SISSA
Entrance
Examination

Elementary
Particles
Sector

Trieste, 15 October 2001

FIVE PROBLEMS are given. You should try and solve as many as you can within three hours. We do not expect anyone to solve them all.

1. On the hyperfine structure of the hydrogen atom

A nucleus having magnetic moment $\vec{M}$, generates a static magnetic field with magnetic potential

$$\vec{A} = \frac{\vec{M} \times \vec{r}}{r^3}$$

1) Write the nonrelativistic Pauli Hamiltonian for an electron moving in this electromagnetic field, keeping at most terms linear in $\vec{M}$.

2) Identify spin-orbit and spin-spin interactions.

3) Estimate the size of these corrections to the energy levels of a Hydrogen atom in the lowest states, that is the size of the ratio of the correction to the unperturbed energy. Remember that the Bohr magneton of a particle with mass m is $e/2m$, that the Bohr radius is $1/m_e e^2$ and the ground state energy of the Hydrogen atom is $m_e e^4$ (in units $\hbar = c = 1$).

Note: the determination of signs and numerical factors is not required, but pay attention to dimensionful parameters.

2. Scattering amplitudes and nonrelativistic potential in scalar theory

Consider a complex scalar field Φ describing a scalar particle of mass M with possible charges $\pm e$ minimally coupled to the electromagnetic field.

1) Write the Lagrangian for the system.

2a) Write the Feynman diagram describing at the lowest order the scattering of two scalar particles of the same charge, and the corresponding amplitude in momentum space.

2b) Suppose that the field Φ also interacts with a neutral massless scalar field φ with an interaction $L_{int} = -\lambda\Phi^*\Phi\varphi$. Compute the lowest order contribution due to this additional interaction to the amplitude of question 2a).

3) In the nonrelativistic Born approximation the scattering amplitude is expressed in terms of the potential describing the interaction.

Find the interaction potential (as a function of the coordinates) which in the Born approximation reproduces the amplitude of question 2) in the limit in which the ratio three-momentum/mass is put to zero.

NOTE: the precise determination of numerical factors or powers of π is not required, but pay attention to the dimensionful parameters.

3. Symmetries of free fermions

Let Ψ^a, $a = 1, \ldots, N$, be a set of free Dirac fermion fields with masses $m_1, \ldots, m_N$.

1) Starting from the general variation of the action under linear transformations of the fields (acting on the index a) find the global internal symmetries of the theory:

- What is the symmetry group if all the masses are different?
- What is the symmetry group if all the masses are equal?
- What happens if some of the masses are zero?

2) Using the canonical equal time anticommutation relations show that the Noether charges generate the symmetry transformations of the fields.

4. Cosmic gamma rays

One of the main sources of the diffuse Galactic γ-ray radiation is the decay of pions produced by the interaction of primary cosmic rays (mainly protons) with the interstellar medium. Once π^0s are produce, they rapidly decay and the dominant process is:

$$\pi^0 \to 2\,\gamma \tag{1}$$

with photons emitted isotropically in the pion rest frame.

1) Derive the minimum total energy of the pion E_π (rest mass plus kinetic energy) needed to produce a photon of energy E_γ.

2) Sketch the photon energy spectrum and show that its main features are independent of details in the chain of π^0 production.

3) Given that the process in Eq. (1) is the dominant π^0 decay mode, how large a branching ratio do you expect for the process $\pi^0 \to 3\,\gamma$? (consider both processes as electromagnetic)

5. Physics of e^+e^- annihilation

Consider the process of electron-positron annihilation into a muon-antimuon pair to the lowest perturbative order in the two ideal situations corresponding to the exchange of a single photon and of a single Z boson respectively. In both cases, consider two different initial conditions that correspond to the presence of right -handed (R) and left-handed (L) electrons in the initial beam.

1) Write the interaction Lagrangian in the four separate cases.

2) Consider the differential cross sections in the center of mass frame, for c.m. energies much larger than the muon mass, for the 4 different cases.

- Which expressions do you expect for the ratios of the left-handed to the right-handed differential cross sections, in the single photon and in the single Z cases? Why?

- What form of angular dependence (versus the cosine of the c.m. scattering angle θ) can exist for the 4 differential cross sections? Why?

3) Following the discussion (2), define at least one, possibly two, experimental observables of the process that will differ essentially in the two ideal cases, namely the exchange of a photon or a Z. Try to find two different c.m. energy values for which the real physical process can be treated to a good approximation, to the lowest perturbative order, as only due to either photon or Z exchange. Try to provide, in the two cases, a simple expression for these special observables and, possibly, stress the presence of relevant features.