# Trails in Quantum Mechanics and Surroundings

January 29 – 30, 2018, Trieste (Italy) SISSA

### **Programme & Abstracts**

### Monday 29/01/2018

- 08:45 09:00 Stefano Ruffo, SISSA director Opening and welcome
- 09:00 09:45 Gianni Dal Maso Homogenisation of free-discontinuity problems
- 09:45 10:30 Fabio Cipriani Logarithmic Sobolev Inequalities seen from Quantum Statistical Mechanics
- 10:30 11:00 Coffee Break
- 11:00 11:45 Riccardo Adami Schrödinger on the grill: nonlinearly, occasionally, vaguely, mildly.
- 11:45 12:30 Giuseppe Gaeta Symmetry of stochastic equations
- 12:30 14:30 Lunch Break
- 14:30 15:15 Fabio Benatti Complete Positivity in Markovian and non-Markovian Quantum Dynamics
- 15:15 16:00 Michele Correggi Anyonic Schrödinger operators
- 16:00 16:30 Coffee Break
- 16:30 17:15 **Gianluca Panati** Optimal decay of Wannier functions in Chern and quantum Hall insulators
- 17:15 18:00 **Rodolfo Figari** Behavior of eigenvalues and resonances of Schrödinger operators under Stark perturbation

### Tuesday 30/01/2018

- 09:00 09:45 **Giuseppe Marmo** The manifold structure of quantum states: tensor fields and evolution
- 09:45 10:30 Andrea Posilicano Asymptotic Completeness and S-Matrix for Singular Perturbations
- 10:30 11:00 Coffee Break
- 11:00 11:45 Giovanni Landi Solitons and Gabor frames over noncommutative tori
- 11:45 12:30 Domenico Finco A model Hamiltonian for Feshbach Resonances
- 12:30 13:15 Sergio Albeverio Quantum mechanical resonances and zeros of exponential sums
- 13:15 13:30 Gianfausto Dell'Antonio Closing remarks

### **Titles and Abstracts**

#### Schrödinger on the grill: nonlinearly, occasionally, vaguely, mildly.

Riccardo Adami DISMA - Politecnico di Torino

We review on some ideas and results obtained in the last years on the problem of the bound states on quantum graphs for the Nonlinear Schrödinger Equation, starting from the case of the line up to the grill and beyond, highlighting new phenomena and drawing possible perspectives.

#### Quantum mechanical resonances and zeros of exponential sums

Sergio Albeverio Bonn Universität

We give a survey of some results concerning resonances (location, distribution, asymptotics) in many centers quantum mechanical models with a particular attention to point interaction models. Methods used involve classical results on the location and distributions of zeros of exponential polynomials, which will be briefly recalled. A comparision with the case of smooth potentials will also be mentioned.

#### Complete Positivity in Markovian and non-Markovian Quantum Dynamics

Fabio Benatti Dipartimento di Fisica, Università di Trieste

The seminar provides an overview of the physical relevance of complete positivity for the state transformations of quantum systems with particular emphasis on its role in the dissipative dynamics of open quantum systems with and without memory effects.

#### Logarithmic Sobolev Inequalities seen from Quantum Statistical Mechanics

Fabio Cipriani

Dipartimento di Matematica - Politecnico di Milano

It is shown how Logarithmic Sobolev Inequalities follow in a concise way from a sub-exponential spectral growth assumption, via two fundamental properties of the Helmholtz Free Energy and Relative Entropy of states. The setting in completely general and examples will range among compact Riemannian manifolds, topological compact groups, dual of discrete groups, noncommutative tori, compact quantum groups and Ideal Bose gas.

#### **Anyonic Schrödinger operators**

Michele Correggi Sapienza Università di Roma, Rome, Italy

We review the main issues concerning the well-posedness (as suitable self-adjoint operators) of the Hamiltonians of anyonic systems in two dimensions. We mostly focus on the simple case of two non-interacting anyons but also discuss some future perspectives about the definition of Schrödinger operators for interacting many-anyon systems.

#### Homogenisation of free-discontinuity problems

Gianni Dal Maso SISSA, Trieste

The stochastic homogenisation of free-discontinuity functionals is studied assuming stationarity of the random volume and surface energy densities. Combining the deterministic results on Gamma-convergence of free-discontinuity functionals with the Subadditive Ergodic Theorem, we characterise the homogenised volume and surface energy densities in terms of limits of the solutions of auxiliary minimum problems on large cubes.

