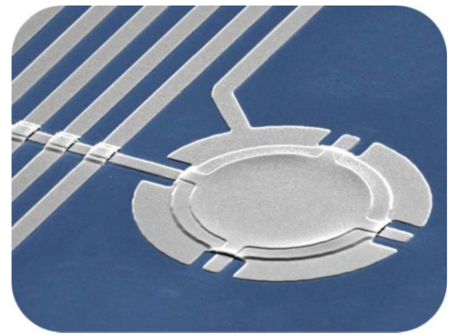
Examples of electro- and opto-mechanical systems



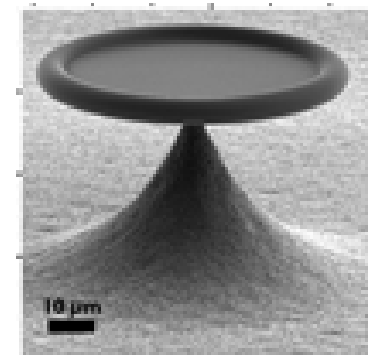
O'Connell et al., (2010) – UCSB



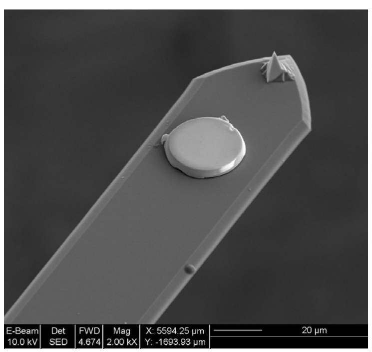
Thompson et al., (2008) – Yale



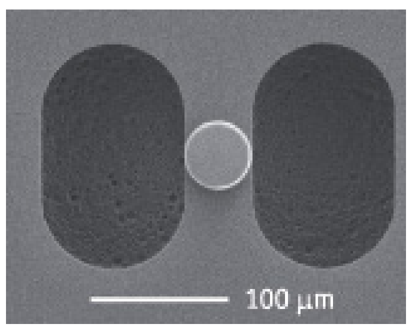
Teufel et al., (2011) – NIST



Schliesser et al., (2009) – EPFL



Kleckner et al, (2006) – UCSB



Gröblacher et al., (2009) – Vienna

- Ability to create and control collective quantum states of massive objects.
  (Vanner, IP et al., PNAS 108, (2011))
- Optical cooling to the ground state.
  (Chan et al., Nature 478 (2011))
- Entanglement between light and matter
- Probing limits of quantum mechanics
  (Marshall et al., PRL 91 (2003))

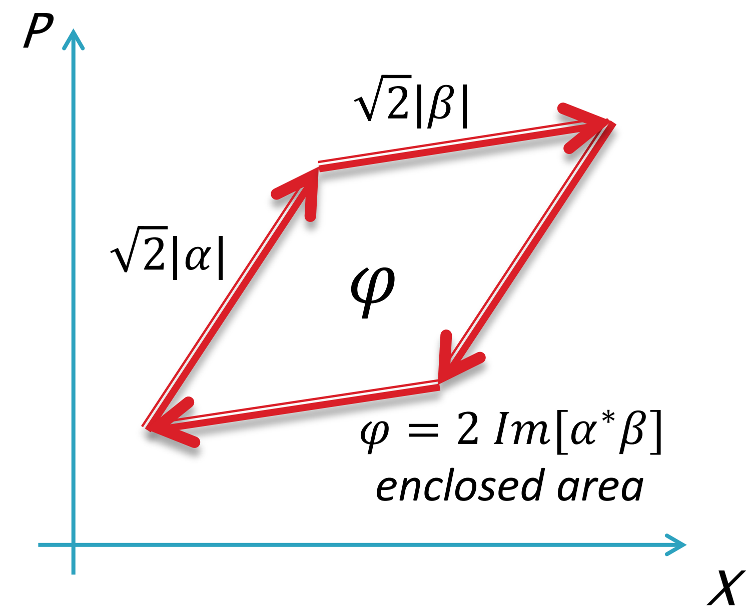
Imprinting a phase Phase Space

Phase space quadratures: $\hat{X} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^{+})$, $\hat{P} = \frac{1}{\sqrt{2}i}(\hat{a} - \hat{a}^{+})$

Displacements in phase space:

$$\hat{D}(\alpha) = e^{\alpha\hat{a}^{+} - \alpha^{*}\hat{a}}$$
$$= e^{i\sqrt{2}\mathrm{Re}[\alpha]\,\hat{X} - i\sqrt{2}\mathrm{Im}[\alpha]\,\hat{P}}$$

$$\hat{D}(\beta)\hat{D}(\alpha) = \hat{D}(\alpha+\beta)\, e^{i\, Im[\alpha^{*}\beta]}$$



$$\hat{D}(-\beta)\hat{D}(-\alpha)\hat{D}(\beta)\hat{D}(\alpha) = e^{2\, i\, Im[\alpha^{*}\beta]}$$

