

SISSA

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Connections Between the High and Low Energy Violation of  
Lepton and Flavor Numbers in  
the Minimal Left-Right Symmetric Model

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# Introduction

A theory behind neutrino masses has not yet been discovered. Soon after the existence of a neutrino was postulated in 1930 by Pauli, the young scientist Ettore Majorana in 1937 offered an alternative way to describe mathematically those particles which are absolutely neutral, i.e., those which are identical to their antiparticles [1]. An immediate phenomenological consequence that could distinguish them from the standard Dirac formulation was found soon after. If neutrinos are Majorana particles then double beta decay may occur without them having to be emitted [2]. This process known as neutrinoless double beta decay ( $0\nu\beta\beta$ ) has not yet been observed. Later on, Bruno Pontecorvo introduced the concept of neutrino oscillations [3] in analogy with the oscillations of neutral kaons and opened a new branch of physics. He understood that if neutrinos are massive they can oscillate among different flavors. It is thanks to neutrino oscillation experiments that we are able obtain information on neutrino masses and mixing, they have established that neutrino masses are small but not zero. For a historical review see [4]. The Standard Model (SM) [5] which describes massless neutrinos thus need to be updated. One natural extension of the SM, in which neutrinos are necessarily massive is given by the Left-Right models [6]. In these models a connection between the large scale of parity restoration and the smallness of neutrino masses can be made [7]. This is nowadays known as the seesaw mechanism [7, 8]. The left-right theories offer plenty of new phenomenology [9] but more important, they offer the possibility to access to the high energy analogous of  $0\nu\beta\beta$  [10]. The connection between the low energy and high energy manifestation of the same lepton number violation was recently addressed in [11] and is the main subject developed in this

thesis.

Before starting with the details of the Left-Right model we comment on the general features of the seesaw mechanism. One way to parametrize the masses of neutrinos is to add to the SM Lagrangian the non-renormalizable Weinberg operator [12]:

$$\mathcal{L}_5 = \frac{c_{\alpha\beta}}{2\Lambda} \ell_L^\alpha \ell_L^\beta \phi \phi \quad (1)$$

where  $\ell_L$  is the lepton doublet and  $\phi$  is the Higgs doublet,  $c_{\alpha\beta}$  is a constant matrix in the flavor space and  $\Lambda/c$  is the scale at which the neutrino masses are generated. The neutrino masses will be generated spontaneously through the Higgs mechanism [13] once the Higgs field takes a vacuum expectation value. The neutrino mass matrix will then be:

$$(M_\nu)_{\alpha\beta} = \frac{c_{\alpha\beta} v^2}{\Lambda} \quad (2)$$

where we have denoted  $v = \langle \phi \rangle$ . The coupling to the Higgs boson can also be obtained from the effective operator. In the mass basis of neutrinos it reads:

$$\mathcal{L}_{Y_\nu} = \frac{h}{\sqrt{2}v} m_{\nu_i} \nu_{iL} \nu_{iL} \quad (3)$$

where  $\phi^0 = v + h/\sqrt{2}$ . But due to the smallness of neutrino masses the direct test of these couplings does not seem to be realistic at present. The only hope to reveal the theory behind neutrino masses is to probe the scale  $\Lambda \sim c \times 10^{14}$  GeV. This can be done essentially in two different ways. Either by low energy high precision experiments or directly by searching new particle states with the help of high energy colliders. At the present time, we can hope to probe  $c$  as large as  $c \lesssim 10^{-10}$ .

The effective Weinberg operator can be realized at tree level in three different ways when one assumes only one kind of new particles. The new degrees of freedom can directly be associated with the scale of new physics. The three ways to do this are:

- Type I seesaw [7, 8]: It assumes the existence of right-handed neutrinos  $\nu_R$  with



zero hypercharge and singlets of  $SU(2)_L$ . These neutrinos are thus completely sterile under the SM gauge group. They are allowed then to have a non-zero mass term  $M_R$  at tree level, and a Dirac Yukawa coupling with the Higgs boson  $y_D \bar{\nu}_L \nu_R H$ . Then the scale of neutrino mass generation will be simply  $\Lambda/c = M_R/y_D^2$  and this means that  $M_R \sim y_D^2 \times 10^{14}$  GeV.

- Type II seesaw[14]: It assumes the existence of a scalar triplet  $\Delta_L = (\Delta_L^{++}, \Delta_L^+, \Delta_L^0)$  of  $SU(2)_L$  with hypercharge 2. They are allowed then to have a non-zero mass term  $M_\Delta$  at tree level, Yukawa coupling with the lepton doublet  $y_\Delta \ell_L \ell_L \Delta_L$ , and also a coupling with the Higgs doublet  $\mu \phi \phi \Delta_L^*$ . The scale of neutrino mass generation will then be  $\Lambda/c = M_\Delta^2/(\mu y_\Delta)$  and therefore  $M_\Delta \sim \sqrt{\mu y_\Delta} \times 10^7$  GeV.
- Type III seesaw[15]: It assumes the existence of a fermion triplet  $\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)$  of  $SU(2)_L$  with zero hypercharge. In analogy to Type I one has  $M_\Sigma \sim y_\Sigma^2 \times 10^{14}$  GeV.

Each of the seesaw types then provides us with a minimum number of fields needed to describe the masses of neutrinos. They have been studied extensively in the literature, for a detailed review see [16]. The type I seesaw has limited phenomenology because if one wishes to produce a  $\nu_R$  with the help of the Yukawa interaction, a large Yukawa coupling is needed, but this will make  $M_R$  out of the reach of any nearby experiment. Some cancellations in the flavor space or approximate lepton number symmetries [17] are also possible which can artificially allow large Yukawas and therefore increase the phenomenological domain of type I seesaw. The type II seesaw, on the contrary, is very attractive. A small enough  $\mu$  allows for a low scale of  $M_\Delta$  with large Yukawa couplings at the same time. From decay of the doubly charged scalar  $\Delta_L^{++}$  into dileptons, these  $y_\Delta$  couplings can be determined and can then be confronted with the mass matrix of light neutrinos [19]. The type III seesaw can also have interesting low scale collider phenomenology because of their gauge interactions [20].

We also address in this thesis a study of the structure of the seesaw mechanism and its connection with the Higgs mechanism. In the Left-Right model the neutrino masses will receive contributions from type I and type II seesaw. The nice feature of the model is

that the right-handed neutrinos cease to be sterile and become fully interacting particles, just like their left-handed counterpart. This leads, as we have said, to the remarkable lepton number violation signature in colliders [10]. Moreover the Yukawa coupling  $y_D$  to the Higgs boson, responsible for type I seesaw contribution turns out to depend only on few observables and is therefore deeply connected to the previous signal of lepton number violation [21].

In short, this work addresses the interconnection between the high energy collider phenomenology and the low energy processes such as  $0\nu\beta\beta$ , lepton flavor violation and the electromagnetic dipole moments of leptons.

# Chapter 1

## The Left-Right Model

The most attractive characteristic of the Left-Right symmetric models comes from the relation between the high and low mass scales of the theory. In the Standard Model parity is maximally violated being essentially the gauging of the V-A theory [22, 23]. Neutrinos are massless because there are only left-handed components charged under the SM gauge group. At first sight there is no relation between the absence of V+A charged currents and the absence of massive neutrinos. Today we know, thanks to neutrino oscillations, that neutrinos have small but not zero masses, and we have no reason to believe that V+A currents do not exist. So the brilliant idea behind the Left-Right models is the connection between the large scale of the V+A currents and the smallness of neutrino masses [7].

We should also keep in mind that the Left-Right model originally described Dirac neutrinos [6], and it was only after the observation made above that the interest in the Majorana nature came back to life once more. Somehow, we believe, the model went back on the right track to the historical developments that date back to 1937, when Majorana introduced the concept of a truly neutral particle in an attempt to describe the neutron and the neutrino.

The Left-Right model is based on the gauge group:

$$G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad (1.1)$$

At low energies, it is required that  $G_{LR}$  breaks spontaneously to the Standard Model gauge group:

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y \quad (1.2)$$

We expect three extra gauge bosons, two charged  $W_R^\pm$  analogous to the  $W^\pm$  boson (which will be denoted hereafter by  $W_L^\pm$  in order to keep the historical symmetric notation), and one neutral  $Z'$  analogous to the  $Z$  boson. The values of new the gauge couplings  $g_R$  and  $g_{B-L}$  are related to the known coupling  $g_Y$  by:

$$\frac{1}{g_R^2} + \frac{1}{g_{B-L}^2} = \frac{1}{g_Y^2} \quad (1.3)$$

thus their values are neither too small nor too large and it is wrong to believe that one may recover the SM by making  $g_R = 0$ . The electric charge generator is given by:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} \quad (1.4)$$

where  $T_{3L}, T_{3R}$  are the diagonal generators of  $SU(2)_L$  and  $SU(2)_R$ , and  $B-L$  is the baryon minus lepton number charge operator. The fermionic particle content is chosen symmetrically as doublet representations of  $SU(2)_L$  and  $SU(2)_R$ :

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \ell_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix}, \quad (1.5)$$

with  $B-L = -1$  for leptons and  $B-L = \frac{1}{3}$  for quarks. We are assuming implicitly the existence of three generations of leptons and quarks. The neutrino field  $N$  has not yet been observed. We believe that its mass is considerably larger than that of light neutrinos  $\nu$ , and for this reason they will be called heavy neutrinos.

## 1.1 The Left-Right symmetry.

The presence of right-handed currents allows us to reconsider the meaning of parity violation in the SM. With the enlarged gauge group and a symmetric particle content one can consider the possibility of having spontaneous violation of parity. In general this extra symmetry has to relate only the left-handed sector with the right handed one. There are two ways of introducing the symmetry: as generalized parity  $\mathcal{P}$  or as generalized charged conjugation  $\mathcal{C}$ . For fermions they coincide with the usual parity and charge conjugation. For gauge boson, they are chosen in such a way as to keep the gauge interactions invariant:

$$\mathcal{P} : \{W_L, q_L, \ell_L\} \leftrightarrow \{W_R, q_R, \ell_R\} \quad (1.6a)$$

$$\mathcal{C} : \{W_L, q_L, \ell_L\} \leftrightarrow \{-W_R^\dagger, (q_R)^c, (\ell_R)^c\} \quad (1.6b)$$

A consequence of these symmetries is the equality of gauge couplings:

$$g_L = g_R = g. \quad (1.7)$$

This can of course be affected by radiative correction. The couplings can be slightly different and formally (1.7) should be taken only as a good approximation.

