QUANTUM GRAVITY EFFECT ON NEUTRINO OSCILLATION

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NOTES ON LANGUAGE

- Strong gravity in the paper referred to the gravitational field that a particle would experience due to a classical mass. Examples would be in the ‘vicinity’ of a black hole or star.
- Decoherence is used with respect to the experimental observation of a given neutrino flux. Neutrino fluxes which no longer demonstrate oscillation demonstrate decoherence. Sources of decoherence can then be compared.

OUTLINE

- Introduction
  - Semi-classical gravity
  - Neutrino physics
  - Propagation decoherence
  - Classical gravity
- Quantum decoherence
- Graviton bremsstrahlung
- Detection
- Conclusions

INTRODUCTION

- Neutrinos are ideal probes of distant 'laboratories' as they interact only via the weak and gravitational forces.
- 3 of 4 forces can be described in QFT framework, 1 (Gravity) is missing (and exp. evidence is missing):
  - semi-classical theory is the best understood
  - graviton interactions suppressed by $(M_{Pl}^2) \sim 10^{38}$ GeV^2
- Many sources of astrophysical neutrinos (SNe, GRB, ..)
- Neutrino states during propagation are different from neutrino states during (weak) interactions

SEMI-CLASSICAL QUANTUM GRAVITY

Class. Quantum Grav. 27 (2010) 145012

- Considered in the limit where one mass is much greater than all other scales of the system.
- Considered in the long range limit.
- Up to loop level, semi-classical quantum gravity and effective quantum gravity are equivalent.
- Produced useful results (Hawking/etc).
- Tree level approximation.

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$$

CLASSICAL NEUTRINO OSCILLATION

- Neutrino Oscillation observed due to Interaction (weak) - Propagation (Inertia) - Interaction (weak)
- Neutrino oscillation depends both on production and detection hamiltonians.
- Neutrinos propagates as superposition of mass states.

$$|\nu_f\,(t)>=\sum_a V_{fa}e^{-iE_a t}|\nu_a>$$

$$\phi_{jk}=\frac{m_j^2-m_k^2}{2E_\nu}L=\frac{\Delta m^2 L}{4E_\nu}$$

$$P_{\nu_f\to\nu_{f'}}\,(E,L)=\sum_{j,k}V_{f'j}V_{f'k}e^{-i\frac{m_j^2}{2E_\nu}L}e^{i\frac{m_k^2}{2E_\nu}L}V_{fj}^*V_{fk}^*$$

MATTER EFFECT

- neutrinos interact due to flavor (via W/Z) with particles (leptons, quarks) as flavor eigenstates
- MSW effect: neutrinos passing through matter change oscillation characteristics due to change in electroweak potential
- effects electron neutrino component of mass states only, due to electrons in normal matter
- neutrino may be in mass eigenstate after MSW effect: resonance
- Expectation of asymmetry for earth MSW effect in Solar neutrinos is ~3% for current experiments.

PROPAGATION DECOHERENCE

M. Beuthe, Phys. Rept. 375, 105 (2003). *Y. Farzan and A. Y. Smirnov, Nucl. Phys. B 805, 356 (2008).*

- During propagation neutrinos may experience dispersion or separation of the eigenstates which are the state of propagation (this changes based on the electromagnetic potential).
- As the neutrino propagates, the coherence (in vacuum) depends on wavepacket size (production process, depending on process can be $\sim 10^{-4}$), neutrino energy, length of propagation, and mass difference:

$$\sigma_x \ll d_L = 3 \times 10^{-3}\mathrm{cm} \frac{L}{100\mathrm{Mpc}} \frac{\Delta m^2}{2.5 \times 10^{-3}\mathrm{eV}^2} \left(\frac{10\mathrm{TeV}}{E}\right)^2$$

- After decoherence due to propagation, a single neutrino still exists as a superposition of mass eigenstates (in vacuum) but has a constant 'phase of oscillation' to give a probability for the neutrino to be detected in a flavor state of:

$$P\left(\nu_\alpha \to \nu_\beta\right) = \sum_i |V_{\beta i}|^2 |V_{\alpha i}|^2$$

CLASSICAL NEUTRINO GRAVITY INTERACTION

D.V. Ahluwalia, and C. Burgard, Phys. Rev. D 57, 4724 (1998).

- Different mass states travel in different geodesics, creating a gravitational phase which builds up over distances.
- In the region of a classical mass, using GR, the propagation can be given in terms of the flat and Schwarzschild metric:

$$e^{-\frac{i}{\hbar}\int_{t_c}^{t_d} H dt + \frac{i}{\hbar}\int_{r_c}^{r_d} P\cdot dx}|\nu_i\rangle = e^{-\frac{i}{\hbar}\int_{R_c}^{R_d}\left(\eta_{\mu\nu}+\frac{1}{2}h_{\mu\nu}\right)dx^{\nu}}|\nu_i\rangle$$

- Giving the standard transition probability but with a extra phase due to the gravitational interaction.

$$P_{\nu_f\to\nu_{f'}}(E,L) = \sum_{j,k} V_{f'j}V_{f'k}e^{i\phi_{k,j}+i\phi^G_{k,j}}V^*_{fj}V^*_{fk} \qquad \phi^G_{k,j} = -\langle\Phi\rangle\phi_{k,j}$$

$$\langle\Phi\rangle = -\frac{1}{L}\int_{r_c}^{r_d} dl\frac{GM}{c^2 r} \qquad \langle\Phi\rangle_{neutronstar} = -0.20$$

