Problem A. Angular Momentum and Spin

Let the unperturbed Hamiltonian of a spin 1/2 quantum rotator be

$$H_0 = \mathbf{L}^2 + \mathbf{S}^2$$

where $\mathbf{L}$ and $\mathbf{S}$ are the orbital angular momentum and spin operator, respectively.

1) Describe the spectrum (eigenvectors, eigenvalues and their degeneracies) of H_0.

2) Let us add to H_0 a spin–orbit coupling term

$$H_1 = \lambda \mathbf{L} \cdot \mathbf{S}$$

λ being a generic real number. Describe the spectrum (eigenvectors, eigenvalues and their degeneracies) of $H_0 + H_1$.