### 1.1.1 Symmetry breaking.

The Left-Right symmetry forces us to have a symmetric particle content. The minimum set of scalar fields needed to accomplish the spontaneous symmetry breaking and to provide fermion masses at tree level are:

$$\begin{aligned} \text{Bidoublet: } & \Phi(1, 2, 2, 0), \\ \text{Triplets Left : } & \Delta_L(1, 3, 1, +2), \\ \text{Triplet Right : } & \Delta_R(1, 1, 2, +2). \end{aligned} \quad (1.8)$$

The bidoublet  $\Phi$  will be responsible for breaking the SM symmetry and giving mass to charged fermions. It is made out of two doublets  $\phi_1$  and  $\phi_2$  of  $SU(2)_L$  and at the same time the rows form two doublets of  $SU(2)_R$ . It is given by:

$$\Phi = \begin{pmatrix} \phi_1 & \phi_2^c \end{pmatrix} = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}. \quad (1.9)$$

The triplets will break the Left-Right symmetry as well as the  $B - L$  thereby giving Majorana mass to neutrinos. They belong to the adjoint representation of  $SU(2)_L$  and  $SU(2)_R$  and are given by:

$$\Delta_L = \begin{pmatrix} \frac{\Delta_L^+}{\sqrt{2}} & \Delta_L^{++} \\ \Delta_L^0 & -\frac{\Delta_L^+}{\sqrt{2}} \end{pmatrix}, \quad \Delta_R = \begin{pmatrix} \frac{\Delta_R^+}{\sqrt{2}} & \Delta_R^{++} \\ \Delta_R^0 & -\frac{\Delta_R^+}{\sqrt{2}} \end{pmatrix} \quad (1.10)$$

The scalar potential is the most general renormalizable potential made out of  $\Phi, \Delta_L$  and  $\Delta_R$ , invariant under  $\mathcal{G}_{LR}$  and the Left-Right symmetry. Under this last symmetry the transformation of the scalar fields follows from the invariance of the Yukawa interactions with leptons and quarks:

$$\mathcal{P} : \{\Phi, \Delta_L, \Delta_R\} \leftrightarrow \{\Phi^\dagger, \Delta_R, \Delta_L\} \quad (1.11a)$$

$$\mathcal{C} : \{\Phi, \Delta_L, \Delta_R\} \leftrightarrow \{\Phi^T, \Delta_R^*, \Delta_L^*\} \quad (1.11b)$$

This imposes additional constraints on the Yukawa couplings and on the parameters of the potential. The most general potential is given in Appendix 1. The minimum of the potential is achieved with:

$$\langle \Phi \rangle = \text{diag}(v_1, -v_2 e^{-ia}), \quad \langle \Delta_L^0 \rangle = v_L = |v_L| e^{i\theta_L}, \quad \langle \Delta_R^0 \rangle = v_R. \quad (1.12)$$

It is possible to keep only two vevs real by using the broken gauge symmetries. In our convention we take  $\langle \phi_1^0 \rangle$  and  $\langle \Delta_R^0 \rangle$  real and instead of  $v_1$  and  $v_2$  we will work with  $v$  and

$\beta$  defined as follows:

$$v_1 = v \cos \beta, \quad v_2 = v \sin \beta \quad (1.13)$$

The vev  $v_L$  will contribute to the masses of neutrinos and  $v_R$  to the scale associated with V+A currents. Therefore, the only physically acceptable vacuum must satisfy the following condition:

$$v_L \ll v \ll v_R. \quad (1.14)$$

The symmetry breaking can be understood in simple terms by imagining that the scale of Left-Right symmetry is much larger than the weak scale. This will allow us to leave aside the bidoublet and concentrate only on the scalar potential of the triplets:

$$\langle V_\Delta \rangle = -\mu^2(v_L^2 + v_R^2) + \lambda(v_L^4 + v_R^4) + \rho v_L^2 v_R^2 \quad (1.15)$$

or alternatively, up to a constant:

$$\langle V_\Delta \rangle = \lambda \left( v_L^2 + v_R^2 - \frac{\mu^2}{2\lambda} \right)^2 + (\rho - 2\lambda) v_L^2 v_R^2 \quad (1.16)$$

It is clear that when  $\rho - 2\lambda < 0$  the vacuum that minimize the potential is Left-Right symmetric:  $v_L = v_R$ . On the contrary, when  $\rho - 2\lambda > 0$  one of the vev's has to be necessarily zero, and thus the vacuum is asymmetric with  $v_L = 0$ ,  $v_R = \sqrt{\frac{\mu^2}{2\lambda}}$ . However, once the SM symmetry is broken a small value of  $v_L$  will be generated. In fact, adding the quartic coupling  $\Delta V \propto \text{tr} \Delta_L \Phi \Delta_R \Phi^\dagger = v_L v_R v^2$  to the potential (1.16) one finds  $v_L \propto \frac{v^2}{v_R}$ . This proves that the Left-Right symmetry can be spontaneously broken in fully agreement with (1.14) .

Once the symmetry is broken, the original neutral gauge fields will mix and produce

$Z', Z, A$ :

$$\begin{pmatrix} Z' \\ Z \\ A \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_W} & -s_{\theta_W} \\ 0 & s_{\theta_W} & c_{\theta_W} \end{pmatrix} \begin{pmatrix} c_{\theta_R} & 0 & -s_{\theta_R} \\ 0 & 1 & 0 \\ s_{\theta_R} & 0 & c_{\theta_R} \end{pmatrix} \begin{pmatrix} W_L^3 \\ W_R^3 \\ B_{B-L} \end{pmatrix} \quad (1.17)$$

The first rotation is necessary to identify  $Z'$  and  $B_Y$  and happens at the first stage when  $\mathcal{G}_{LR}$  breaks to  $\mathcal{G}_{SM}$ . The second rotation through the weak mixing angle  $\theta_W$  takes place when the  $\mathcal{G}_{SM}$  breaks to  $U(1)_{em}$  and serves to identify the photon  $A$  and the  $Z$  boson. The Left-Right symmetry implies that:

$$\sin \theta_R = \frac{\sin \theta_W}{\sqrt{\cos \theta_W}}. \quad (1.18)$$

The resulting gauge bosons will not be mass eigenstates and an additional rotation will be necessary. To reduce the clutter we skip writing down the mass matrices and present only well known results. For the charged gauge bosons we will need the following rotation:

$$\begin{pmatrix} W^- \\ W_R^- \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \xi^* \\ -\xi & 1 \end{pmatrix} \begin{pmatrix} W^- \\ W_R^- \end{pmatrix} \quad (1.19)$$

where the complex angle  $\xi = |\xi|e^{ia}$  contains the same phase of the bidoublet. For the neutral gauge bosons we will need:

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \zeta \\ -\zeta & 1 \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix} \quad (1.20)$$

with  $\zeta$  real. The gauge boson masses are given by:

$$M_W^2 \simeq \frac{1}{2}g^2v^2, \quad M_Z^2 \simeq \frac{M_W^2}{c_W^2}, \quad (1.21a)$$

$$M_{W_R}^2 \simeq g^2v_R^2, \quad M_{Z'}^2 \simeq \frac{2c_W^2}{c_W}M_{W_R}^2, \quad (1.21b)$$



and the gauge mixing angles by:

$$\sin \xi \simeq \frac{M_W^2}{M_{W_R}^2} \sin 2\beta, \quad \sin \zeta \simeq \frac{M_W^2}{M_{W_R}^2} \frac{c_{2W}^{\frac{3}{2}}}{2c_W^4}. \quad (1.22)$$

### 1.1.2 Gauge interactions of fermions

Using (1.17) one can write the gauge interaction of fermions in a familiar form:

$$\begin{aligned} \sum i\bar{\Psi}\gamma^\mu D_\mu \Psi &= \sum \bar{\Psi}i\partial \Psi - eA_\mu J_{em}^\mu - \frac{g}{c_{\theta_W}}(Z_\mu J_Z^\mu + \frac{c_{\theta_W}^2}{\sqrt{c_{2\theta_W}}}Z'_\mu J_{Z'}^\mu) \\ &- \frac{g}{\sqrt{2}}(W_{L\mu}^+ J_{W_L}^\mu + W_{R\mu}^+ J_{W_R}^\mu) - \frac{g}{\sqrt{2}}(W_{L\mu}^- J_{W_L}^{\mu\dagger} + W_{R\mu}^- J_{W_R}^{\mu\dagger}) \end{aligned} \quad (1.23)$$

where the fermionic currents are given by:

$$J_{em}^\mu = -\bar{e}\gamma^\mu e + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d \quad (1.24a)$$

$$J_Z^\mu = \sum \bar{\Psi}\gamma^\mu (T_L^3 - s_{\theta_W}^2 Q)\Psi \quad (1.24b)$$

$$J_{Z'}^\mu = \sum \bar{\Psi}\gamma^\mu \left(T_R^3 - \frac{s_{\theta_W}^2}{c_{\theta_W}^2}(Q - T_{3L})\right)\Psi \quad (1.24c)$$

$$J_{W_L}^\mu = \bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L \quad (1.24d)$$

$$J_{W_R}^\mu = \bar{N}_R\gamma^\mu e_R + \bar{u}_R\gamma^\mu d_R \quad (1.24e)$$

These currents will get mixed after rotating the gauge bosons to their mass eigenstates by means of (1.19) and (1.20). This will result in mixed interactions of the form  $W_{L\mu} J_{W_R}^\mu$ ,  $Z_\mu J_{Z'}^\mu$  and so on.

### 1.1.3 Yukawa interactions of leptons

The interactions between leptons and scalars are governed by the following Lagrangian:

$$L_\Phi = \bar{\ell}_L(y_1\Phi + y_2\Phi^c)\ell_R + \text{h.c.} \quad (1.25a)$$

$$L_\Delta = -\frac{1}{2}y_{\Delta_L}\bar{\ell}_L^c\epsilon\Delta_L\ell_L - \frac{1}{2}y_{\Delta_R}\bar{\ell}_R^c\epsilon\Delta_R\ell_R + \text{h.c.} \quad (1.25b)$$

Here  $\Phi^c = i\sigma_2\Phi^*i\sigma_2$ . The Yukawa matrices are given in term of the vevs (1.12) and (1.13) and the mass matrices as follows:

$$y_1 = -\frac{1}{vc_{2\beta}}(c_\beta M_D^\dagger + e^{ia}s_\beta M_\ell), \quad y_{\Delta_L} = \frac{M_L}{v_L} \quad (1.26a)$$

$$y_2 = \frac{1}{vc_{2\beta}}(e^{-ia}s_\beta M_D^\dagger + c_\beta M_\ell), \quad y_{\Delta_R} = \frac{M_R^*}{v_R} \quad (1.26b)$$