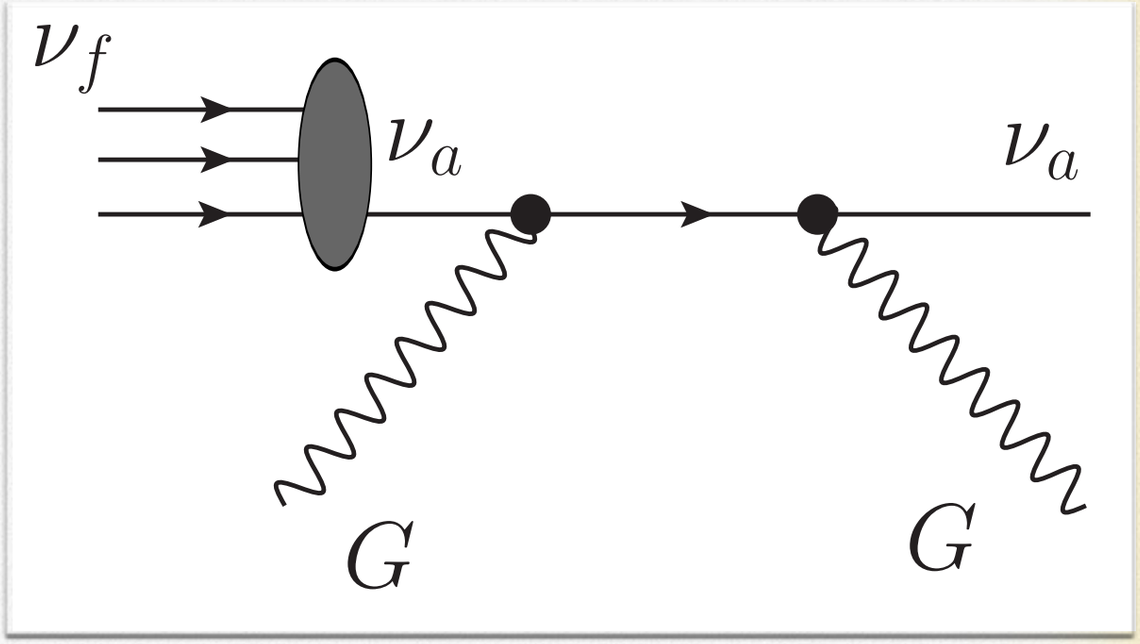
- Penrose decoherence for neutrinos is this combined with quantum collapse models (CSL-like).

OUTLINE

- Introduction
- Quantum decoherence
  - Neutrino-graviton scattering
- Graviton bremsstrahlung
- Detection
- Conclusions

NEUTRINO-GRAVITON SCATTERING

Analogical to Compton Scattering



"Graviton physics", BR Holstein, 2006.

$$\sigma \sim \frac{E_\nu^2}{M_{Pl}^2}$$

GRAVITON INTERACTION

- Neutrinos interact due to mass (via gravitons) with particles (for example: solar masses) as mass eigenstates.
- Propagating neutrino is 'observed' by hard graviton, has definitive mass, propagates in a mass eigenstate.
- Neutrino in definitive mass state due to Interaction (weak) - Propagation (inertia) - Graviton Interaction (gravitation) - Propagation (inertia) - Interaction (weak)
- Neutrino ceases to demonstrate oscillation phenomena or effects depending on being in a superposition of mass states.

$$\Psi_{\nu_f \to \nu_a} = e^{-i \frac{m_a^2}{2E_\nu} L} V_{af}$$

GRAVITON INTERACTION: DETAILS

- The probability can be given in terms of the transition amplitude squared. In any real measurement, energy is integrated over (Detection, Production, momentum):

$$\int dT A_{K',K} A^*_{M',M} \sim \int dE E^2 \sum_{K',K,M',M} V^*_{\alpha,K} \tilde{V}_{\beta,K'} V_{\alpha,M} \tilde{V}^*_{\beta,M'} \Phi_D(p'_{K'}) \Phi_P(p_K) \Phi^*_D(p'_{M'}) \Phi^*_P(p_M) F_{K,K'} F^*_{M,M'}$$

- For the graviton interaction, V is diagonal.
- The condition on the graviton energy:

$$E_G > F(\Delta m)$$

NEUTRINO DETECTION

- Probability for initial electron neutrino to be in mass eigenstate depends on PMNS matrix element.
  - Independent of energy, distance travelled, phase.
- Flavor measurement depends on P_G, The probability for neutrino to have interacted with graviton

$$P_{e\to 1} = \cos^2\theta_{12}\cos^2\theta_{13}$$

$$P_{e\to 2} = \cos^2\theta_{13}\sin^2\theta_{12}$$

$$P_{e\to 3} = \sin^2\theta_{13}$$

$$\frac{N_{\mathrm{e,det}}}{N_{\mathrm{e,init}}} = P_{\mathrm{ee}}^{\mathrm{vac}}(1-P_G)+$$

