Problem B. Lagrangian Description of Scalar Particles

Consider two real scalar fields φ_1 and φ_2 with Lagrangian

$$L = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + b\,\partial_\mu\varphi_1\partial^\mu\varphi_2 - \frac{1}{2}a_1\varphi_1^2 - \frac{1}{2}a_2\varphi_2^2 - a_{12}\varphi_1\varphi_2\ . \tag{1}$$

1) Find a transformation $\varphi_1 = \varphi_1(\phi_1, \phi_2)$, $\varphi_2 = \varphi_2(\phi_1, \phi_2)$ which brings the Lagrangian to the form

$$L = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}A_1\phi_1^2 - \frac{1}{2}A_2\phi_2^2 - A_{12}\phi_1\phi_2\ . \tag{2}$$

Does the transformation exist for all values of the parameter b? Is it unique?

2) Depending on the value of b, how many propagating particles are described by the Lagrangian (1)? What are the masses of the particles described by the Lagrangian (2) when $A_1 = A_2 = \frac{3}{2}M^2$ and $A_{12} = -\frac{1}{2}M^2$?

3) What is the symmetry group of the Lagrangian (2) with the above values of A_1, A_2 and A_{12}? What is the symmetry group of the Lagrangian (2) when $A_1 = A_2 = M^2$ and $A_{12} = 0$?