We can insert these relations back in (1.25a) to get a more physical insight. For this, we first notice that the bidoublet is made out of two  $SU(2)_L$  doublets  $\Phi = (\phi_1, \phi_2^c)$ . These fields are mixed and so it is desirable to work in the physical basis. For this let us call  $\varphi_1$  the field that takes the vev  $\langle\varphi_1\rangle = v$  and  $\varphi_2$  the field which does not and so  $\langle\varphi_2\rangle = 0$ . We can then write:

$$\varphi_1 = c_\beta\phi_1 + e^{-ia}s_\beta\phi_2, \quad \varphi_2 = -e^{ia}s_\beta\phi_1 + c_\beta\phi_2 \quad (1.27)$$

where we have introduced the shorthand notation  $c_\beta = \cos\beta$ , etc. Inserting (1.27) and (1.26) in (1.25a) we obtain:

$$\begin{aligned} L_\Phi = & -\bar{\ell}_L \left[ \frac{M_D^\dagger}{v}\varphi_1 - \frac{M_\ell + e^{-ia}s_{2\beta}M_D^\dagger}{vc_{2\beta}}\varphi_2 \right] N_R \\ & + \bar{\ell}_L \left[ \frac{M_\ell}{v}\varphi_1^c - \frac{M_D^\dagger + e^{ia}s_{2\beta}M_\ell}{vc_{2\beta}}\varphi_2^c \right] e_R + \text{h.c.} \end{aligned} \quad (1.28)$$

In the same way, we can expand (1.25b) and write:

$$\begin{aligned} L_\Delta = & -\frac{1}{2}\frac{M_L}{v_L}(\Delta_L^0\bar{\nu}_L\nu_L - \sqrt{2}\Delta_L^+\bar{\nu}_Le_L - \Delta_L^{++}\bar{e}_L^c e_L) \\ & -\frac{1}{2}\frac{M_R^*}{v_R}(\Delta_R^0\bar{N}_R N_R - \sqrt{2}\Delta_R^+\bar{N}_R e_R - \Delta_R^{++}\bar{e}_R^c e_R) + \text{h.c.} \end{aligned} \quad (1.29)$$

### 1.1.4 Yukawa interactions of Quarks

In the same way as before we rewrite the quarks bidoublet interactions analogously to (1.28):

$$L_\Phi = \overline{q_L} (y_1^q \Phi + y_2^q \Phi^c) q_R + \text{h.c.} \quad (1.30a)$$

$$= -\overline{q_L} \left[ \frac{M_u}{v} \varphi_1 - \frac{M_d + e^{-ia} s_{2\beta} M_u}{v c_{2\beta}} \varphi_2 \right] u_R \quad (1.30b)$$

$$+ \overline{q_L} \left[ \frac{M_d}{v} \varphi_1^c - \frac{M_u + e^{ia} s_{2\beta} M_d}{v c_{2\beta}} \varphi_2^c \right] d_R + \text{h.c.} \quad (1.30c)$$

### 1.1.5 The Left-Right symmetry and the mass matrices

The Yukawa couplings are not arbitrary complex matrices. The Left-Right symmetry imposes the following relations among them:

$$\mathcal{P} : \quad y_{1,2} = y_{1,2}^\dagger, \quad y_{\Delta_L} = y_{\Delta_R}, \quad y_{1,2}^q = y_{1,2}^{q\dagger} \quad (1.31a)$$

$$\mathcal{C} : \quad y_{1,2} = y_{1,2}^T, \quad y_{\Delta_L} = y_{\Delta_R}^*, \quad y_{1,2}^q = y_{1,2}^{qT} \quad (1.31b)$$

In the  $\mathcal{C}$  case we have symmetric quark and charged lepton matrices and also the Dirac neutrino mass matrix. In the  $\mathcal{P}$  case these matrices are almost hermitian. Hermiticity is lost because of the complex phase  $e^{ia}$  in the bidoublet vev.

We are interested in the relations resulting on the mixing matrices. In the quark sector, when diagonalizing the mass matrices  $M_u$  and  $M_d$  by an appropriate rotations of the quark fields we will encounter a mixing matrix in the charged current. We write the quark mass matrices in term of the mass eigenvalues:

$$M_u = U_L m_u U_R^\dagger, \quad M_d = D_L m_d D_R^\dagger \quad (1.32)$$

where  $m_u, m_d$  are the diagonal quark masses and  $U_L, U_R, D_L, D_R$  are the rotation matrices of the fields  $u_L, u_R, d_L, d_R$  necessary to achieve the diagonalization. The mixing matrices

that will appear in the charged V-A and V+A currents are:

$$V - A \text{ quark mixing : } V_L^q = U_L^\dagger D_L \quad (1.33a)$$

$$V + A \text{ quark mixing : } V_R^q = U_R^\dagger D_R \quad (1.33b)$$

Now when  $M_u, M_d$  are almost hermitian then  $U_R = U_L S_u$  and  $D_R = D_L S_d$ , with  $S_u, S_d$  diagonal sign matrices<sup>1</sup>. This implies that:

$$\mathcal{P} : V_R^q \simeq S_u V_L^q S_d \quad (1.34)$$

When  $M_u$  and  $M_d$  are symmetric we have  $U_R = U_L^* K_u^*$ ,  $D_R = D_L^* K_d$ , with  $K_u, K_d$  diagonal matrices of phases<sup>2</sup> and therefore:

$$\mathcal{C} : V_R^q = K_u (V_L^q)^* K_d \quad (1.35)$$

Analogously, in the lepton sector we will encounter leptonic mixing matrices when diagonalizing the mass matrices of charged leptons, light neutrinos and heavy neutrinos. The light neutrino mass matrix  $M_\nu$  is diagonalized by  $V_L$ , the heavy neutrino mass  $M_N$  by  $V_R$ , and we can write:

$$M_\nu = V_L^* m_\nu V_L^\dagger, \quad M_N = V_R m_N V_R^T \quad (1.36)$$

In the basis in which the charged leptons matrix is diagonal, the rotation matrices coincide with the leptonic mixing matrices:

$$V - A \text{ leptonic mixing : } V_L \quad (1.37a)$$

$$V + A \text{ leptonic mixing : } V_R \quad (1.37b)$$

Due to the nature of the seesaw formula which contains an extra matrix  $M_D$ , no connection

<sup>1</sup>For example,  $m_u = X m_u X$  has  $X = U_L^\dagger U_R = S_u$  as solution.

<sup>2</sup>The condition reduces to  $m_u = X^* m_u X$  and therefore  $X = U_L^T U_R = K_u^*$ .

between the left and right-handed leptonic mixing matrices follows directly from  $\mathcal{C}$  or  $\mathcal{P}$ .

## 1.2 Seesaw mechanism

The mass matrix of neutrinos is given in the  $(\nu_L, N_L)$  basis by the following  $6 \times 6$  matrix:

$$\begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} \quad (1.38)$$

By an appropriate rotation of the neutrino fields it can be brought to a block diagonal form. To do this, let us imagine that the mass scale of  $M_R$  is much larger than that of  $M_L$  and  $M_D$ . More precisely, we will only consider the first terms in the expansion in  $M_R^{-1}$  when performing the diagonalization. We thus expect the heavy neutrino mass at the large scale to be approximately unperturbed:

$$M_N = M_R \quad (1.39)$$

The rotation is then made through a “small” mixing matrix  $\Theta$  as follows:

$$\begin{pmatrix} \nu_L \\ N_L \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \Theta^\dagger \\ -\Theta & 1 \end{pmatrix} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} \quad (1.40)$$

The matrix  $\Theta$  is given by:

$$\Theta = M_N^{-1} M_D \quad (1.41)$$

and mass matrix of light neutrinos is given by the canonical seesaw formula:

$$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D \quad (1.42)$$

Recalling that  $\nu_L \propto v^2/v_R$ , we observe that both terms in the neutrino mass matrix are in general of the same order. Therefore the largeness of  $v_R$  ensures small neutrino masses.

This is known as the seesaw mechanism [7, 8]. But we remark that the scale of  $N$  may be well below the scale of  $W_R$ . The same is also true for  $M_D$ , which can be many orders of magnitude below  $M_W$ . Just like it happens in the standard model if we compare the mass of  $W$  with the mass of the electron. Meaning that Yukawas can be arbitrary small, and yet we do not understand how the mass hierarchies are generated. But this by no means lowers the stature of the seesaw mechanism. There is a large portion of the parameter space with low scale of heavy neutrinos, susceptible of experimental verification, that must be taken seriously.

### 1.2.1 Light-Heavy Neutrino mixing - Inverting the seesaw formula

The light neutrino mass matrix has almost been reconstructed by oscillation experiments. Only the mass scale of the lightest neutrino, their hierarchy and the CP violating phases need to be determined. The LHC offers the possibility to measure the heavy neutrino masses and right handed mixing matrix in the near future. We then may consider inverting the seesaw formula to hopefully obtain some knowledge of the Dirac mass matrix itself, and in turn predict the coupling of neutrinos to the Higgs boson.

In this section we show that, thanks to the Left-Right symmetry responsible for making  $M_D$  symmetric, the Dirac matrix can indeed be determined [21]. The left-right symmetry  $\mathcal{C}$  provides us with the following relations:

$$M_D = M_D^T \tag{1.43a}$$

$$M_L = \epsilon M_N \tag{1.43b}$$

where  $\epsilon$  has been defined as the ratio of the vevs of the triplets:

$$\epsilon = \frac{v_L}{v_R}. \tag{1.44}$$

Inserting (1.43) in the seesaw formula (1.42) and writing it in terms of  $\Theta$  we obtain:

$$M_\nu = M_N(\epsilon - \Theta^2) \quad (1.45)$$

Then we can invert this equation and solve for  $\Theta$  [21]:

$$\Theta = \sqrt{\epsilon - M_N^{-1}M_\nu} \quad (1.46)$$

The mixing matrix  $\Theta$ , which in general is undetermined [24], turns out to depend only on the parameter  $\epsilon$  and on the mass matrices of light and heavy neutrinos. We recall that the square root of a matrix has a certain number of discrete solutions and, only in some degenerate cases, the root may contain arbitrary coefficients. Nevertheless these arbitrariness should be regarded as unphysical. To understand what happened, we shall count the number of parameters before and after the diagonalization. We started with two complex symmetric matrices  $M_N$  and  $M_D$ , and with one complex parameter  $\epsilon$ . We ended with two complex symmetric matrices  $M_\nu$  and  $M_N$  and with one complex matrix  $\Theta$ . This last matrix  $\Theta$  can have only one free parameter  $\epsilon$  and not  $n^2$  if the degrees of freedom are to be conserved. The matrix  $\Theta$  found in (1.46) have this remarkable property. The Dirac mass matrix is given simply by:

$$M_D = M_N \sqrt{\epsilon - M_N^{-1}M_\nu} \quad (1.47)$$

The physical matrix that couples the mass eigenstates of light and heavy neutrinos to the Higgs boson is given by:

$$M_D^{\text{phys}} = V_R^\dagger M_D V_L \quad (1.48)$$

The remarkable fact is that once  $M_\nu$ ,  $M_N$  and  $\epsilon$  are measured, the coupling  $M_D^{\text{phys}}$  is predicted. This allows a verification of the seesaw mechanism.

### 1.2.2 Examples of $M_D$

The general procedure of extracting the root of a matrix is outlined in Appendix 2. Here we present two cases in which the Dirac mass matrix takes a simple form.