$$P_G \sum_{i=1,2,3} V_{ei}V_{ie}^{*}V_{ei}V_{ie}^{*}$$

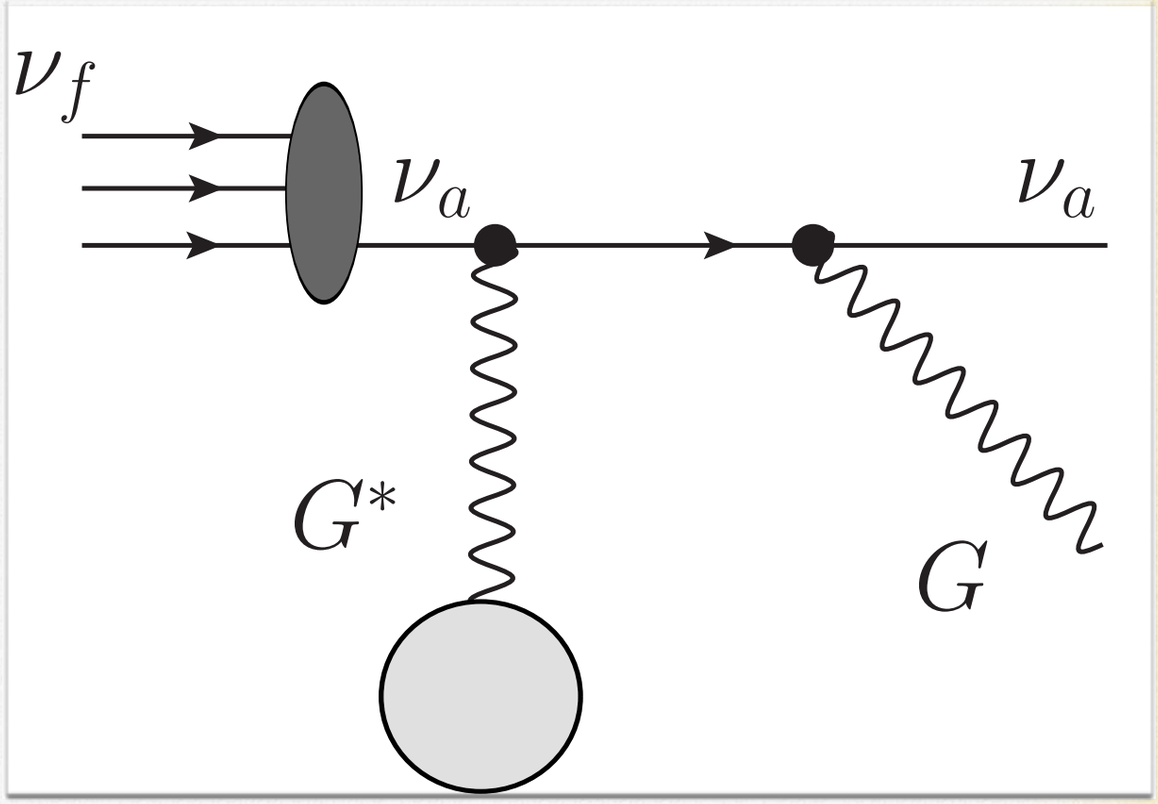
OUTLINE

- Introduction
- Quantum decoherence
- Graviton bremsstrahlung
  - First order calculation
  - Photon bremsstrahlung
  - Towards second order calculation
- Detection
- Conclusions

GRAVITON BREMSSTRAHLUNG

NEUTRINO-MASSIVE SOURCE SCATTERING

Analogical to Photon Bremsstrahlung on Nucleus



B. M. Barker, S. N. Gupta, J. Kaskas, Phys. Rev. 182 (1969) 1391-1396.

GRAVITON BREMSSTRAHLUNG

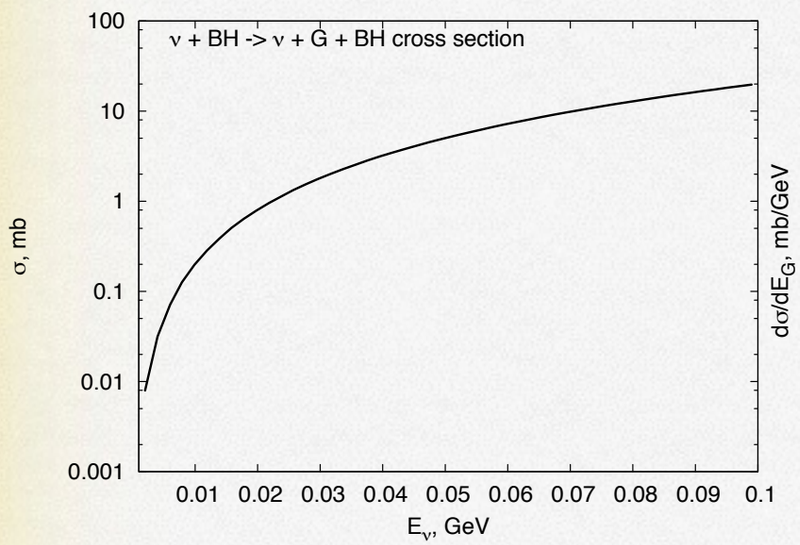
$$\frac{d\sigma}{dk_0 d\Omega_k d\Omega_{p'}} = \frac{\kappa^6 M^2}{(4\pi)^5} \frac{|\mathbf{p}'|}{|\mathbf{p}|} \frac{k_0 \left(I\left(\mathbf{p}, \mathbf{k}, \mathbf{p}'\right)\right)^2}{\left(\mathbf{k} + \mathbf{p}' - \mathbf{p}\right)^4}$$

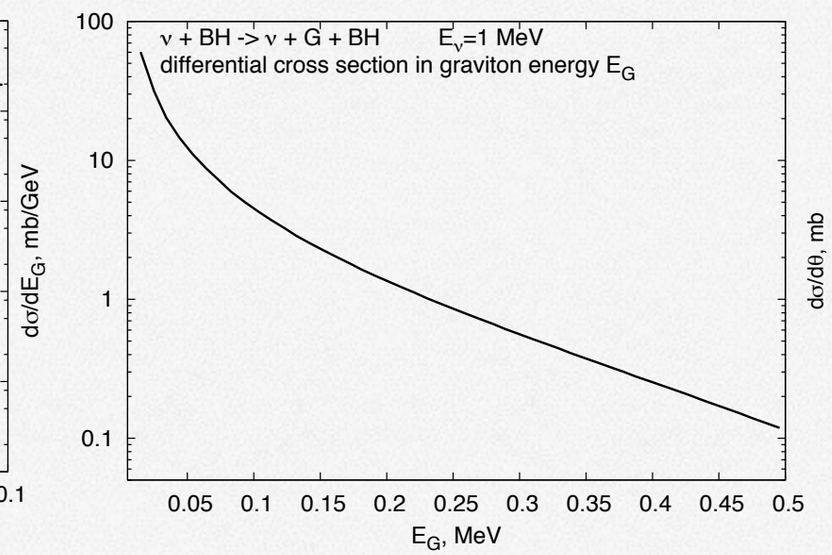
- scattering of small mass (m) off large mass (M)
- more correct then considering external field
- spinless approximation, result after summation over polarization