- Results in an overall phase
- State independent
- No classical analogue
- Arises due to $[x, p] \neq 0$

Can be used for quantum computing
Milburn, Schneider, James, Fortschr. Phys. 48, 801(2000),
Sørensen, Mølmer, Phys. Rev. A 62, 022311 (2000)

Implemented to create a phase gate for ions
Leibfried et al., Nature 422, 27 (2003)

Displacements via an ancilla

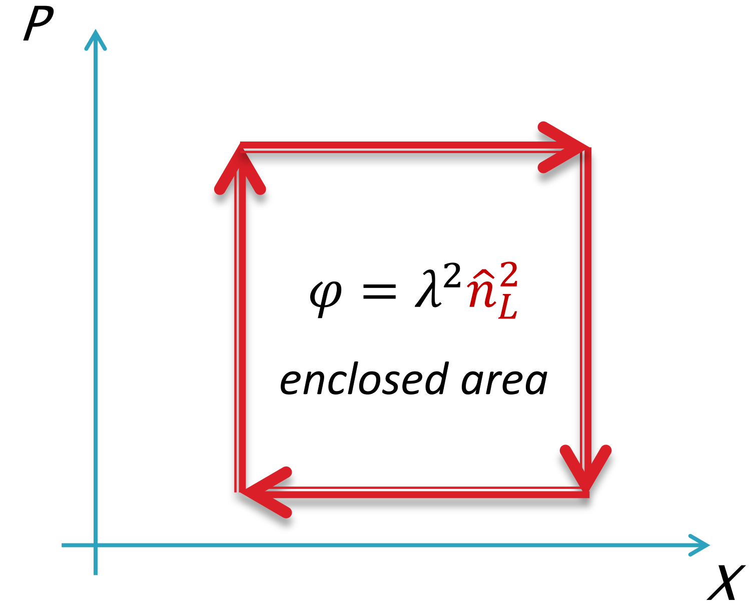
Displacements of a quantum system around a loop in phase space via an ancillary system (photon):

$$\hat{\xi} = e^{i\lambda \hat{n}_L \hat{P}}\, e^{-i\lambda \hat{n}_L \hat{X}}\, e^{-i\lambda \hat{n}_L \hat{P}}\, e^{i\lambda \hat{n}_L \hat{X}}$$

$$= e^{-i\,\lambda^2 \hat{n}_L^2}$$

- Resulting phase changes the ancilla, but is state-independent
- System remains unaffected, and is fully disentangled from the light

⇒ The system remains quantum system and does not decohere



Generalization: Measurement of the canonical commutator

$$\hat{\xi} = e^{i\lambda\hat{n}_L\hat{P}}\, e^{-i\lambda\hat{n}_L\hat{X}}\, e^{-i\lambda\hat{n}_L\hat{P}}\, e^{i\lambda\hat{n}_L\hat{X}} = e^{\sum_{k=1}^{\infty}\frac{(-i\lambda\hat{n}_L)^{k+1}}{k!}\underbrace{[\hat{X},\hat{P}]_k}_{[\hat{X},[\hat{X},\dots,\hat{P}]}}$$

- In quantum mechanics: $[\hat{X},\hat{P}]=\mathrm{i} \quad \Rightarrow \quad \hat{\xi}_{QM} = e^{-i\lambda^2\hat{n}_L^2}$
- Alternative theories: $[\hat{X},\hat{P}]=\mathrm{i}\,\mathrm{F}(\hat{X},\hat{P}) \quad \Rightarrow \quad \hat{\xi} = e^{-i\lambda^2\hat{n}_L^2+\epsilon(\hat{n}_L)}$

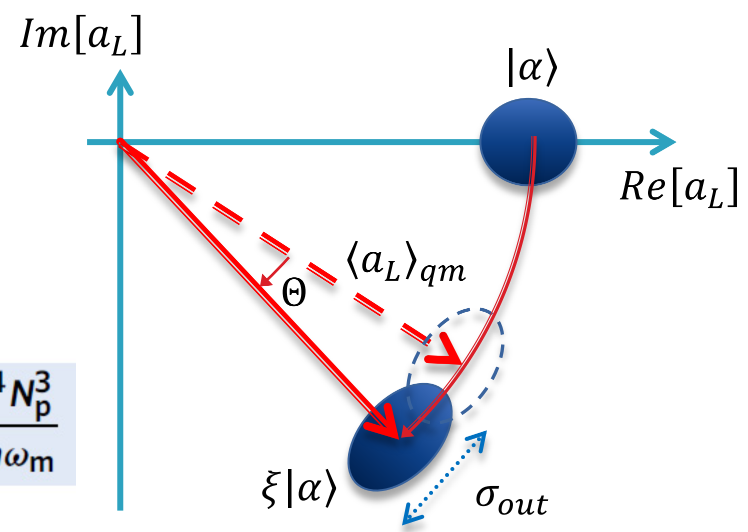
By measuring the ancilla one can obtain a measure of the commutator.