#### Case $2 \times 2$

In the  $2 \times 2$  case there exists an exact formula for the square root of a matrix<sup>3</sup>. In this case (1.47) reduces to:

$$M_D = \pm \frac{1}{\sqrt{\tau + 2\delta}} [(\epsilon + \delta)M_N - M_\nu], \quad (1.49)$$

with

$$\tau = \text{tr}(\epsilon - M_N^{-1}M_\nu), \quad \delta = \pm \sqrt{\det(\epsilon - M_N^{-1}M_\nu)}. \quad (1.50)$$

There are four solutions in total.

#### Case $V_R = V_L^*$

This choice of the leptonic mixing angles is analogous to what happens in the quark sector where, up to phases, the  $\mathcal{C}$  symmetry implies that  $V_R^q = V_L^{q*}$ . In the lepton sector this is not mandatory but is a clean example which we believe worth mentioning. Using (1.36) in (1.47) together with  $V_R = V_L^*$  it is easy to find<sup>4</sup>:

$$M_D = V_L^* m_N \sqrt{\epsilon - \frac{m_\nu}{m_N}} V_L^\dagger. \quad (1.51)$$

Each eigenvalue comes with a  $\pm$  sign in the front:

$$\pm \left\{ m_{N_1} \sqrt{\epsilon - \frac{m_{\nu_1}}{m_{N_1}}}, m_{N_2} \sqrt{\epsilon - \frac{m_{\nu_2}}{m_{N_2}}}, m_{N_3} \sqrt{\epsilon - \frac{m_{\nu_3}}{m_{N_3}}} \right\}, \quad (1.52)$$

---

<sup>3</sup>The square root of a  $2 \times 2$  matrix is given by  $\sqrt{A} = \pm_1 \frac{A \pm_2 \sqrt{\det(A)} I_{2 \times 2}}{\sqrt{\text{tr}(A) \pm_2 2\sqrt{\det(A)}}$ .

<sup>4</sup>The mixing matrix is  $\Theta = V_L \sqrt{\epsilon - \frac{m_\nu}{m_N}} V_L^\dagger$



In our convention  $m_{N_2}$  and  $m_{N_3}$  carry complex Majorana phases, and for this reason the  $\pm$  signs are not very important. Moreover, in this case the number of families is also irrelevant and we are dealing with a trivial generalization of the one dimensional case.

The physical coupling to the Higgs is diagonal:

$$(M_D^{\text{phys}})_{ij} = \delta_{ij} m_{N_j} s_j \sqrt{\epsilon - \frac{m_{\nu_j}}{m_{N_j}}} \quad (1.53)$$

with  $s_i = \pm$ . There are 8 solutions in total for three generations.

### 1.2.3 General Parametrization of $M_D$ .

In this section we comment about an attempt that was made to invert the seesaw formula in favor of the Dirac mass matrix. This was done for the cases in which  $M_D$  possesses no symmetries. Consider for simplicity the case in which  $M_L$  is set to zero: <sup>5</sup>

$$M_\nu = -M_D^T \frac{1}{M_N} M_D \quad (1.54)$$

The so called Casas-Ibarra parametrization of the Dirac mass matrix reads [24]:

$$M_D = iV_R \sqrt{m_N} O \sqrt{m_\nu} V_L^\dagger \quad (1.55)$$

with  $O$  an orthogonal complex matrix:

$$OO^T = 1. \quad (1.56)$$

Therefore (1.55) provides in general no unique solution but a family of  $M_D$  parametrized by three complex parameters in the orthogonal complex matrix  $O$ .

The point is that when  $M_D$  is symmetric, and we believe this is a well motivated consequence of the Left-Right symmetry, we have 3 additional complex relations which fix

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<sup>5</sup>The case with nonzero  $M_L$  was discussed in [25].

the orthogonal complex matrix completely. The seesaw formula leads us directly to:

$$M_D = iM_N \sqrt{M_N^{-1} M_\nu} \quad (1.57)$$

with only a finite number of discrete solutions.

### 1.3 Summary

In Table 1.1 we collect the interactions of leptons with gauge bosons and with physical scalar fields. In Table 1.2 we do the same for quarks. In Table 1.3 we write all the physical observables of the theory appearing in Table 1.1 and 1.2. In the next chapter part of the phenomenological consequences of these interactions, in particular the leptonic ones, will be discussed in detail.

leptons &	interaction Lagrangian
$W_L$	$-\frac{g}{\sqrt{2}} \left[ \bar{e}_L \not{W}_L V_L \nu_L + \bar{e} \not{W}_L (\Theta^\dagger V_R^*) \gamma_L N \right] + \text{h.c.}$
$W_R$	$-\frac{g}{\sqrt{2}} \bar{e}_R \not{W}_R V_R N_R + \text{h.c.}$
$Z$	$-\frac{g}{c_W} \left\{ \bar{e} \not{Z} \left[ \left(-\frac{1}{2} + s_W^2\right) \gamma_L + s_W^2 \gamma_R \right] e + \frac{1}{2} \bar{\nu}_L \not{Z} \nu_L \right\}$ $-\frac{g}{c_W} \frac{1}{2} \bar{\nu} \not{Z} (V_L^\dagger \Theta^\dagger V_R^*) \gamma_L N + \text{h.c.}$
$Z'$	$-\frac{g}{c_W \sqrt{c_{2W}}} \left\{ \bar{e} \not{Z}' \left[ \frac{1}{2} s_W^2 \gamma_L + \left(-\frac{1}{2} + \frac{3}{2} s_W^2\right) \gamma_R \right] e + \frac{s_W^2}{2} \bar{\nu}_L \not{Z}' \nu_L \right.$ $\left. + \frac{c_W^2}{2} \bar{N}_R \not{Z}' N_R - \frac{c_{2W}}{2} \left[ \bar{\nu} \not{Z}' (V_L^\dagger \Theta^\dagger V_R^*) \gamma_L N + \text{h.c.} \right] \right\}$
Higgs	$-\frac{H}{v} (\bar{e}_L m_\ell e_R + \bar{N}_R M_D^{\text{phys}} \nu_L) + \text{h.c.}$ $-\frac{H}{v} \left[ \bar{\nu} (V_L^T M_I V_L) \gamma_L \nu + \bar{N} (V_R^\dagger \delta M_N V_R^*) \gamma_L N \right] + \text{h.c.}$
neutral $\Delta_L, \Delta_R$	$-\frac{1}{2} \frac{1}{v_L} \left[ H_L^0 \bar{\nu} (V_L^T M_{II} V_L) \gamma_L \nu + i A_L^0 \bar{\nu} (V_L^T M_{II} V_L) \gamma_L \nu \right] + \text{h.c.}$ $-\frac{1}{2} \frac{m_N}{v_R} H_R^0 \bar{N} N$
$\Delta_L^+, \Delta_L^{++}, \Delta_R^{++}$	$\frac{1}{2} \frac{M_N}{v_R} \left[ \sqrt{2} \Delta_L^+ \bar{\nu} \gamma_L e + \Delta_L^{++} \bar{e}^c \gamma_L e \right] + \frac{1}{2} \frac{M_N^*}{v_R} \Delta_R^{++} \bar{e}^c \gamma_R e + \text{h.c.}$
neutral $\varphi_2$	$\frac{V_L^\dagger m_\ell V_R + e^{-ia} s_{2\beta} (M_D^{\text{phys}})^\dagger}{v c_{2\beta}} H_2^0 \bar{\nu}_L N_R + \text{h.c.}$ $\frac{V_L (M_D^{\text{phys}})^\dagger V_R^\dagger + e^{ia} s_{2\beta} m_\ell}{v c_{2\beta}} H_2^{0*} \bar{e}_L e_R + \text{h.c.}$
charged $\varphi_2$	$\frac{m_\ell V_R + e^{-ia} s_{2\beta} V_L (M_D^{\text{phys}})^\dagger}{v c_{2\beta}} H_2^- \bar{e}_L N_R + \text{h.c.}$ $-\frac{(M_D^{\text{phys}})^\dagger V_R^\dagger + e^{ia} s_{2\beta} V_L^\dagger m_\ell}{v c_{2\beta}} H_2^+ \bar{\nu}_L e_R + \text{h.c.}$

Table 1.1: Summary of the leptonic gauge and scalar interactions in the mass basis. Here  $M_I$  and  $M_{II}$  are the seesaw contributions to neutrino masses  $M_\nu = M_I + M_{II}$ . The matrix  $\delta M_N$  is the correction to the heavy right handed neutrino mass matrix  $\delta M_N = \frac{1}{2}(M_D \Theta^\dagger + \Theta^* M_D)$

quarks &	interaction Lagrangian
$W_L$	$-\frac{g}{\sqrt{2}}\bar{d}_L W_L V_L^q u_L + \text{h.c.}$
$W_R$	$-\frac{g}{\sqrt{2}}\bar{d}_R W_R V_R^q u_R + \text{h.c.}$
$Z$	$-\frac{g}{c_W}\bar{u}\not{Z}\left[\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\gamma_L-\frac{2}{3}s_W^2\gamma_R\right]u$ $-\frac{g}{c_W}\bar{d}\not{Z}\left[\left(-\frac{1}{2}+\frac{1}{3}s_W^2\right)\gamma_L+\frac{1}{3}s_W^2\gamma_R\right]d$
$Z'$	$-\frac{g}{c_W\sqrt{c_{2W}}}\bar{u}\not{Z}'\left[-\frac{1}{6}s_W^2\gamma_L+\left(\frac{1}{2}-\frac{7}{6}s_W^2\right)\gamma_R\right]u$ $-\frac{g}{c_W\sqrt{c_{2W}}}\bar{d}\not{Z}'\left[-\frac{1}{6}s_W^2\gamma_L+\left(-\frac{1}{2}+\frac{5}{6}s_W^2\right)\gamma_R\right]d$
Higgs	$-\frac{H}{v}(\bar{u}_L m_u u_R + \bar{d}_L m_d d_R) + \text{h.c.}$
netural $\varphi_2$	$\frac{V_L^{q\dagger} m_d V_R^q + e^{-ia} s_{2\beta} m_u}{vc_{2\beta}} H_2^0 \bar{u}_L u_R + \text{h.c.}$ $\frac{V_L^q m_u V_R^{q\dagger} + e^{ia} s_{2\beta} m_d}{vc_{2\beta}} H_2^{0*} \bar{d}_L d_R + \text{h.c.}$
charged $\varphi_2$	$\frac{m_d V_R^q + e^{-ia} s_{2\beta} V_L^q m_u}{vc_{2\beta}} H_2^- \bar{d}_L u_R + \text{h.c.}$ $-\frac{m_u V_R^{q\dagger} + e^{ia} s_{2\beta} V_L^{q\dagger} m_d}{vc_{2\beta}} H_2^+ \bar{u}_L d_R + \text{h.c.}$

Table 1.2: Summary of the quark gauge and scalar interactions in the mass basis.