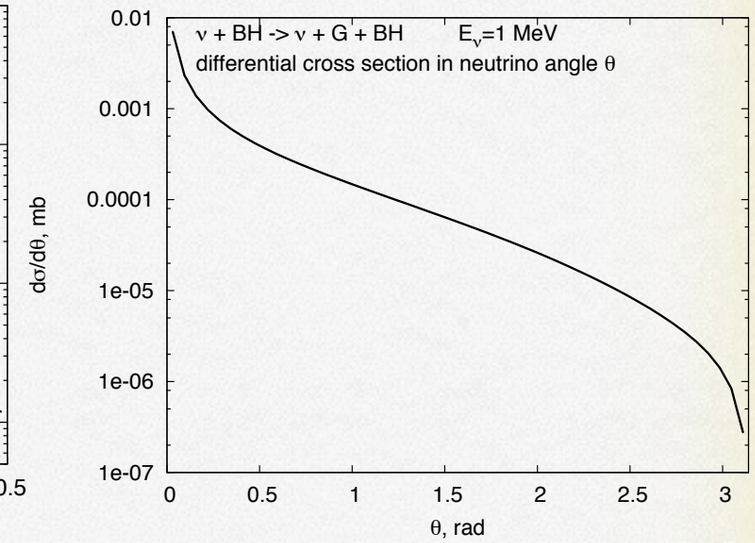
$$\sigma_{\mathrm{GBH}} \sim \frac{M^2 E_\nu^2}{M_{Pl}^6}, \quad M \gg E_\nu \gg m_\nu$$

$$M \sim 10^{57}\ \mathrm{GeV}\ \text{gives}\ M^2/M_{Pl}^6 \sim 1\ \mathrm{GeV}^{-4}$$

GRAVITON BREMSSTRAHLUNG







The bulk of the cross section comes form soft graviton emission in forward limit.
The cross section is larger than electron-neutrino cross section by 16-18 order of magnitudes.
Real, hard gravitons may be produced.
Cross section with respect to E_ν may be measurable.
May be sensitive to next order corrections.

PHOTON BREMSSTRAHLUNG

- Comparisons between tree level and more accurate models have been made for photon bremsstrahlung.
- Tree level is accurate to first order, corrections of less than factor of ~10.
- Requires theory input for quantum gravity.

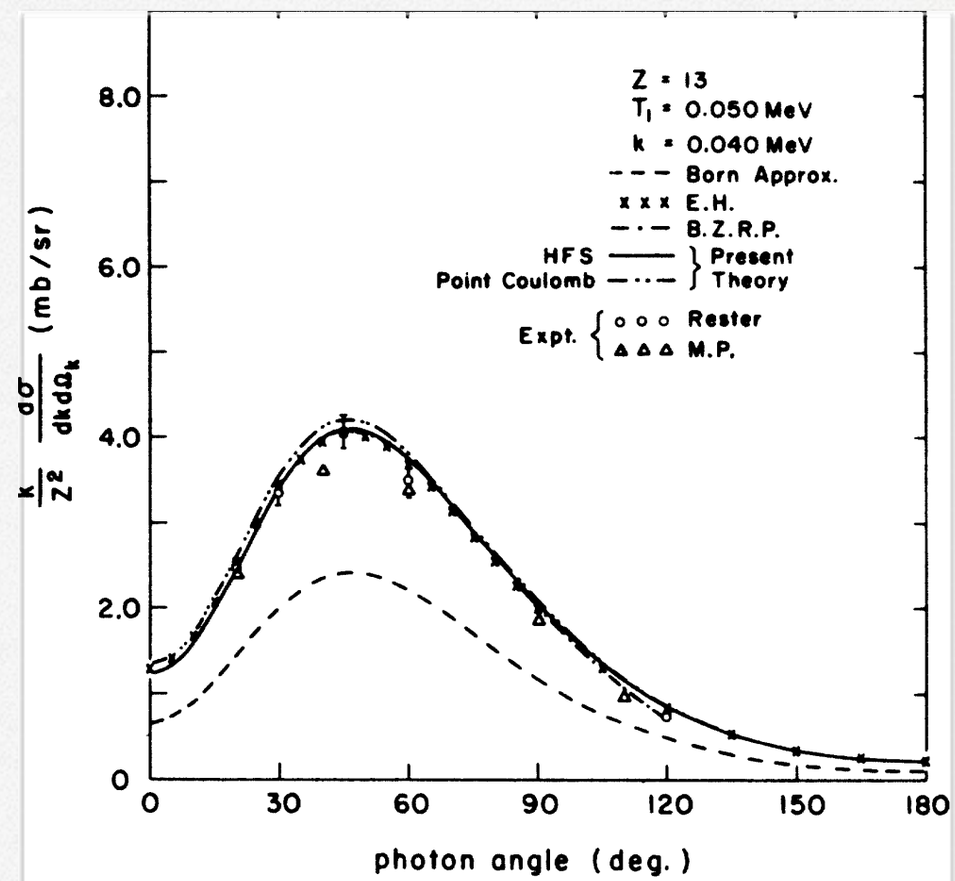


FIG. 6. Comparisons of present results (solid line for HFS field, double-dotted-broken line for point-Coulomb field) with the Born-approximation results (broken line), the results of BZRP (dotted-broken line), the results of EH (crosses), and the experimental data of MP (triangles) and of Rester (circles) for the case $Z=13$, $T_1=0.050$ MeV, $k=0.040$ MeV.

H. K. Tseng and R. H. Pratt, Phys. Rev. A 3 (1971) 100-115.

TOWARDS SECOND ORDER CALCULATION

- Loop corrections are important but won't change first order result ('large').
  - Effective quantum gravity is equivalent to semi-classical quantum gravity at 2-loop.
- Overall differential cross section will be sensitive to Quantum Gravity models.

