Problem C. Stress-Energy Tensor of Quantum Field Theory between two planes

Consider two infinite planes which are parallel and at distance **a** apart along the z-axis. Suppose that a massless Quantum Field Theory is defined in the region between the two planes. The boundary conditions are such that the stress-energy tensor $T_{\mu,\nu}$ has a vacuum expectation value $t_{\mu,\nu}(t,\vec{x}) \equiv < 0 \mid T_{\mu,\nu}(t,\vec{x}) \mid 0 >$ different from zero. By virtue of the symmetry of this time-independent problem, $t_{\mu,\nu}$ must be written in terms of $g_{\mu,\nu}$ and tensors composed from the unit vector $\hat{z}_\mu = (0,0,0,1)$.

1. Write down the most general form of $t_{\mu,\nu}$ based upon based the considerations above.

2. Show that the conservation law $\partial^\mu T_{\mu,\nu}(t,\vec{x}) = 0$ and the condition that the trace of $T_{\mu,\nu}$ vanishes (which expresses the massless nature of the Quantum Field Theory) uniquely determine $t_{\mu,\nu}$ up to a constant. Use dimensional analysis to fix this constant up to a numerical factor in terms of a single parameter of the problem that bears a dimension.

3. Using the form of $t_{\mu,\nu}$ so obtained, compute the force per unit surface between the two planes.