$g$	gauge coupling
$M_W, M_{W_R}$	mass of $W$ and $W_R$
$v = \frac{\sqrt{2}M_W}{g}$	physical vev breaking the SM
$v_R = \frac{M_{W_R}}{g}$	physical vev breaking $SU(2)_R \times U(1)_{B-L}$
$V_L^q, V_R^q$	quark mixing matrices
$V_L, V_R$	leptonic mixing matrices
$m_u, m_d, m_\ell$	diagonal mass matrices of quarks and charged leptons
$M_\nu = V_L^* m_\nu V_L^\dagger$	light left-handed neutrino mass matrix
$M_N = V_R m_N V_R^T$	heavy right-handed neutrino mass matrix
$\Theta = \sqrt{\epsilon - M_N^{-1} M_\nu}$	light-heavy neutrino mixing matrix
$M_D^{\text{phys}} = V_R^\dagger M_N \Theta V_L$	higgs coupling to light and heavy neutrinos
$v_L = \langle \Delta_L^0 \rangle$	complex vev of $\Delta_L^0$
$\beta = \arctan \frac{\langle \phi_1 \rangle}{\langle \phi_2^c \rangle}$	$\beta$ angle of the doublets inside the bidoublet
$e^{ia}$	complex phase of $\langle \phi_2 \rangle$

Table 1.3: Summary of the physical observables appearing in Table 1.1 and Table 1.2

## Chapter 2

# Phenomenology

In this chapter we perform a study of the main phenomenological implications of the Left-Right model. We aim here to establish a connection between the high energy physics of colliders and the low energy physics of neutrinoless double beta decay, electromagnetic dipole moments and lepton flavour violating processes.

### 2.1 Measuring the scale of parity restoration

Colliders offer the best tool to access the new physics. The LHC can hopefully sit on the resonance of  $W_R$  as shown in Figure 2.1. Once produced,  $W_R$  will decay  $\sim 75\%$

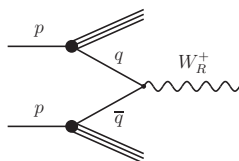


Figure 2.1: The production of  $W_R$  at the LHC.

into quarks and  $\sim 25\%$  into charge leptons and heavy neutrinos. The decay channel into heavy neutrinos is open only for those  $m_N < M_{W_R}$ . These Majorana fermions will in turn decay 50% into leptons and 50% into anti-leptons plus two jets. The same-sign lepton decay of  $W_R$  will signal the violation of lepton number by two units [10]. The corresponding Feynman diagram is shown in Figure 2.2. The KS and also the lepton

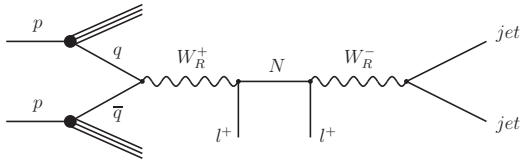


Figure 2.2: The Keung and Senjanovic production of same-sign charged lepton pairs.

flavour violating channel will allow the determination of the right handed leptonic mixing matrix  $V_R$ . Therefore a direct reconstruction of the heavy neutrino mass matrix  $M_N$  is promising. A full Monte Carlo simulation was done in [26]. The LHC at 14 TeV will be able to reach up to  $\sim 6$  TeV with an integrated luminosity of  $\sim 300 \text{ fb}^{-1}$ .

## 2.2 Testing the Higgs mechanism.

The scalar interactions allow us to test the multiple Higgs mechanism present in the theory. On one side we have the Majorana nature of heavy neutrinos with  $\Delta_R$  being the Higgs responsible for its mass generation and on the other hand we have the Dirac nature of charged leptons and quarks in which the masses are governed by the Standard Model Higgs boson. The light neutrino is particularly interesting because it is receiving, through the seesaw mechanism, contributions from both sides. In the following, we outline how to probe experimentally this interrelated Higgs and seesaw mechanism.

The Yukawa couplings of leptons to the left and right doubly-charged triplets are tied to the masses of the heavy neutrinos:

$$Y_L = Y_R^* = \frac{g}{M_{W_R}} V_R m_N V_R^T \quad (2.1)$$

The decay signatures of  $\Delta_L^{++}$  and  $\Delta_R^{++}$  into pairs of same-sign leptons will therefore provide information about the heavy neutrino mass matrix. If one could in principle measure all the channels into dilepton one would get 6 parameters  $(Y_\Delta)_{ij}$ . Combining this with the  $m_N$  and  $V_R$  from the same-sign charge lepton signal we could in the best scenario get information on the Majorana phases. In any case the confirmation of (2.1) will be a direct



test of the Majorana Higgs mechanism.

In the Dirac Higgs mechanism, it is a well know fact that the couplings of charged fermions to the Higgs boson are predicted to be proportional to the masses of the same fermions. The same would be true if we were dealing only with Dirac neutrinos. For Majorana neutrinos the coupling to the Higgs boson is essentially different. Nevertheless, in the left-right model this coupling is predicted, meaning that the seesaw mechanism by no means overshadows the Higgs mechanism [21]. This coupling  $M_D^{\text{phys}}$  was given in (1.48). Here we write it explicitly in term of neutrino masses and mixing:

$$M_D^{\text{phys}} = m_N V_R^T V_L \sqrt{\frac{g v_L}{M_{W_R}} - (V_R^T V_L)^\dagger \frac{1}{m_N} (V_R^T V_L)^* m_\nu} \quad (2.2)$$

It depends only on  $V_R, m_N$  and  $\epsilon$ . We have already seen that the first two can be obtained in principle from the KS signal and from the decay of the doubly charged scalars. The vev  $v_L = \langle \Delta_L \rangle$  can also be hunted in the colliders provided it is not too small. It parametrizes the strength of the interaction between  $\Delta_L^{++}$  and the  $W_L$  boson. With the predicted coupling of neutrinos to the Higgs boson we need only to wait for the experimental confirmation.

## 2.3 Experimental limits on particle masses

In order to have a picture on the mass scale of the particles that we will be dealing in the following sections, we present in Table 2.1 the experimental limits on particle content of the theory masses that arise from direct searches in the collider. The theoretical limit on the heavy doublet arises from the precision measurements on the mass difference of  $K_L$ - $K_S$  mesons.

## 2.4 Lepton flavour violation

If neutrinos were massless, we could have absorbed the leptonic mixing matrix by a redefinition of the neutrino fields. This could have been done by rotating the neutrino fields

Particle	Lower limit	ref.
$W_R$	2.9 TeV	[27]
$Z'$	3 TeV	[28]
$A_\phi^0$	$\sim 10$ TeV (Theory)	[29]
$H_\phi^0$	$\sim 10$ TeV (Theory)	[29]
$H_\phi^+$	$\sim 10$ TeV (Theory)	[29]
$A_L^0$	45 GeV	[28]
$H_L^0$	45 GeV	[28]
$\Delta_L^+$	70-90 GeV	[30]
$\Delta_L^{++}$	100-355 GeV	[31]
$H_R^0$	-	-
$\Delta_R^{++}$	113-251 GeV	[31, 32]

Table 2.1: Experimental limits on the particle masses adapted from [33].

in the same way charged leptons were rotated. In that case the individual flavor lepton number would be conserved. We thus see that flavour conservation depends only on whether neutrinos are massive particles or not. Today we know that neutrinos of different flavour oscillate into each other. This is interpreted as neutrinos being created as flavour eigenstates, instead of mass eigenstates. Neutrino oscillation experiments provide us with useful information about the left-handed leptonic mixing matrix  $V_L$  and the mass squared difference of light neutrinos  $\Delta m_\nu^2$  [16]. In Table 2.2 we collect the oscillation parameters obtained from recent experiments.

However, there are many other processes that violate the lepton flavor number. Table 2.3 shows the experimental limits on rare leptonic decay modes of the muon and tau leptons. Also Table 2.4 shows limits on LFV decay modes for light mesons and the  $Z$  boson. The most sensitive limits are given by  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e^+ee$ ,  $K_L^0 \rightarrow e^\pm\mu^\mp$ . This last one can be mediated by the heavy doublet at tree level, but we know that it has to have large mass from the limits on  $K_L-K_S$  mass difference. Thus the most relevant processes are provided by the leptonic rare decays of the muon.

	Central value	99% CL Range
$\sin^2 \theta_{12}$	0.320	$31.3^\circ < \theta_{12} < 37.5^\circ$
$\sin^2 \theta_{13}$	N : 0.0246, I : 0.0245	$7.5^\circ < \theta_{13} < 10.5^\circ$
$\sin^2 \theta_{23}$	N : 0.613, I : 0.600	$36.8^\circ < \theta_{23} < 55.5^\circ$
$\Delta m_{21}^2$ in $\text{eV}^2$	$7.62 \times 10^{-5}$	$(7.12 - 8.20) \times 10^{-5}$
$\Delta m_{31}^2$ in $\text{eV}^2$	N : $2.55 \times 10^{-3}$ , I : $2.43 \times 10^{-3}$	$(2.21 - 2.74) \times 10^{-3}$

Table 2.2: Oscillation parameters. Table adapted from [34]. There is a small difference between the normal (N) and the inverted (I) hierachy.

The contribution of light neutrino masses to  $\mu \rightarrow e\gamma$  was found to be [49]:

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{3\alpha}{32} \left| (V_L)_{ei} (V_L)_{\mu i}^* \frac{m_{\nu_i}^2}{M_W^2} \right|^2 \quad (2.3)$$

And the contribution to  $\mu \rightarrow e^+ee$  contains an additional factor of  $\alpha$ . Plugging numbers we see that both branching ratios are predicted to be smaller than  $10^{-50}$ . This is a general result, any contribution of light neutrinos in LFV other than oscillations can, most of the time, be safely be neglected.