OUTLINE

- Introduction
- Quantum decoherence
- Graviton bremsstrahlung
- Detection
  - Detection
  - Detection with Earth MSW
  - Analytical formula
  - Story
- Conclusions

DETECTION

- For certain neutrino production mechanisms (pion or muon), neutrinos of high energy (>10 TeV) won't experience propagation decoherence:

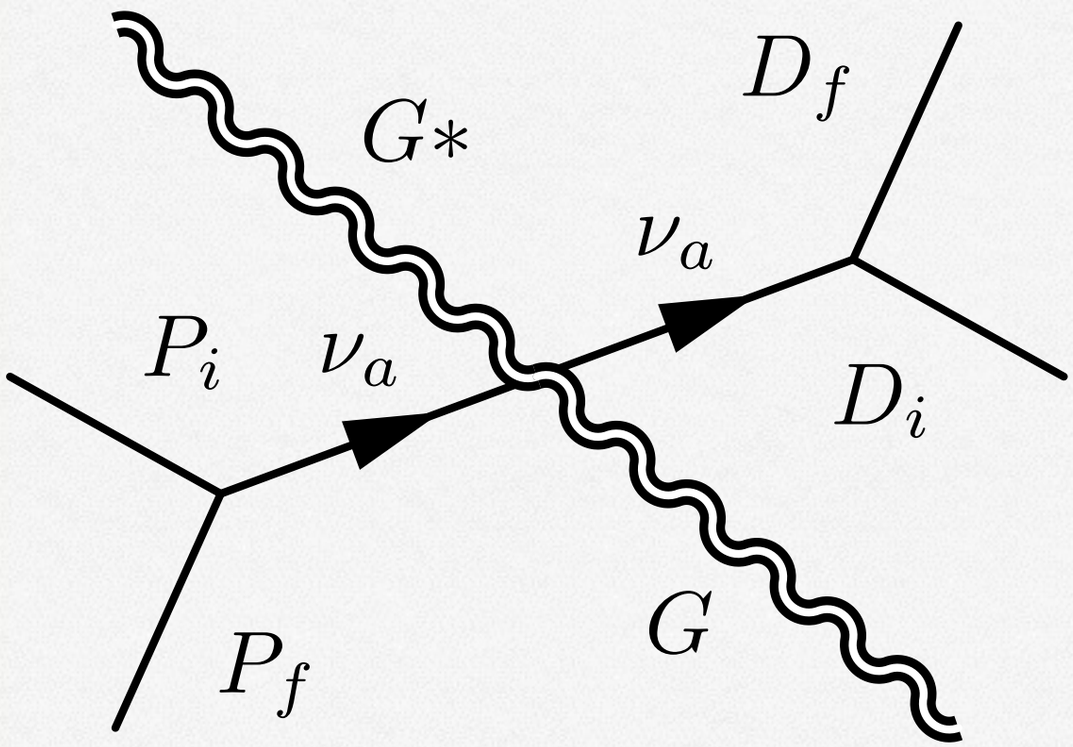
$$\frac{d_L}{\sigma_x} \sim .1 \frac{L}{100\text{Mpc}} \frac{\Delta m^2}{8 \times 10^{-5}\text{eV}^2} \left(\frac{10\text{TeV}}{E}\right) \frac{3 \times 10^{-8}\text{sec}}{\tau_{muon}}$$

- However, even for coherent fluxes, due to detector limitations, no measurable difference can be seen between coherent fluxes and decoherent fluxes (without the consideration of Earth MSW).

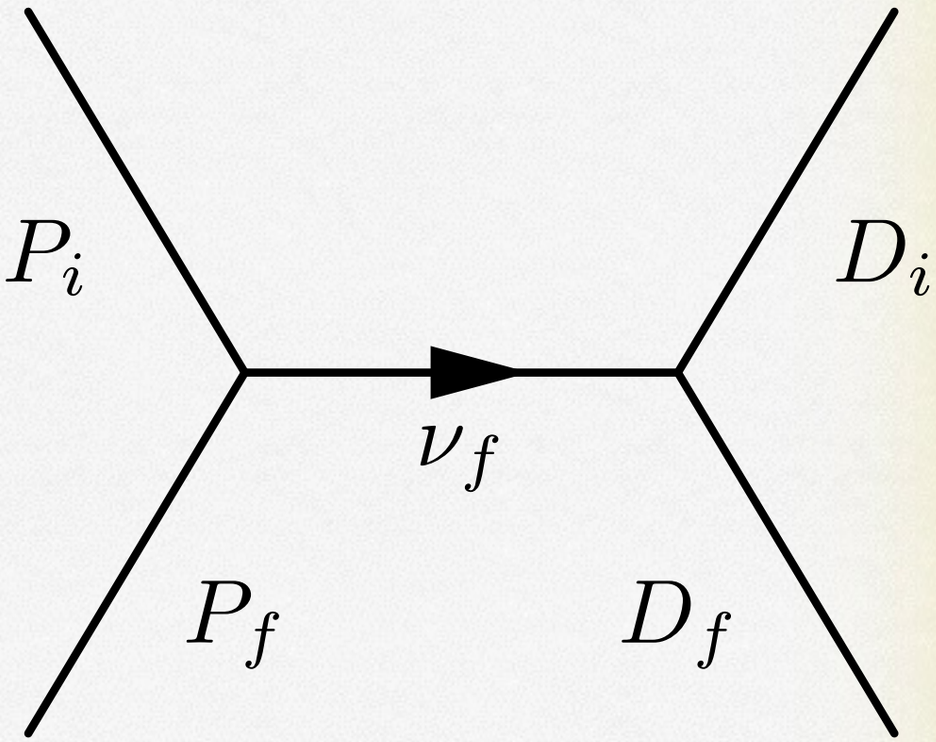
$$\phi_{jk} = \frac{\Delta m^2 L}{4E_\nu}$$

- For extremely high energy (10 EeV) and close (10 kpc), the phase may be small and the flux coherent. In this case the coherent flux may be compared to the decoherent flux due to graviton interaction.

NEUTRINO PROPAGATION



Neutrino produced and detected in detector in flavor state, exists as a definite mass state (due to graviton 'observation'), decoherence due to only single neutrino state propagating.

