Alexey Golovnev

On gravitational collapse in MOND

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The Dark Sectors in Cosmology

- The general relativity is a very successful theory, and the standard cosmological model is also confirmed with a high precision
- However, sticking to the standard paradigm, a bit less than five percent of the energy budget of the Universe represents some well-established physics
- Dark Energy (and the huge problem of the cosmological constant), Dark Matter...
- The need to postulate a period of inflation with an unknown nature of the inflaton, or something even more crazy...
- And some less striking but very important problems on top of that. The low multipole anomalies in CMB – the lack of correlations at the largest angular scales, statistical anisotropy...

It is reasonable to look for solutions in modifying the GR itself – “dark gravity”. These are infrared modifications – very popular now.

In a sense, MOND goes the same path, assuming also that the whole modification, at least to some approximation, is rooted in the Newtonian sector.

MOND

The peripheral parts of galactic rotation curves are surprisingly flat.
There are seemingly large amounts of hidden mass in galaxy clusters, too.

Why is that? What is the reason?

The standard paradigm is Dark Matter.
Nicely fits the rest of the cosmological data.
But evades any sort of direct detection.

Another idea is to modify the laws of gravity and/or inertia such that the centripetal acceleration of a test body orbiting around a central mass M is

$$a = \frac{\sqrt{GMa_0}}{r}$$

where $a_0 \approx 1.2 \cdot 10^{-8} \frac{cm}{s^2}$ (if Newtonian acceleration falls below a_0)

Note the $\sqrt{M}$ -dependence – possible re-entrance to Newtonian regime.
Negative phantom mass in MOND.
For galaxy clusters some amount of Dark Matter is needed, even with MOND

Dark Matter has problems at galactic scales

• Local Group Structure – too empty a Local Void, too many large galaxies on its outskirts, missing satellites problem...
• The problems with formation of observed structures in standard paradigm – cuspy cores prediction, existence of pure disk (bulgeless) galaxies...
• Some less sound problems – Dark Flow...

• Many of them might be due to an incomplete understanding of non-linearities and baryonic physics.
• Also warm Dark Matter, interacting Dark Matter...

• Bullet Cluster – separation of lensing signal and X-luminous hot gas consistent with Newtonian gravity and DM, though not impossible in MOND. However, the collisional velocities are too high for CDM.

• The most puzzling for DM are apparent regularities – acceleration scale, constant Dark Matter halo central surface density...
•If not MOND, then why?

Spherical collapse

The time of collapse of a spherical dust cloud is

in Newtonian gravity $T_N = \sqrt{\dfrac{3\pi}{32\,G\rho}}$

in MOND $T_M = \sqrt[4]{\dfrac{3\pi R}{16 G\rho a_0}}$

$$\frac{T_M}{T_N} = \sqrt[4]{\frac{16}{\pi^2} \cdot \frac{a_N}{a_0}}$$

A sparse cloud with $a_N \ll a_0$ at the periphery collapses much faster in MOND. Probably, this is the reason for which numerical simulations would rather give too much structures in MONDian universes despite the absence of Dark Matter.

The layer-wise dynamics of collapse is obviously different from that of Newtonian gravity.

Realisations of MOND

Outside spherical symmetry, simply rescaling the acceleration is a pathological idea – bad momentum non-conservation

The standard substitute is Modified Poisson equation – let's call it canonical MOND

$$\nabla\left(\mu\left(\frac{\nabla\phi}{a_0}\right)\cdot\nabla\phi\right)=4\pi G\rho$$

$$\mu(x)\to x, \qquad x\to 0$$

$$\mu(x)\to 1, \qquad x\to\infty$$

very complicated non-linear problem

There is a simpler (quasi-linear) realisation – QUMOND. First find the Newtonian potential, and then solve a new inhomogeneous Poisson equation

$$\Delta\phi=\nabla\left(\nu\left(\frac{\nabla\phi^{(N)}}{a_0}\right)\cdot\nabla\phi^{(N)}\right)$$

In N-body simulations all these were used.

In cosmology it would be highly desirable to have a good relativistic realisation – much less clarity and agreement on that, sadly.

In TeVeS the k-essence type scalar field is constructed such as to reproduce the two asymptotics by hand. Cannot by itself produce additional gravitational lensing – vectorial part of an Einstein-aether type is tuned to do so. Not without problems for cosmology.

But everything can be ascribed to realisations and not the paradigm...

MOND is intrinsically non-linear

In Newtonian gravity the outer layers of a spherically symmetric mass distribution do not affect the gravitational field inside themselves. This is because the force law $\sim \frac{1}{r^2}$ nicely corresponds to the growth of surface areas $\sim r^2$ within a given solid angle, so that the opposite sides compensate each other.
It is not the case in MOND. Moreover, with the $\sqrt{M}$ dependence of the force, the superposition principle can not work.
If instead we assume $a = M \cdot \sqrt{\frac{Ga_0}{M_g}} \cdot \frac{1}{r}$ with some new mass constant, then the spherical shell of width ΔR would contribute to centripetal acceleration of a test particle at distance r from the centre as

$\delta a \approx 8\pi r \sqrt{\frac{Ga_0}{M_g}} \rho \Delta R$ irrespective of its radius R.

Being cut-off at the Hubble scale, it makes the contribution of order a_0.
One needs some (yet non-relativistic realisation) without this effect. The most commonly accepted one is $\nabla\left(\mu\left(\frac{\nabla\phi}{a_0}\right)\cdot\nabla\phi\right) = 4\pi G\rho$

The problems of non-linearity

Consider an over-density inside a spherical cloud. Which acceleration should it experience towards the centre?

If the gravitational gradient is larger than a_0 , then the acceleration must be Newtonian, otherwise we expect the MONDian regime to be in operation. However, the central part might produce a weak field while the self-fields inside the over-density can be larger than the MOND limit. Naively, the external force will also be driven to another value.

In canonical MOND for a small inhomogeneity, the center-of-mass motion is governed by the MONDian force as for a test particle (Bekenstein, Milgrom). But what about the tidal effects?

To which scales should we take these effects into account? At the Compton wavelength this self-Newtonisation effect occurs for masses

$m < m_0 \approx 2 \cdot 10^{-25} g$ Note that $m_{e^-} < m_0 < m_{p^+}$

What effects might these averaging problems have on the structure and formation of galaxy clusters? This is a very hard question for the modified Poisson equation of MOND. However, in simplified quasi-linear model (QUMOND), it seems feasible, and is under investigation now.

Perspectives and outlook

MOND is often claimed to be more successful than the CDM. However, while CDM fails in one corner of the observational landscape being confirmed by lots of precision data in modern cosmology, the new paradigm perfectly describes just what it was invented for with unresolved problems in cosmology and with not so many details about the small scale structures. But still requires some mild amounts of dark matter.
Moreover, any agreement on its relativistic realisation is lacking. And any cosmological problem can be referred to this fact. At the end of the day, is that satisfactory?

There is more agreement on the non-relativistic limit. In my opinion, it is necessary to pay attention to analytic understanding of the non-linearities and interplay of different scales in MOND. It might be hard in canonical (modified Poisson) MOND, but seems feasible in quasi-linear approach.

On the other hand, it may teach us how a modified gravity alternative to the Dark Matter might look like, at least in certain regimes...

Conclusions

• MOND is an independent paradigm for explaining the dynamics of structures at galactic scales.

• Cosmological embeddings are possible but somewhat problematic yet

• And indeed, even the correct relativistic realisation is far from being clear and unequivocal

•However, some non-relativistic conceptual issues are ought to be investigated, anyway

•In progress...