The effective operator involved in  $\mu \rightarrow e\gamma$  is of dimension 5, and involves a generic coupling  $em_\mu G_F^2 M_W^2 / \Lambda^2$  with  $\Lambda$  the scale of new physics. The branching ratio is effectively given by:

$$B(\mu \rightarrow e\gamma)_{NP} \propto \alpha \frac{M_W^4}{\Lambda^4} \quad (2.4)$$

Meaning that the experiments on  $\mu \rightarrow e\gamma$  are probing the scale  $\Lambda \sim 20\text{TeV}$ . But of course if there is a loop involved the scale will be an order of magnitude smaller. In the same way, the effective operator of  $\mu \rightarrow e^+ee$  is of dimension 6 and we can write:

$$B(\mu \rightarrow e^+ee)_{NP} \propto \frac{M_W^4}{\Lambda^4} \quad (2.5)$$

And the experiments on  $\mu \rightarrow e^+ee$  are in principle probing the scale  $\Lambda \sim 80\text{TeV}$ . From

LFV process	Upper Limits on BR	Reference
$\mu \rightarrow e\gamma$	$2.4 \times 10^{-12}$	[39]
$\tau \rightarrow e\gamma$	$3.3 \times 10^{-8}$	[40]
$\tau \rightarrow \mu\gamma$	$4.4 \times 10^{-8}$	
$\mu \rightarrow e^+ee$	$1.0 \times 10^{-12}$	[38]
$\tau \rightarrow e^+ee$	$2.7 \times 10^{-8}$	[41]
$\tau \rightarrow e^+\mu\mu$	$1.7 \times 10^{-8}$	
$\tau \rightarrow e^+e\mu$	$1.8 \times 10^{-8}$	
$\tau \rightarrow \mu^+ee$	$1.5 \times 10^{-8}$	
$\tau \rightarrow \mu^+\mu\mu$	$2.1 \times 10^{-8}$	
$\tau \rightarrow \mu^+e\mu$	$2.7 \times 10^{-8}$	

Table 2.3: Leptonic LFV decay modes

this point of view, all the physics that generates the masses of light neutrinos will therefore be somehow restricted by these processes. In the next section we show how the interplay of the different scales of the model can produce small rates of LFV while allowing, at the same time, a low scale of  $W_R$  of no more than few TeV. It turns out that the  $\Lambda$  used above is in fact  $\Lambda \simeq \frac{m_S}{m_N} M_{W_R}$  where  $S$  is one of the scalar fields and  $N$  is a heavy neutrino. By having  $m_N$  below  $m_S$  we can generate an artificially large  $\Lambda$  and can therefore comply with the experimental constraints.

### 2.4.1 LFV decay modes of Leptons

#### Tree level decay $\mu \rightarrow e^+ee$

We begin by writing the effective Hamiltonian for the tree level exchange of  $\Delta_L^{++}$  and  $\Delta_R^{++}$ . From (1.29) one obtains the following effective four-fermions interaction:

$$H = c_{ijkl}^L \bar{\ell}_i^c \gamma_L \ell_j \bar{\ell}_k \gamma_L \ell_l^c + c_{ijkl}^R \bar{\ell}_i^c \gamma_R \ell_j \bar{\ell}_k \gamma_R \ell_l^c \quad (2.6)$$

LFV process	Upper Limits on BR	Reference
$\pi^0 \rightarrow e^+ \mu^-$	$3.4 \times 10^{-9}$	[42]
$\pi^0 \rightarrow e^- \mu^+$	$3.8 \times 10^{-10}$	[43]
$K^+ \rightarrow \pi^+ \mu^+ e^-$	$1.3 \times 10^{-11}$	[44]
$K^+ \rightarrow \pi^+ \mu^- e^+$	$5.2 \times 10^{-10}$	[42]
$K_L^0 \rightarrow e^\pm \mu^\mp$	$4.7 \times 10^{-12}$	[45]
$K_L^0 \rightarrow \pi^0 e^\pm \mu^\mp$	$7.6 \times 10^{-11}$	[46]
$Z \rightarrow e^\pm \mu^\mp$	$1.7 \times 10^{-6}$	[47]
$Z \rightarrow e^\pm \tau^\mp$	$9.8 \times 10^{-6}$	[47]
$Z \rightarrow \mu^\pm \tau^\mp$	$1.2 \times 10^{-5}$	[48]

Table 2.4: LFV decay modes of light mesons and the  $Z$  boson.

where the coefficients  $c_{ijkl}^{L,R}$  are found to be:

$$c_{ijkl}^L = \sqrt{2} G_F \frac{M_W^2}{M_{W_R}^2} \frac{(M_N^*)_{ij} (M_N)_{kl}}{m_{\Delta_L^{++}}^2} \quad (2.7)$$

$$c_{ijkl}^R = \sqrt{2} G_F \frac{M_W^2}{M_{W_R}^2} \frac{(M_N)_{ij} (M_N^*)_{kl}}{m_{\Delta_R^{++}}^2} \quad (2.8)$$

Assuming only that the mass of the decaying particle  $m_{\ell_1}$  is much larger than the masses of the decay products we obtain the following three-body decay rate:

$$\begin{aligned} \Gamma(\ell_j \rightarrow \ell_i^c \ell_k \ell_l) &= \frac{m_{\ell_j}^5}{768\pi^3} (|c_{ijkl}^L|^2 + |c_{ijkl}^R|^2) \\ &= \frac{G_F^2 m_{\ell_j}^5}{384\pi^3} \frac{M_W^4}{M_{W_R}^4} \left| \frac{(M_N)_{ij} (M_N)_{kl}}{m_{\Delta^{++}}^2} \right|^2 \end{aligned} \quad (2.9)$$

where  $m_{\Delta^{++}}^{-4} = m_{\Delta_L^{++}}^{-4} + m_{\Delta_R^{++}}^{-4}$ . For the muon only the decay  $\mu \rightarrow e^+ ee$  is kinematically allowed. For the tau there are six channels allowed  $\tau \rightarrow \{e^+ ee, e^+ e\mu, e^+ \mu\mu\}$  and those with  $e^+$  replaced by  $\mu^+$ . The strongest constraint will arise from  $\mu \rightarrow e^+ ee$ . The branching

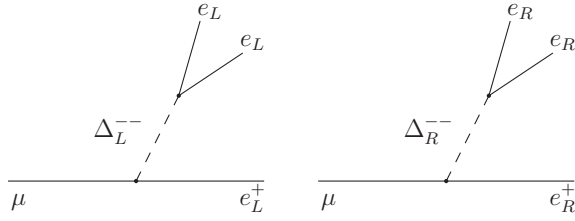


Figure 2.3: Feynman diagrams of  $\mu \rightarrow e^+ee$

ratio normalized to the standard muon decay is given by:

$$\text{BR}(\mu \rightarrow e^c ee) \simeq 10^{-7} \left( \frac{3.5 \text{TeV}}{M_{W_R}} \right)^4 \left| (V_R)_{\mu i} (V_R)_{ei} (V_R)_{ej} (V_R)_{ej} \frac{m_{N_i} m_{N_j}}{m_{\Delta^{++}}^2} \right|^2 \quad (2.10)$$

When  $M_{W_R}$  is not more than few TeV, there are only three ways that can make this branching ratio small enough to comply with the experimental constraint given in Table 2.3:

$$\text{BR}^{\text{exp}}(\mu \rightarrow e^+ ee) < 10^{-12} \quad (2.11)$$

For example, an almost diagonal right-handed mixing matrix  $V_R \simeq 1$ , or an almost degenerate heavy neutrino spectra  $m_{N_1} \simeq m_{N_2} \simeq m_{N_3}$  are enough to make (2.10) vanish. But far more interesting is when the ratio  $m_{N_i}/m_{\Delta^{++}}$  is small itself. Indeed, for  $M_{W_R}$  at the reach of LHC, the following ratio:

$$\frac{m_{N_i}}{m_{\Delta^{++}}} \lesssim \frac{1}{10} \quad (2.12)$$

will ensure that for any  $V_R$  and any heavy neutrino hierarchy the branching ratio agrees with the experimental limit given in Table 2.3.

### Radiative decay $\mu \rightarrow e\gamma$

We now calculate the radiative decay rate of  $\ell_1 \rightarrow \ell_2 \gamma$  with the help of [50] and update the formulas presented in [51] by including the mixing of light and heavy neutrinos. The relevant Feynman diagrams are shown in Figure 2.4. The effective Hamiltonian reads as follows:

$$H_{\ell_1 \rightarrow \ell_2 \gamma} = \bar{\ell}_2 \sigma_{\mu\nu} [(\sigma_{LL} + \sigma_{LR})_{21} \gamma_R + (\sigma_{RR} + \sigma_{RL})_{21} \gamma_L] \ell_1 F^{\mu\nu} \quad (2.13)$$

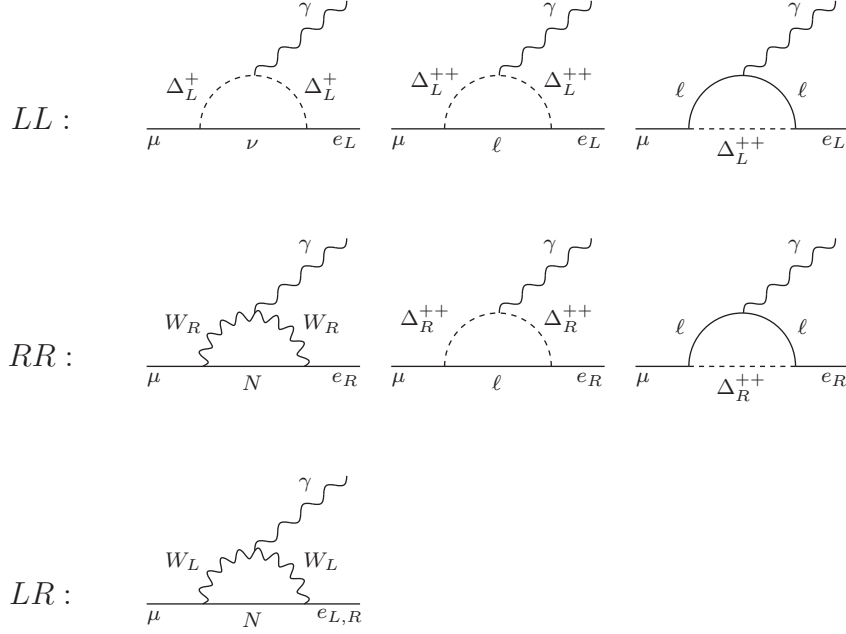


Figure 2.4: Feynman diagrams of  $\mu \rightarrow e\gamma$ .

and total decay rate is given by the following generic formula:

$$\Gamma(\ell_1 \rightarrow \ell_2 \gamma) = \frac{m_{\ell_1}^3}{16\pi} \left( |(\sigma_{LL} + \sigma_{LR})_{21}|^2 + |(\sigma_{RR} + \sigma_{RL})_{21}|^2 \right) \quad (2.14)$$