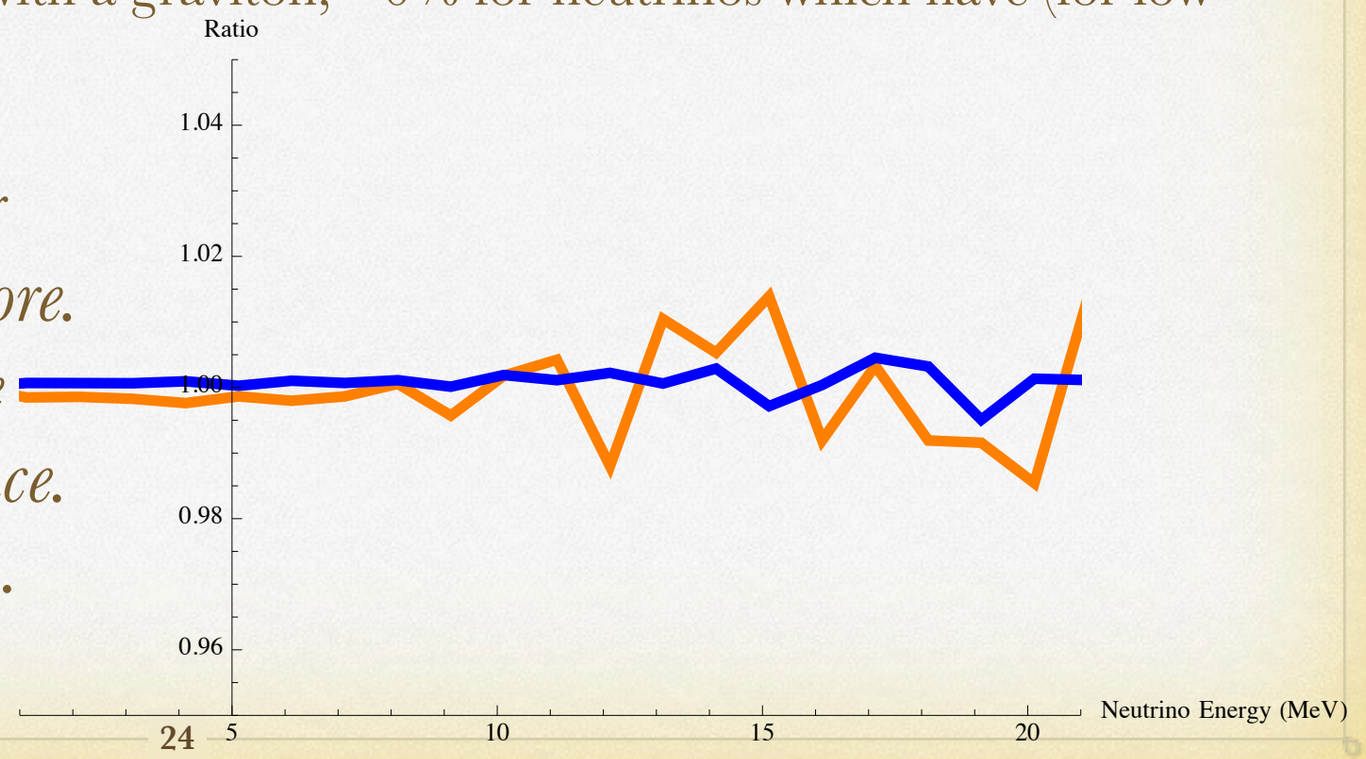


Neutrino produced and detected in flavor state, exists in superposition of mass states, decoherence caused by separation of mass states.

DETECTION: EARTH MSW

- Neutrinos which are decoherent due to propagation exist as a decoherent superposition of mass eigenstates.
- Neutrinos which are decoherent due to quantum gravity exist as a single mass eigenstate.
- For significant changes of matter potential, the neutrino state's flavor is matched for both propagating bases.
- Expect an (energy and location dependent) >10% effect for decoherent electron neutrinos which have not had an interaction with a graviton, ~0% for neutrinos which have (for low energy).

Analytic calculation in 2 flavor approximation without Earth's Core. Orange is neutrinos which have experienced propagation decoherence. Constant density approximation.



DETECTION: EARTH MSW (CONST. DENSITY, 2 REGION, 2 FLAVOR APPROXIMATION)

Space	*Flavor Match*	*Earth*
ν_e	$\cos^2(\theta)\, e^{i\phi} + \sin^2(\theta)\, e^{-i\phi}$	$\cos^2(\theta_m)\, e^{i\phi_m} + \sin^2(\theta_m)\, e^{-i\phi_m}$ ν_e
	$\sin(\theta)\cos(\theta)\left(e^{-i\phi} - e^{i\phi}\right)$	$\sin(\theta_m)\cos(\theta_m)\left(e^{-i\phi_m} - e^{i\phi_m}\right)$
	Coherent	
ν_e	$\cos^2(\theta)\, e^{i\frac{3\pi}{4}} + \sin^2(\theta)\, e^{-i\frac{3\pi}{4}}$	$\cos^2(\theta_m)\, e^{i\phi_m} + \sin^2(\theta_m)\, e^{-i\phi_m}$ ν_e
	$\sin(\theta)\cos(\theta)\left(e^{-i\frac{3\pi}{4}} - e^{i\frac{3\pi}{4}}\right)$	$\sin(\theta_m)\cos(\theta_m)\left(e^{-i\phi_m} - e^{i\phi_m}\right)$
	Propagation Decoherence	
$P_{e1} = \cos^2(\theta)$ ν_1	$\cos(\theta)$	$\cos^2(\theta_m)\, e^{i\phi_m} + \sin^2(\theta_m)\, e^{-i\phi_m}$ ν_e
Or	$-\sin(\theta)$	$\sin(\theta_m)\cos(\theta_m)\left(e^{-i\phi_m} - e^{i\phi_m}\right)$
ν_e $P_{e2} = \sin^2(\theta)$ ν_2	$\sin(\theta)$	$\cos^2(\theta_m)\, e^{i\phi_m} + \sin^2(\theta_m)\, e^{-i\phi_m}$
	$\cos(\theta)$	$\sin(\theta_m)\cos(\theta_m)\left(e^{-i\phi_m} - e^{i\phi_m}\right)$
	Quantum Gravity Decoherence	

ANALYTICAL RESULT

- The ratio of neutrinos which have experienced propagation decoherence:

$$\frac{\left(\cos\left(\tilde{x}\right)^2\left(3+\cos\left(4\theta\right)\right)+\left(2+\cos\left(4\theta_m-8\theta\right)+\cos\left(4\theta_m-4\theta\right)\right)\sin\left(\tilde{x}\right)^2-2\sin\left(2\tilde{x}\right)\sin\left(2\theta_m-2\theta\right)\sin\left(2\theta\right)\right)}{3+\cos\left(4\theta\right)}$$