For initial coherent state (Np: # photons):

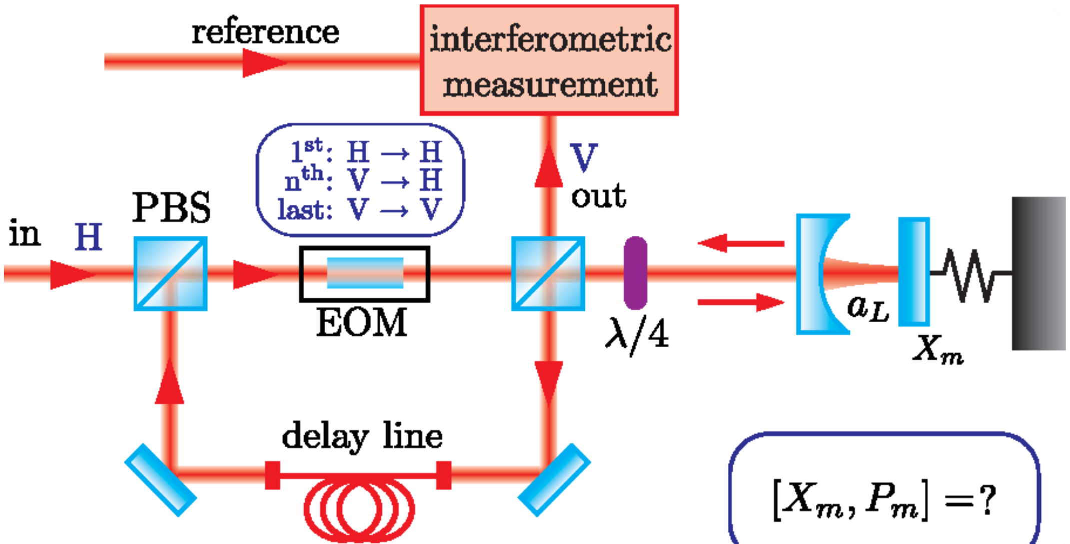
$$\langle\hat{a}_L\rangle = \left\langle\alpha\middle|\hat{\xi}^{+}\hat{a}_L\hat{\xi}\middle|\alpha\right\rangle \cong \langle\hat{a}_L\rangle_{QM}\, e^{-i\,\Theta([\hat{X},\hat{P}]_{mod})}$$

$$\Downarrow$$

$$\mu_0\,\frac{32\hbar\mathcal{F}^2 mN_p}{M_P^2\lambda_L^2\omega_m} \qquad \gamma_0\,\frac{96\hbar^2\mathcal{F}^3N_p^2}{M_P c\lambda_L^3 m\omega_m} \qquad \beta_0\,\frac{1024\hbar^3\mathcal{F}^4N_p^3}{3M_P^2c^2\lambda_L^4 m\omega_m}$$



Possible implementation



- Pulsed interactions (duration $\tau \ll {\omega_m}^{-1}$) *(Vanner, IP et al., PNAS 108, (2011))*: $\hat{H} \approx -\hbar g_0 \hat{n}_L \hat{X}_m$
- Harmonic evolution: $\hat{X}_m(t) = \hat{X}_m \cos(\omega_m t) - \hat{P}_m \sin(\omega_m t)$
- Four roundtrips seperated by $\omega_m t = \pi/2$:

$$\hat{\xi} = e^{i\lambda\hat{n}_L\hat{P}}\, e^{-i\lambda\hat{n}_L\hat{X}}\, e^{-i\lambda\hat{n}_L\hat{P}}\, e^{i\lambda\hat{n}_L\hat{X}}$$

Noise and losses analyzed (theoretically):

- Optical losses
- Cavity dynamics and temporal mode mismatching
- Coupling of the mechanics to external environment
- Finite harmonic evolution during interaction

Testing quantum gravitational corrections

Is it possible to see quantum gravitational modifications of the commutator?