The matrices  $\sigma_{LL}$ ,  $\sigma_{LR}$ ,  $\sigma_{LR}$  and  $\sigma_{RL}$  are found to be:

$$\sigma_{LL} = m_{\ell_1} \frac{eG_F}{4\sqrt{2}\pi^2} \frac{M_W^2}{M_{W_R}^2} \left( \frac{1}{24} \frac{M_N^* M_N}{m_{\Delta_L^+}^2} + \frac{1}{3} \frac{M_N^* M_N}{m_{\Delta_L^{++}}^2} \right) \quad (2.15a)$$

$$\sigma_{RR} = m_{\ell_1} \frac{eG_F}{4\sqrt{2}\pi^2} \frac{M_W^2}{M_{W_R}^2} \left( \frac{1}{8} \frac{M_N M_N^*}{M_{W_R}^2} + \frac{1}{3} \frac{M_N M_N^*}{m_{\Delta_R^{++}}^2} \right) \quad (2.15b)$$

$$\sigma_{LR} = m_{\ell_1} \frac{eG_F}{4\sqrt{2}\pi^2} \left( \Theta^\dagger V_R^* F_1(t) V_R^T \Theta - \xi^* \Theta^\dagger V_R^* F_2(t) V_R^T \frac{M_N^*}{m_{\ell_1}} \right) \quad (2.15c)$$

$$\sigma_{RL} = m_{\ell_1} \frac{eG_F}{4\sqrt{2}\pi^2} \left( |\xi|^2 V_R^* F_1(t) V_R^T - \frac{M_N}{m_{\ell_1}} V_R^* F_2(t) V_R^T \xi \Theta \right) \quad (2.15d)$$

where the loop functions entering in  $\sigma_{LR}$  and  $\sigma_{RL}$  are:

$$F_1(t) = \frac{-4t^3 + 45t^2 - 33t + 10}{12(t-1)^3} - \frac{3t^3 \log t}{2(t-1)^4} \quad (2.16)$$

$$F_2(t) = \frac{t^2 - 11t + 4}{2(t-1)^2} + \frac{3t^2 \log t}{(t-1)^3} \quad (2.17)$$

where  $t = m_N^2/M_W^2$ . Moreover, as we are interested in a low scale of heavy neutrinos, we have taken the limit  $m_N \ll M_{W_R}$  and simplified an extra loop function present in the gauge component of  $\sigma_{RR}$ . The branching ratio  $\mu \rightarrow e\gamma$  normalized to the standard muon decay can be expressed ignoring square of  $\xi$  and  $\Theta$  and after some simplifications as:

$$\text{BR}(\mu \rightarrow e\gamma) \simeq 10^{-9} \left( \frac{3.5\text{TeV}}{M_{W_R}} \right)^4 \left( \left| \frac{1}{3} \frac{M_N^* M_N}{m_L^2} - e^{-ia} s_{2\beta} \frac{M_D^\dagger}{m_\mu} V_R F_2(t) V_R^\dagger \right|_{e\mu}^2 + \left| \frac{1}{3} \frac{M_N M_N^*}{m_R^2} - e^{ia} s_{2\beta} V_R F_2(t) V_R^\dagger \frac{M_D}{m_\mu} \right|_{e\mu}^2 \right) \quad (2.18)$$

where we have shortened the expression defining:

$$\frac{1}{m_L^2} = \frac{1}{24} \frac{1}{m_{\Delta_L^+}^2} + \frac{1}{3} \frac{1}{m_{\Delta_L^{++}}^2}, \quad \frac{1}{m_R^2} \simeq \frac{1}{8} \frac{1}{M_{W_R}^2} + \frac{1}{3} \frac{1}{m_{\Delta_R^{++}}^2} \quad (2.19)$$

The mass of  $\Delta_L^+$  can not be much lower than the mass of  $\Delta_L^{++}$ . To see this we refer to the Appendix A. From Table A.1 the following mass difference can be obtained:

$$M_{\Delta_L^{++}} - M_{\Delta_L^+} = \frac{\alpha_3}{2} v^2 \cos 2\beta \quad (2.20)$$

where  $\alpha_3$  is a dimensionless parameter of the scalar potential. This means that during the symmetry breaking of  $\mathcal{G}_{SM}$  the masses of the scalar triplet will get split. Coming back to (2.19), we can safely ignore the singly charged scalar contribution. Moreover the scale of the doubly charged scalar  $\Delta_R^{++}$  is phenomenologically less constrained and can therefore be much lighter than that of the right-handed gauge boson  $W_R$ . We thus follow



$X$	Au	Al	Ti	Pb	Cu	S
$B(\mu X \rightarrow eX) <$	$7 \times 10^{-13}$	-	$4.3 \times 10^{-12}$	$4.6 \times 10^{-11}$	$1.6 \times 10^{-8}$	$7 \times 10^{-11}$

Table 2.5: Limits on muon conversion.

the scenario with light  $\Delta_R^{++}$ . From now on, we will always use approximately:

$$\frac{1}{m_L^2} \simeq \frac{1}{3} \frac{1}{m_{\Delta_L^{++}}^2}, \quad \frac{1}{m_R^2} \simeq \frac{1}{3} \frac{1}{m_{\Delta_R^{++}}^2}, \quad (2.21)$$

in the branching ratio (2.18).

By comparing this branching ratio (2.18) with that of  $\mu \rightarrow e^+ee$  found in the previous section in (2.10) we see that roughly the branching ratio of  $\mu \rightarrow e\gamma$  is expected to be smaller than that of  $\mu \rightarrow e^+ee$  by a factor of  $10^{-2}$  because of the loop suppression. The rate of both processes is controlled mainly by the mass of  $W_R$ , and secondly by the lightest of the doubly charged particles  $\Delta_R^{++}, \Delta_L^{++}$ . Both branching ratios have a strong flavor dependence on the right-handed mixing matrix, with the right handed neutrino mass matrix controlling the flavor structure. The process  $\mu \rightarrow e^+ee$  is controlled by  $(M_N)_{e\mu}(M_N)_{ee}$  whereas  $\mu \rightarrow e\gamma$  is controlled by  $(M_N M_N^\dagger)_{e\mu}$  which has the extra feature of being independent of the Majorana phases. Interestingly the Dirac mass matrix contribution is relevant when the following generic relation is satisfied:

$$\frac{m_D}{m_\mu} \sim \frac{m_N^2}{m_{\Delta^{++}}^2} \quad (2.22)$$

Thus, for the parameter spaces accessible to the LHC (for example  $m_N \sim 100$  GeV and  $m_{\Delta^{++}} \sim 1$  TeV), a Dirac mass  $m_D$  of the order few MeV would be on the border limit.

#### 2.4.2 Muon-electron conversion

The lepton flavor can also be violated in the process  $\mu + X \rightarrow e + X$  [53], where  $X$  is a given muonic atom. The muon instead of decaying naturally into an electron, neutrino and antineutrino, gets transformed into an electron. The observation of an electron with

fixed energy  $m_\mu - B$ , with  $B$  the binding energy of the muonic atom, will signal the muon conversion and the violation of lepton flavor number. Table 2.5 shows the experimental upper limits on the branching ratio normalized to the standard muon capture in the same nuclei.

In the left-right model the process can be mediated by the photon or by any of the two heavy neutral gauge bosons  $Z, Z'$ . The photonic interaction involves on-shell photons which are contained in the electric field of the atom [54]. Their contribution to amplitude will be the same as we used before for  $\mu \rightarrow e\gamma$  and the rate will contain an extra factor  $\alpha$ . Therefore these photonic on-shell interactions will not give any new constraint. The interesting contribution then comes from off-shell photonic interactions. It turns out that the contribution of the doubly charged particles  $\Delta_{L,R}^{++}$  is always logarithmically enhanced [55]:

$$\text{enhancement factor} \sim \log \frac{m_\mu^2}{m_{\Delta^{++}}^2} \quad (2.23)$$

Figure 2.5 shows the corresponding Feynman diagrams. This enhancement allows one to ignore the contribution of the  $Z$  boson<sup>1</sup> and focus only on the doubly charged particles. The decay rate for a general effective Hamiltonian can be found in [52]. In our case we have for the branching ratio normalized with respect to the capture rate the following expression [51]:

$$B(\mu N \rightarrow e N) = \frac{512\pi\alpha^2}{3} \frac{\Gamma_\mu}{\Gamma_{\text{cap}}} \frac{(V^{(p)})^2}{m_\mu^5} \frac{M_W^4}{M_{W_R}^4} \left( \log \frac{m_\mu^2}{m_{\Delta^{++}}^2} \right)^2 \left| \frac{M_N M_N^*}{m_{\Delta^{++}}} \right|_{e\mu}^2 \quad (2.24)$$

where  $V^{(p)}$  (of dimension  $[m]^{5/2}$ ) takes into account the effect of the electric charge density of the proton on the outgoing electron, and  $\Gamma_{\text{cap}}$  is the capture rate. These values can be found tabulated for every element in [52]. For example, for Au we have  $V_{\text{Au}}^{(p)} = 0.0974 m_\mu^{5/2}$  and  $\Gamma_{\text{cap}}^{\text{Au}} \simeq 6 \Gamma_\mu$ . Taking the enhancement in the region of phenomenologically interest

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<sup>1</sup>However they were calculated in [51]

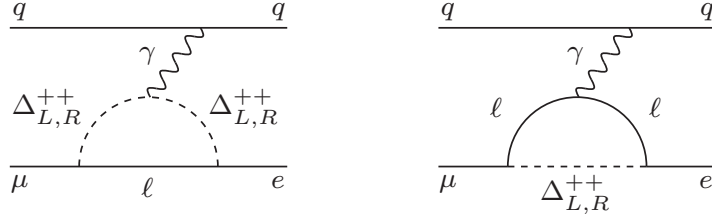


Figure 2.5: Feynman diagrams of muon conversion.

$\log^2 \sim 2 \times 10^2$  one finds:

$$B(\mu Au \rightarrow e Au) \simeq 5 \times 10^{-10} \left( \frac{3.5 \text{ TeV}}{M_{W_R}} \right)^4 \left| \frac{M_N M_N^*}{m_{\Delta}^{++}} \right|_{e\mu}^2 \quad (2.25)$$

The muon conversion mediated by the doubly charged scalars is therefore slightly more constrained than the decay  $\mu \rightarrow e\gamma$ . Both branching ratios are theoretically of the same order because the additional factor of  $\alpha$  in the muon conversion gets balanced by the large logarithm coming from the loop, but at the same time the limits on the conversion are roughly one order of magnitude stronger.

## 2.5 Interplay between different LFV processes

Lepton flavor violation could vanish if the right-handed mixing matrix is nearly diagonal. So a definite prediction can not be made and the different rates depend crucially on  $V_R$ . For a generic  $V_R$  one could also have a particular mass spectrum of right-handed heavy neutrinos which could make the rate of one or more processes vanish as well. The interesting processes with high sensitivity are  $\mu \rightarrow e^+ee$ ,  $\mu \rightarrow e\gamma$ , and  $\mu$ - $e$  in the nuclei. The common flavor dependence of  $\mu \rightarrow e\gamma$  and  $\mu$ - $e$  conversion is given by:

$$(M_N M_N^*)_{e\mu} = \Delta m_{12}^2 c_{13} s_{12} (c_{12} c_{23} - e^{-id} s_{12} s_{13} s_{23}) + e^{-id} \Delta m_{13}^2 s_{13} c_{13} s_{23} \quad (2.26)$$

with  $\Delta m_{12}^2$ ,  $\Delta m_{13}^2$  the mass squared difference of the heavy neutrinos and with  $s_{12} = \sin \theta_{12}^R$ , etc. This factor can vanish for some  $\Delta m_{13}^2$  and  $\Delta m_{12}^2$ . For  $\mu \rightarrow e^+ee$  one have

LFV process	Future sensitivity	ref.
$\mu \rightarrow e^+ ee$	$10^{-15}$	Mu3e
$\mu \rightarrow e\gamma$	$10^{-15}$	Meg
$\mu Al \rightarrow eAl$	$10^{-17}$	Meco

Table 2.6: Upcoming LFV experiments.

additionally the freedom of the Majorana phases and they can easily make the rate vanish as well. So without the masses  $m_N$  we can not constrain our parameter space either. The problem is that we have only three competitive processes. We could have done more if we would have the three radiative decay of  $\ell_1 \rightarrow \ell_2\gamma$  and the seven three body decay  $\ell_i \rightarrow \ell_j\ell_k$  with similar experimental sensitivity. They would make 10 constraints on 3 masses  $m_{N_i}$ , 3 angles and 4 phases in  $V_R$ . But this scenario is far from being realistic. Table 2.6 shows some future experiments and their experimental reach. When combining this future prospects from LHC searches on  $W_R$  and the heavy neutrino mass matrix we could then translate the LFV limits into quantitative constraints on the scalar masses.