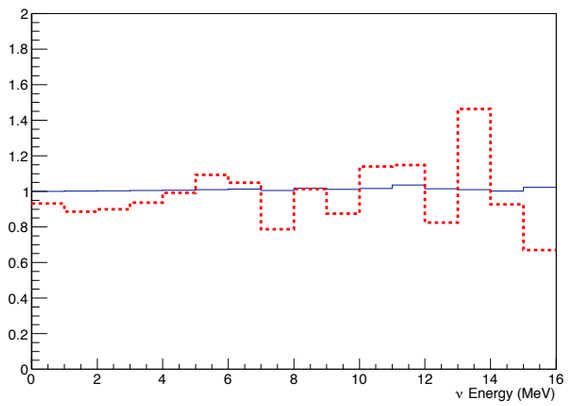
- The ratio of neutrinos which have experienced quantum gravity decoherence:

$$\frac{5+\cos\left(4\theta_m\right)+\cos\left(4\theta_m-4\theta\right)+\cos\left(4\theta\right)+4\cos\left(2\tilde{x}\right)\cos\left(2\theta\right)\sin\left(2\theta_m-2\theta\right)}{6+2\cos\left(4\theta\right)}$$

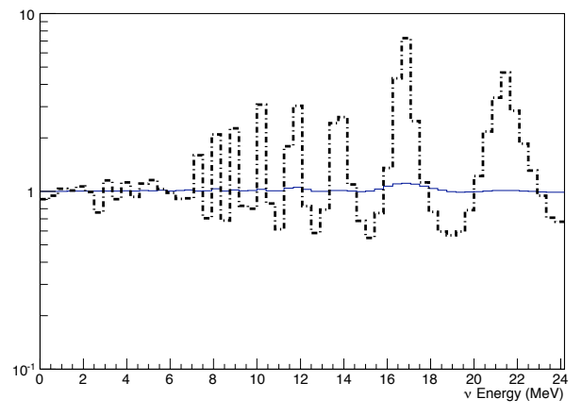
- This is for 2 region vacuum/constant density and 2 flavor approximation. Here

$$\tilde{x}=x\sqrt{\sin\left(2\theta\right)^2+\left(\cos\left(2\theta\right)-\frac{2AE_\nu}{\Delta m^2}\right)^2} \qquad \sin\left(2\theta_m\right)^2=\frac{\sin\left(2\theta\right)^2}{\sin\left(2\theta\right)^2+\left(\cos\left(2\theta\right)-\frac{2AE_\nu}{\Delta m^2}\right)^2}$$

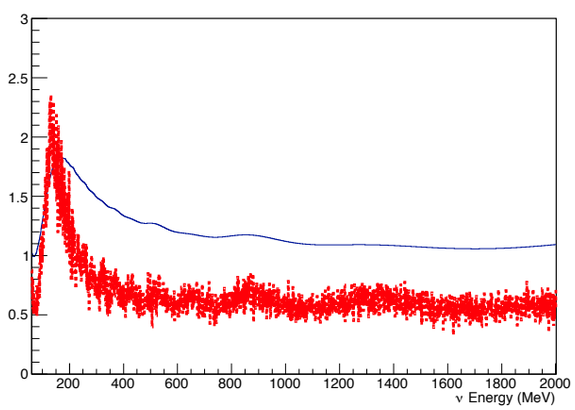
WITH CORE - 3 FLAVOR



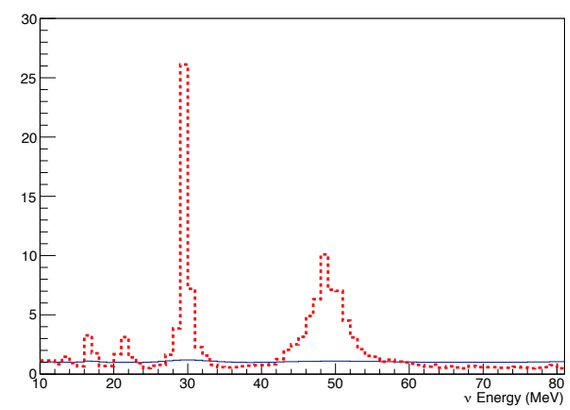
(a) Earth MSW (500 keV)



(b) Earth MSW (50 keV)

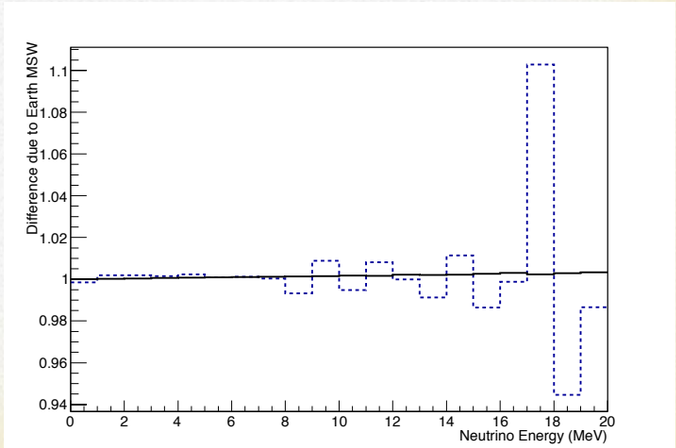


(d) Earth MSW (500 keV)



