Problem B. Implications of Discrete Symmetries in Elementary Particle Physics.

1. Consider the positronium atom (e^+e^-) at rest in a state $|e^+e^-; L, S>$ with orbital momentum L and spin S. The P- and C- parities of this state are respectively equal to $\eta_P(e^+e^-) = (-1)^{L+1}$ and $\eta_C(e^+e^-) = (-1)^{L+S}$:

$$U_P|e^+e^-; L, S>= \eta_P(e^+e^-)\ |e^+e^-; L, S>,$$

$$U_C|e^+e^-; L, S>= \eta_C(e^+e^-)\ |e^+e^-; L, S>,$$

where U_P and U_C are the unitary operators of space reflection and charge conjugation.

i) What are the constraints the charge conjugation symmetry of the electromagnetic interaction imposes on the electromagnetic decay of the positronium atom in a state $|e^+e^-; L, S>$ into n photons, $|e^+e^-; L, S>\rightarrow n\gamma$? What is the minimal value of n in the cases of S = 0 (parapositronium) and S = 1 (orthopositronium)?

ii) Consider the case of decay of positronium in a state with $L = 0$ into the minimal allowed number, n_{min}, of photons:

$$|e^+e^-; 0, S> \quad \rightarrow n_{min}\ \gamma.$$

What are the allowed values of the total spin S_γ and the total orbital momentum L_γ of the photons in the final state? (Use the constraints following from the space-reflection symmetry, angular momentum conservation and the fact that the photons are bosons.)

2. Consider the strong interaction mediated annihilation of a proton p and antiproton $\bar{p}$ into n pseudo-scalar π^0-mesons:

$$p + \bar{p} \rightarrow n\pi^0.$$

Denoting the total angular momentum and the total spin of the p–$\bar{p}$ system by $L_{p\bar{p}}$ and $S_{p\bar{p}}$, find for which values of $L_{p\bar{p}} \leq 2$ and $S_{p\bar{p}}$ annihilation into two π^0-mesons can take place. (Use the constraints following from the charge conjugation and space reflection symmetries of the strong interaction and the fact that $\eta_C(\pi^0) = 1$.)