Requires different parameters for different theories:

- $[\hat{X},\hat{P}]_\beta = i(1+\beta_0 \frac{p_0^2 \hat{P}^2}{M_{Pl}^2 c^2})$
- $[\hat{X},\hat{P}]_\mu = i\sqrt{1+\mu_0 \frac{(p_0\hat{P}/c)^2+m^2}{M_{Pl}^2}}$
- $[\hat{X},\hat{P}]_\gamma = i(1-\gamma_0 \frac{p_0 \hat{P}}{M_{Pl} c}+\gamma_0 \frac{p_0^2 \hat{P}^2}{M_{Pl}^2 c^2})$

Finesse	$F \sim 10^5 - 10^6$	Bath temperature	$T < 100mK$
Mass	$\mathrm{m} \sim 0.1 - 1ng$	Qualtiy factor	$\mathrm{Q} > 10^7$
Frequency	$\omega_m \sim 1 - 100kHz$	Opt. cooling	$n_m < 30$
Intentisity	$N_p \sim 10^8 - 10^{12}$	Opt. loss	$\eta > 0.9$

Challenging, but realistic parameters.

Deformations of center-of-mass observable even for $\beta_0, \mu_0, \gamma_0 \lesssim 1$

Improvement by up to 33 orders of magnitude, Possibility to probe possible Planck-scale deformations.

Ambiguity in such deformation models

The analysis relied on modifications of the center-of-mass mode of a quantum mechanical system. Maybe modification apply only to „elementary“ particles? Then (simple argument):

Model: N identical particles, composition rule unmodified:

$$\hat{x}_{cm} = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i, \quad \hat{p}_{cm} = \sum_{i=1}^{N}\hat{p}_i$$

$$\rightarrow \qquad [\hat{x}_{cm}, \hat{p}_{cm}]_\beta = \mathrm{i}\left(1 + \frac{1}{N}\frac{\beta_0}{M_{Pl}^2 c^2}\left(\hat{p}_{cm}^2 - \sum_{i\neq j}\hat{p}_i\hat{p}_j\right)\right)$$

Compare to direct center-of-mass deformation:

$$[\hat{x}_{cm}, \hat{p}_{cm}]_\beta = \mathrm{i}\left(1 + \frac{\beta_0}{M_{Pl}^2 c^2}\hat{p}_{cm}^2\right)$$

If only „elementary“ particles are modified, then composite systems have a modification with an an effective strength $\beta_0 \rightarrow \frac{\beta_0}{N}$ or $\frac{\beta_0}{N^2}$ (depending on momentum correlations of the elemntary particles).

Experiment would still put bounds, but is less sensitive

Where to apply the deformations?

Currently no theory predicts the „correct" approach. Good reasons for either assumption:

Elementary particles:

- "Natural building blocks"
- No extensions into macroscopic and classical domain (effect remains hard to detect)

Wavefunction of composite system:

- "Natural" in quantum domain, independent of composition
- Deformation-induced restriction to localization applies universally

Present in many other proposals for lab tests of quantum gravity phenomenology, e.g.:

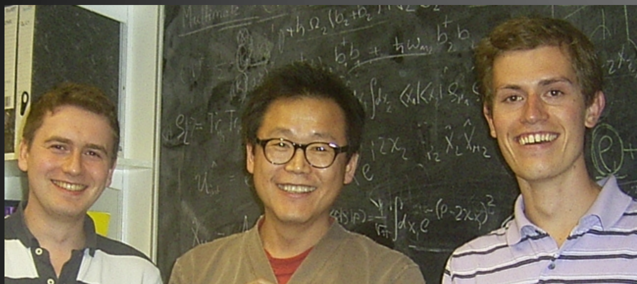
C. Hogan, Phys. Rev. D 77, 104031 (2008);
J. D. Bekenstein, Phys. Rev. D, 86, 124040 (2012);
F. Marin et al., Nature Phys. 9, 71 (2013)

Summary

- Overlap between quantum mechanics and general relativity is accessible in the lab, even at low energies
- Phenomenological predictions of quantum gravity can be tested or constrained using quantum optics
- Assumptions:
  - Quantization of space-time leads to deformations of the canonical commutator
  - Deformations apply to the center-of-mass
- Instead of Planck-scale position measurements, use a pulsed opto-mechanical scheme to amplify the effect and imprint information of commutator deformations onto optical field

I. Pikovski, M. Vanner, M. Aspelmeyer, M. S. Kim, Č. Brukner.
Probing Planck-Scale Physics with Quantum Optics.
Nature Physics 8, 393 (2012); arxiv:1111.1979.

Opto-mechanics and tests of quantum gravity phenomenology



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- **Pulsed opto-mechanics** (with G. Cole, K. Hammerer, G. Milburn): *PNAS 108, 16182-16187 (2011)*
- **Probing Planck-Scale physics with quantum optics**: *Nature Physics 8, 393-397 (2012)*

Time dilation in quantum mechanics and implications for photons, matter waves and decoherence

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- **Quantum interferometric visibility as a witness of general relativistic proper time**: *Nature Communications 2, 505 (2011)*
- **General relativistic effects in quantum interference of photons** (with T. C. Ralph): *Class. Quantum Grav. 29, 224010 (2012)*
- **Universal decoherence due to gravitational time dilation**: *arxiv:1311.1095*

Thank you for your attention!