#### Behavior of eigenvalues and resonances of Schrödinger operators under Stark perturbation

Rodolfo Figari

Dipartimento di Scienze Fisiche, Università di Napoli Federico II

In recent yet unpublished work K. Yajima analyzed the stability of resonances under weak Stark perturbations. Yajima found that resonances are strongly unstable even for very weak Stark potentials. I will introduce Yajima's results and comment about the possibility of investigating stability of resonances, of eigenvalues embedded in the continuum and of discrete eigenvalues in models of multiple channels point interaction hamiltonians perturbed by weak Stark potentials.

#### A model Hamiltonian for Feshbach Resonances

Domenico Finco UniNettuno, Roma

We discuss a model to describe magnetically induced Feshbach Resonances, that is how a closed scattering channel can interact with an open scattering channel such that the scattering length can be tuned to any value.

#### Symmetry of stochastic equations

Giuseppe Gaeta Dipartimento di Matematica dell' Università degli Studi di Milano

Symmetry methods proved over the years to be invaluable in the study of nonlinear (deterministic) differential equations. Recently there has been a surge in activity devoted to extending them to stochastic equations as well. I will explain what are the conceptual problems in this direction, and illustrate what results are available.

#### Solitons and Gabor frames over noncommutative tori

Giovanni Landi Dipartimento di Matematica e Informatica, Università degli Studi di Trieste

We study of solitons over noncommutative tori from the perspective of time-frequency analysis and treat the case of general topological charge. Solutions are associated with vector bundles of higher rank over noncommutative tori. We express these vector bundles in terms of vector-valued Gabor frames and apply the duality theory of Gabor analysis to show that Gaussians are solitons of general topological charge over noncommutative tori. An energy functional for projections over noncommutative tori is the basis for the self and anti-self duality equations of the solitions which turns out to have a reformulation in terms of Gabor atoms and we prove that projections generated by Gaussians minimize this energy functional. Finally we comment on the case of the Moyal plane and the associated continuous vector-valued Gabor frames and show that Gaussians are the only class of solitons.

## The manifold structure of quantum states: tensor fields and evolution

Giuseppe Marmo Dipartimento di Scienze Fisiche, Università di Napoli Federico II

Usual applications of differential geometry to physics deal with smooth manifolds. The space of quantum states is not smooth but carries a stratified manifold structure, this requires additional care when tensor fields and transformation groups (or semigroups) are considered. In this talk we consider in particular infinitesimal generators of trace-preserving completely positive maps.

## Optimal decay of Wannier functions in Chern and quantum Hall insulators

Gianluca Panati Sapienza Università di Roma

We investigate the localization properties of independent electrons in a periodic background, possibly including a periodic magnetic field, as e.g. in Chern insulators and in Quantum Hall systems. Since, generically, the spectrum of the Hamiltonian is absolutely continuous, localization is characterized by the decay, as  $|x| \rightarrow \infty$ , of the composite Wannier functions associated to the Bloch bands below the Fermi energy, which is supposed to be in a spectral gap. We prove the validity of a localization dichotomy, in the following sense: either there exist exponentially localized composite Wannier functions, and correspondingly the system is in a trivial topological phase with vanishing Hall conductivity, or the decay of any composite Wannier function is such that the expectation value of the squared position operator, or equivalently of the Marzari-Vanderbilt localization functional, is  $+\infty$ . In the latter case, the Bloch bundle is topologically non-trivial, and one expects a non-zero Hall conductivity. The talk is based on a joint paper with Domenico Monaco, Adriano Pisante, and Stefan Teufel.

#### Asymptotic Completeness and S-Matrix for Singular Perturbations

Andrea Posilicano

Dipartimento di Scienza e Alta Tecnologia, Università degli Studi dell'Insubria

We give a criterion of asymptotic completeness and provide a representation of the scattering matrix for the scattering couple  $(A_0, A)$ , where  $A_0$  and A are semi-bounded self-adjoint operators in  $L^2(M, \mathscr{B}, m)$  such that the set  $\{u \in D(A_0) \cap D(A) : A_0u = Au\}$  is dense. No sort of trace-class condition on resolvent differences is required. Applications to the case in which  $A_0$  corresponds to the free Laplacian in  $L^2(\mathbb{R}^n)$  and A describes the Laplacian with self-adjoint boundary conditions on rough hypersurfaces are given. The proofs use a combination of different techniques in scattering theory as the Cook-Kato-Kuroda theory, the limiting absorption principle, the Birman-Kato invariance principle and the Birman-Yafaev stationary approach.