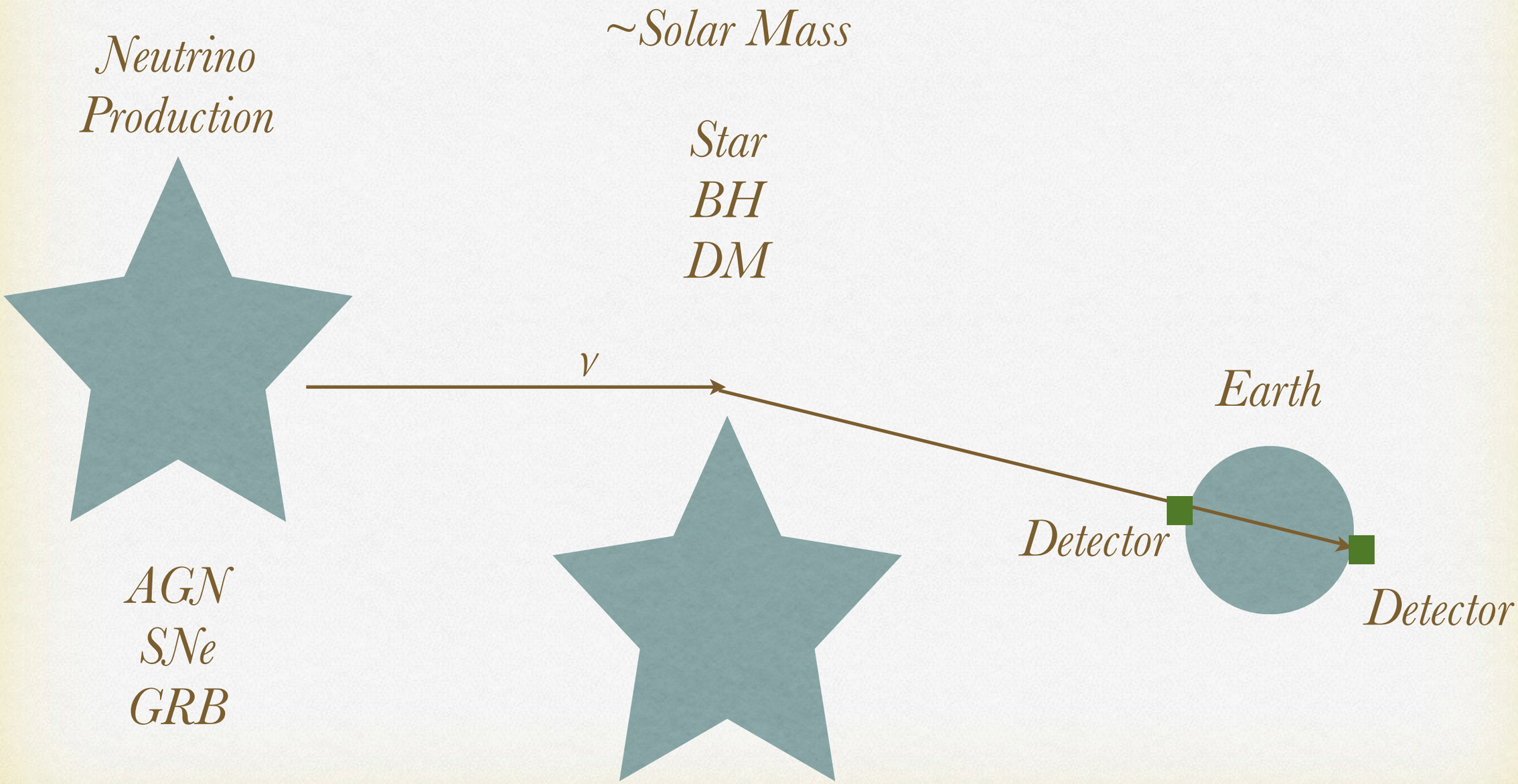
(c) Earth MSW (500 keV)

Ratio with and without Earth MSW effect due to initial electron neutrino.



Without Core

STORY


~Solar Mass
Neutrino Production
Star
BH
DM
ν
Earth
Detector
Detector
AGN
SNe
GRB

OUTLINE

- Introduction
- Quantum decoherence
- Graviton bremsstrahlung
- Detection
- Conclusions
  - Interaction conclusions
  - Neutrino observatories
  - Detection conclusions
  - Summary

INTERACTIVE CONCLUSIONS

- Gravity QFT interactions, due to hard graviton emission, have reasonably high cross-section in the presence of classical masses.
  - Effective versus fundamental QFT?
- Neutrino oscillation behavior can be used as an observable to measure this cross-section.
- Due to the difference between quantum gravity induced decoherence and decoherence due to propagation, existing in a single mass eigenstate versus a decoherent superposition of mass eigenstates, measuring the Earth MSW effect can provide an energy dependent >10% difference in expectations.
- **Quantum Gravity can be measured!** For astrophysical neutrinos a point source is required (SNe/GRB/AGN) and large (low energy?) neutrino detectors at multiple points around the world (Chile, Japan, North America, Europe, Antarctica) to differentiate the quantum gravity induced decoherence from other sources of decoherence.

NEUTRINO OBSERVATORIES

LOW ENERGY NEUTRINO OBSERVATORIES

Observatory	Size + Type	Location	near SNe #
Super-K	32 kT (Water)	Japan - Now	7000
Borexino	0.3 kT (Scint)	Italy - Now	100
SNO+	0.8 kT (Scint)	Canada-Now*	300
LBNE(1)	10 kT (Argon)	USA-201X	1000
HALO	0.08 kT (Lead)	Canada-Now*	30
LAGUNA**	100 kT (Mixed)	Europe - 202X	15000
ANDES***	3 kT (Scint)	Chile - 202X	1000
Hyper-K	530 kT (Water)	Japan -202X?	110000
Beyond DC	1 MT (Ice)	Antarctica-203X?	??

DETECTION CONCLUSIONS

- Neutrino observatories in South America, South Africa, and Australia will improve chance to measure response to the Earth matter effect (with core). Especially interested in < 1 GeV (below IceCube).
- Need to investigate possible source spectrums (GRB, DM annihilation/decay, SuperNova, etc).
- Need Quantum Gravity theorists to improve theory so that proper calculation can be done for structure (point versus star/DM halo/etc).

SUMMARY

- Neutrino travelling through a massive field experiences radically different effects depending on describing the interaction in QFT framework or in GR framework.
- In STG, cross section for graviton bremsstrahlung may be large to first order. STG and ETG are equivalent in far field, high mass limit. Corrections needed.
- Decoherent neutrino fluxes due to propagation decoherence (where each neutrino is decoherent) and quantum gravity decoherence (where each neutrino is coherent) may be discriminated using the Earth MSW.
- Thank you for listening.