Corso di Laurea Magistrale in Matematica
Istituzioni di Fisica Matematica A
Prof. Ludwik DABROWSKI

Linear partial differential equations of mathematical physics

Program:

1. Linear partial differential operators:

Definitions and main examples. - Principal symbol of a linear differential operator. - Change of independent variables. - Canonical form of linear differential operators of order 1 and of order 2, with constant coeffcients. - Characteristics. Elliptic and hyperbolic operators. - Reduction to a canonical form of second order linear differential operators in a two-dimensional space. Parabolic operators. - General solution of a second order hyperbolic equation with constant coefficients in the two-dimensional space.

2. Wave equation:

Vibrating string. - Cauchy problem. D'Alembert formula. - Some consequences of the D'Alembert formula. - Semi-infinite vibrating string. - Periodic problem for wave equation. - Introduction to Fourier series. - Finite vibrating string. Standing waves. - Energy of vibrating string. - Solutions in dimension 2 and 3. - Solutions of the inhomogeneous problem.

3. Laplace equation:

Ill-posedness of Cauchy problem for Laplace equation. - Dirichlet and Neumann problems for Laplace equation on the plane. - Properties of harmonic functions: mean value theorem, the maximum principle. - Harmonic functions on the plane and complex analysis.

4. Heat equation:

Derivation of heat equation. - Main boundary value problems for heat equation. - Fourier transform. - Solution of the Cauchy problem for the heat equation on the line. - Mixed boundary value problems for the heat equation. - More general boundary conditions. - Solution of the inhomogeneous heat equation.

5. Statement of the Cauchy-Kowalewska theorem. Abstract Cauchy problem. One-parameter evolution semigroups.

6. Notes on Schroedinger equation, Maxwell equation and Dirac equation.

References:

Lecture Notes by Boris Dubrovin, http://people.sissa.it/~dubrovin/bd_courses.html

L.C. Evans, Partial differential equations, Providence, AMS, 1998; (mainly Chapter 2.4)

H.O. Fattorini, The Cauchy problem (Enc. Math. Appl. vol 18) Addison-Wesley, 1983; (mainly Chapter 1)

W. Thirring, A course in mathematical physics, vol. 3, Springer 1981; (mainly Sect. 2.4, 2.5, 3.3).

A. N. Tikhonov, A. A. Samarskij, Equazioni della fisica matematica, Moscow Mir 1981, (e.g. p. 722).