Problem D. Elementary Particle Physics

Consider the decay of the pseudo-scalar π^0-meson into two real photons, generated by the electromagnetic interaction: $\pi^0(q) \to \gamma(k_1) + \gamma(k_2)$, where q, k_1 and k_2 are the 4-momenta of the π^0 and the two photons, $q^2 = m_\pi^2$, $k_1^2 = k_2^2 = 0$, m_π being the mass of the pion. The amplitude of the decay, $A(\pi^0 \to 2\gamma)$, can be written in the form:

$$A(\pi^0 \to 2\gamma) =< \gamma(k_1)\ \gamma(k_2) \mid S \mid \pi^0(q) >=$$

$$= e^2\ (2\pi)^{-9/2}\ (8k_{10}k_{20}q_0)^{-1/2}\ e_\mu^{\lambda_1}(k_1)\ e_\nu^{\lambda_2}(k_2)\ M^{\mu\nu}(k_1, k_2;\ q)\ (2\pi)^4\ \delta(k_1 + k_2 - q), \qquad (1)$$

where S is the S-matrix, e is the proton charge, $e_\mu^{\lambda_1}(k_1)$ and $e_\nu^{\lambda_2}(k_2)$ are the 4-vectors of polarization of the photons ($\lambda_i = 1, 2$), $M^{\mu\nu}(k_1, k_2;\ q)$ is the matrix element which contains the whole information about the dynamics of the decay, the $\delta-$function expresses the energy–momentum conservation law, and the other factors in (1) are simply normalization factors, k_{10}, k_{20} and q_0 being the energies of the two photons and the π^0-meson.

1. What are the constraints on $M_{\mu\nu}(k_1, k_2;\ q)$ following from:

i) identity of the two photons in the final state?

ii) Lorentz invariance and parity conservation?

iii) the gauge invariance of electromagnetic interaction?

2. Using the results obtained by solving the above problem and the energy–momentum conservation, construct the most general explicit expression for $M^{\mu\nu}(k_1, k_2;\ q)$. On how many constant parameters does it depend?

3. Exploiting the general properties of $M^{\mu\nu}(k_1, k_2;\ q)$ and of the photon polarization vectors $e_\mu^{\lambda_i}(k_i)$, $i = 1, 2$ one can reduce the differential rate of the $\pi^0 \to 2\gamma$ decay in the π^0 rest frame to the form:

$$d\Gamma(\pi^0 \to 2\gamma) = e^4\ \frac{1}{(2\pi)^2}\ \frac{1}{16m_\pi}\ \sum_{\mu,\nu=0}^{3} [M^{\mu\nu}(k_1, k_2;\ q)\ (M_{\mu\nu}(k_1, k_2;\ q))^*]\ \delta(k_1+k_2-q)\ \frac{d\mathbf{k}_1}{k_{10}}\ \frac{d\mathbf{k}_2}{k_{20}}, \qquad (2)$$

where the star * means complex conjugation and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the photons. Using eq. (2) and the expression for $M_{\mu\nu}(k_1, k_2;\ q)$ derived by solving problem 2. find the total $\pi^0 \to 2\gamma$ decay rate: $\Gamma(\pi^0 \to 2\gamma) = \int d\Gamma(\pi^0 \to 2\gamma)$. Try to estimate $\Gamma(\pi^0 \to 2\gamma)$.