Quantum Field Theory (Serone/Gambassi) **Exam Summer Session**

Academic Year 2012-2013 **05.07.2013**

Solve the following three exercises:

Exercise 1.

Discuss the notion of renormalization group from the original point of view developed by Wilson.

Exercise 2.

Consider the chiral Lagrangian describing the low energy dynamics of the 3 pions π^+, π^- and π^0:

$$\mathscr{L}_{Kin} = \frac{f_\pi^2}{4}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right), \tag{1}$$

where U is the 3×3 matrix of the Nambu Goldstone pions, corresponding to the

$$SU(2)_L \times SU(2)_R \to SU(2)_V \tag{2}$$

spontaneous symmetry breaking pattern.

(a) Write the expession for U.

(b) Neglect in the underlying QCD the mass terms for the up and down quarks. What are in this limit the pions ? Find how the pion masses depend on the quark masses.

(c) Introduce the electromagnetic interactions in eq.(1) by gauging a $U(1)_V$ subgroup of $SU(2)_V$. Expand explicitly the Lagrangian (1) and check that the leading order terms have the expected form.

(d) Estimate the effect of the electromagnetic interactions on the pion masses by considering a one-loop graph with photon exchange. Give a numerical rough estimate of $\Delta m \equiv m_{\pi^+} - m_{\pi^0}$, neglecting the effect of the quark masses.

Hint: in (d) naively use cut-off regularization and identify Λ with $\Lambda_{QCD} \sim 1$ GeV.

Exercise 3.

Consider a theory of N real scalar fields $\{\psi_a\}_{a=1,\dots,N}$ and a real scalar field ϕ in d spatio-temporal dimensions,with Lagrangian[1]

$$\mathscr{L} = \sum_{a=1}^{N}\left[\frac{1}{2}\partial_\mu\psi_a\partial^\mu\psi_a - \frac{1}{2}M^2\psi_a^2\right] + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{2}\phi\sum_{a=1}^{N}\psi_a^2. \tag{3}$$

(a) Identify the symmetries of $\mathscr{L}$ and determine the dimensionality d_c at which the theory is renormalizable.

[1] This Lagrangian has a potential unbounded from below and cannot be physical. Forget about this issue and pretend it is physical, as well as its completion in point (b).

(b) Complete $\mathscr{L}$ by adding all the terms which are marginal at d_c and are compatible with its symmetries. Discuss both the case $N = 1$ and $N \neq 1$.

(c) For $N \neq 1$, calculate the finite non-analytic term in the one-loop effective potential for ϕ setting $\psi_a = m = 0$ in $\mathscr{L}$ and adding the terms discussed in (b), in dimensional regularization with $\varepsilon \equiv d_c - d$.

Hint: Possibly useful formulae:

$$a^{-1} = \int_0^\infty dt\, e^{-at}, \quad \Gamma(x) = \int_0^\infty dt\, e^{-t} t^{x-1} \tag{4}$$