Quantum Field Theory (Serone/Marzocca) **Exam Winter Session**

Academic Year 2019-2020 **19.12.2019**

Solve the following two/three exercises:

Exercise 1.

Discuss the notion of redundant operators.

Exercise 2.

Consider scalar QED in four space-time dimensions:

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - m^2 |\phi|^2 \,. \tag{1}$$

(a) Show that $\mathscr{L}$ is not power counting renormalizable and draw the leading order Feynman diagrams that lead to a new interaction $\delta\mathscr{L}$ in the Lagrangian.

(b) Compute the one-loop anomalous dimension of the field ϕ.

(c) Compute the one-loop anomalous dimension of the composite operator $|\phi|^2 = \phi^\dagger \phi$, considering only the contribution of electromagnetic interactions.

(d) What do you expect about the ξ-dependence of γ_ϕ and $\gamma_{\phi^\dagger\phi}$? Motivate your answer.

[The formula below might be useful:

$$\int \frac{d^d q_E}{(2\pi)^d} \frac{1}{(q_E^2 + a)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - d/2)}{\Gamma(n)} a^{d/2-n} \,. \qquad] \tag{2}$$

Exercise 3. *(Mandatory for TPP students only.)*

In the Standard Model baryon and lepton number, $U(1)_B \times U(1)_L$, are classically conserved accidental symmetries. Each generation of (left handed) fermions is given by these representations under $SU(3)_c \times SU(2)_L \times U(1)_Y$:

	q	u^c	d^c	ℓ	e^c
SM	$(3,2)_{\frac{1}{6}}$	$(\bar{3},1)_{-\frac{2}{3}}$	$(\bar{3},1)_{\frac{1}{3}}$	$(1,2)_{-\frac{1}{2}}$	$(1,1)_1$
B	$1/3$	$-1/3$	$-1/3$	0	0
L	0	0	0	1	-1

where we also show the values of baryon and lepton number for each fermion. Consider only one generation of fermions in what follows.

(a) Compute all anomalies of $X = B + \eta L$ with the SM gauge currents and find η such that all vanish.

(b) Assuming we wanted to gauge X, compute also the X^3 anomaly and the $X \times (\text{gravity})^2$ one. Find a fermion representation, to be added to the Standard Model, which makes them both vanish while avoiding all the others from point (a).