Quantum Field Theory (Serone/Gambassi) **Exam Winter Session**

Academic Year 2013-2014 **10.01.2014**

Solve the following two/three exercises:

Exercise 1.

Discuss the renormalization of quantum field theories, focusing in particular on how local counterterms can cancel divergencies.

Exercise 2. *(Scalar theory in six dimensions)*

Consider the Lagrangian density (with $\varepsilon = 6 - d$)

$$\begin{aligned}\mathscr{L} = &\frac{1}{2}Z_\phi \partial_\mu \phi \partial^\mu \phi - \frac{1}{2}Z_m m^2 \phi^2 - Z_g \frac{g\mu^{\varepsilon/2}}{3!}\phi^3 \\ &+ \frac{1}{2}Z_\sigma \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2}Z_M M^2 \sigma^2 - Z_\lambda \frac{\lambda \mu^{\varepsilon/2}}{2}\phi \sigma^2.\end{aligned} \tag{1}$$

(a) Discuss the discrete symmetries of this theory.

(b) In dimensional regularization, calculate Z_λ from the *divergent part* of the one-loop effective potential $V_{eff}(\phi_0, \sigma_0)$, neglecting g.
[Hint: $\ln(1+x) = x - x^2/2 + O(x^3)$ *and*

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 - X)^3} = -i\frac{a}{\varepsilon} + \textit{finite} \quad \textit{with} \quad a = \frac{1}{(4\pi)^3}]$$

(c) Knowing that

$$Z_\phi = 1 - \frac{a}{6}\frac{g^2 + \lambda^2}{\varepsilon} \quad \text{and} \quad Z_\sigma = 1 - \frac{a}{3}\frac{\lambda^2}{\varepsilon} \tag{2}$$

at one-loop level, determine the beta function of the coupling constant λ, again neglecting g.

(d)* Which is the minimal value $g_{min} = g(\lambda)$ that the coupling constant g can take such that it is stable under radiative corrections?

Exercise 3.* *(Composite operators)*

Calculate the anomalous dimension γ_{ϕ^2} of the operator ϕ^2 in the $\lambda\phi^4$ theory at one loop.

$$\textit{[Hint:} \quad \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 - X)^2} = i\frac{b}{\varepsilon} + \textit{finite} \quad \textit{with} \quad b = \frac{2}{(4\pi)^2} \quad \textit{and} \quad \varepsilon = 4 - d]$$

(*) Mandatory for TPP students only.