The prospects on processes  $\mu \rightarrow e\gamma$  and  $\mu$ - $e$  in the nuclei are particularly encouraging. The ratio of both processes can be used to get useful information on the masses of the doubly charged scalar particle. One has for doubly charged contributions the following ratio [51]:

$$R_X = \frac{B(\mu X \rightarrow eX)}{B(\mu \rightarrow e\gamma)} = c_X \times \alpha \times \frac{\frac{1}{m_{\Delta_L^{++}}^4} \left( \log \frac{m_\mu^2}{m_{\Delta_L^{++}}^2} \right)^2 + \frac{1}{m_{\Delta_R^{++}}^4} \left( \log \frac{m_\mu^2}{m_{\Delta_R^{++}}^2} \right)^2}{\frac{1}{m_{\Delta_L^{++}}^4} + \frac{1}{m_{\Delta_R^{++}}^4}} \quad (2.27)$$

where  $c_X$  depends only on the particular muonic atom:

$$c_X = \frac{\Gamma_\mu}{\Gamma_{\text{cap}}} \frac{(32\pi V^{(p)})^2}{m_\mu^2} \quad (2.28)$$

The value of  $c_X$  is shown for some interesting atoms in Table 2.7. The ratio  $R_X$  is therefore

$X$	Au	Al	Ti	Pb	Cu	S
$c_X$	3.3	1.7	2.8	2.4	2.7	1.9

Table 2.7:  $c_X$  defined in the text basically governs the ratio  $B(\mu X \rightarrow eX)/B(\mu \rightarrow e\gamma)$ .

predicted by the theory to be close to one. Its precise value will then permit to probe the scalar sector of the theory and can be used to complement collider searches in the hunt for the doubly charged particles.

## 2.6 Neutrinoless double beta decay

Apparently Majorana himself was thinking about an experiment that could distinguish “truly” neutral particles, which are identical to its antiparticles, from “normal” neutral particles which have distinct antiparticles [4]. At that time the only known electrically neutral particles were the photon, the neutron and the hypothetical neutrino. It was clear the photon having only real components was a truly neutral particle in that sense. Now we know that the neutron is made out of three electrically charged quarks, and therefore is not equal to the antineutron. The question whether or not the neutrino is a “truly” neutral particle is still open. The experimental idea came shortly after, if neutrinos are Majorana particles then double beta decay may occur without the emission of neutrinos [2].

The standard double beta decay, can only be seen in those nuclei in which single beta decay is kinematically forbidden. It has been experimentally observed for many nuclei:  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{100}\text{Mo}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ,  $^{150}\text{Nd}$ , etc. The outgoing electrons have a continuum energy spectrum due to the presence of two antineutrinos. The sum of their energies are less than the available energy  $Q$  which is of the order of few MeV. Table 2.8 shows the double beta decay half-life and the current experimental limits on  $0\nu\beta\beta$  for some interesting nuclei.

The contribution of light neutrino masses to the  $0\nu\beta\beta$  decay rate is proportional to

	$^{76}\text{Ge}$	$^{82}\text{Se}$	$^{100}\text{Mo}$	$^{130}\text{Te}$	$^{136}\text{Xe}$	$^{150}\text{Nd}$
$\frac{T_{2\nu\beta\beta}^{1/2}}{10^{21}\text{yr}}$	1.74 [56]	0.096 [57]	0.007[58]	0.7 [59]	2.11 [60]	0.009 [61]
$\frac{T_{0\nu\beta\beta}^{1/2}}{10^{25}\text{yr}}$	2.2 [62]	0.036 [63]	0.11 [63]	0.013 [59]	0.006 [64]	0.002 [61]

Table 2.8: Double beta decay half-life and limits on neutrinoless double beta decay

the 1-1 element of the neutrino mass matrix  $(M_\nu)_{ee}$  and can be written as follows:

$$\mathcal{A}_\nu \propto G_F^2 \frac{(M_\nu)_{ee}}{k^2} \quad (2.29)$$

where  $k \sim 100\text{ MeV}$  is of the order of the Fermi momentum of the nucleus and may be considered a measure of the neutrino virtuality. The absolute value of  $(M_\nu)_{ee}$  is plotted in Figure 2.6 as a function of the lightest neutrino mass [65]. The indeterminacy of the Majorana phases is causing the slight dispersion of the points. Current experimental constraints on the decay rate translate generically into [16]:

$$(M_\nu)_{ee} \lesssim \text{eV} \quad (2.30)$$

Recently a strong limit from searches on  $^{136}\text{Xe}$  has been reported in [66]:

$$(M_\nu)_{ee} \lesssim (0.140 - 0.380)\text{eV} \quad (2.31)$$

up to the uncertainty of the nuclear matrix elements.

One may attempt to draw some premature conclusions from Figure 2.6 without considering possible new physics effects. For example, one may conclude that the hierarchy of light neutrino masses can be probed by  $0\nu\beta\beta$ . Future experiments on  $0\nu\beta\beta$  with sensitivity less than  $0.1\text{ eV}$  will then be probing the inverted hierarchy. The normal hierarchy still needs to wait for better sensitivity and in some cases the rate may even vanish completely because of freedom on the Majorana phases. So the hope for a future observation relies

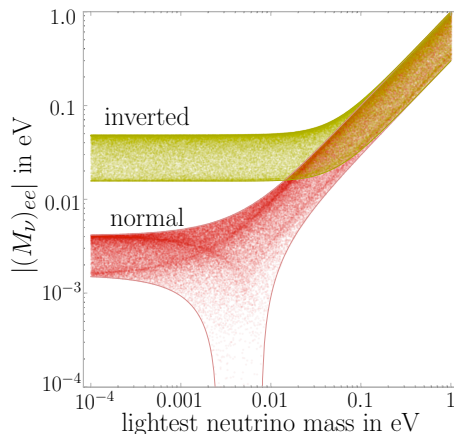


Figure 2.6: The canonical contribution from light neutrino masses. The mixing angles are fixed at  $\{\theta_{12}, \theta_{23}, \theta_{13}\} = \{35^\circ, 45^\circ, 9^\circ\}$ , while the Dirac and Majorana phases vary in the interval  $[0, 2\pi]$ .

mainly on the inverted hierarchy mass spectrum.

Nevertheless, cosmology is pushing from above the sum of all neutrino masses [68], which can be translated into the lightest neutrino mass less than  $\sim 0.1$  eV. This is in apparent contradiction with the recent limit (2.31). There is also another claim that  $0\nu\beta\beta$  decay have been seen for  $^{76}\text{Ge}$  [67] and that it corresponds to  $(M_\nu)_{ee} \simeq 0.4\text{eV}$ . Together these three facts can be interpreted as the manifestation of new physics.

In any case, in a complete theory of neutrino masses all the conclusions one can draw from Figure 2.6 are not necessary valid. The simplest addition of heavy right handed neutrinos, for example, through the heavy Majorana mass matrix can also produce sizeable rates of  $0\nu\beta\beta$  as was noticed in [69], some years before the seesaw mechanism was discovered. In short, Figure 2.6 represents only one part of the story.

In general all the physics behind the generation of light Majorana neutrino masses, provided its scale is not so high, can cause  $0\nu\beta\beta$ . We can estimate the size of this scale. The operator contributing to the decay rate is of dimension 9, and therefore the new physics will produce the following amplitude:

$$\mathcal{A}_{\text{NP}} \propto G_F^2 \frac{M_W^4}{\Lambda^5} \quad (2.32)$$

where  $\Lambda$  is the scale of new physics. When the scale  $\Lambda$  is close to the TeV scale, the new physics starts competing with the contribution of light neutrinos. In the following section we analyse the rate produced by the Left-Right model. Specially attractive is the right-handed neutrino contribution, analogous to the left-handed neutrinos, which goes through the  $W_R$  boson instead of the  $W_L$  boson. In this case the scale  $\Lambda$  will split into the scale  $M_{W_R}$  and  $m_N$  and the theory produces for  $m_N$  of few GeV considerable rates of  $0\nu\beta\beta$ .

### 2.6.1 Decay Rate

One can write the effective Hamiltonian of  $0\nu\beta\beta$  in the following form<sup>2</sup>:

$$H_{\text{eff}} = -\frac{G_F^2}{k^2} \left( m_{LL}^{ee} \bar{e}_L e_L^c J_L^2 - m_{RR}^{ee} \bar{e}_R e_R^c J_R^2 - m_{LR}^{ee} \bar{e}_L \gamma^\mu \gamma^\alpha \gamma^\nu e_R^c J_L^\mu \hat{k}^\alpha J_R^\nu \right) \quad (2.33)$$

Here  $J_L, J_R$  denote the left- and right-handed quark currents respectively and  $k$  is a constant vector needed to get the right dimension. It is expected to be of the order  $\sim 100$  MeV as it is describing the virtuality of light neutrinos. The constant  $k$  has been factorized for convenience in order to have all the coefficients normalized to the canonical contribution. The diagrams contributing to the effective Hamiltonian are shown in Figure 2.7.

We have omitted few diagrams which are irrelevant. For example, the mixing  $\xi$  between  $W_L$ - $W_R$  does not play any significant role for the substitution of a heavy  $W_R$  by a light  $\xi W_L$  will only constraint  $\sin 2\beta$  to be smaller than one. This is because the mixing is already predicted to be small  $\xi \simeq M_W^2/M_{W_R}^2 \sin 2\beta$ . We have neglected the contributions coming from the heavy doublet [70] because the limits on its mass have to be larger than 10 TeV [71]. The heavy neutrino masses were assumed to lie above the Fermi momentum of the nucleus which is around 100 MeV, and so the momentum dependence of their propagator can be neglected. Nevertheless, it can be restored at the end if needed [72].

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<sup>2</sup>Strictly speaking one should not write effective interactions for light neutrinos which are propagating. But this step will be justified in a moment.



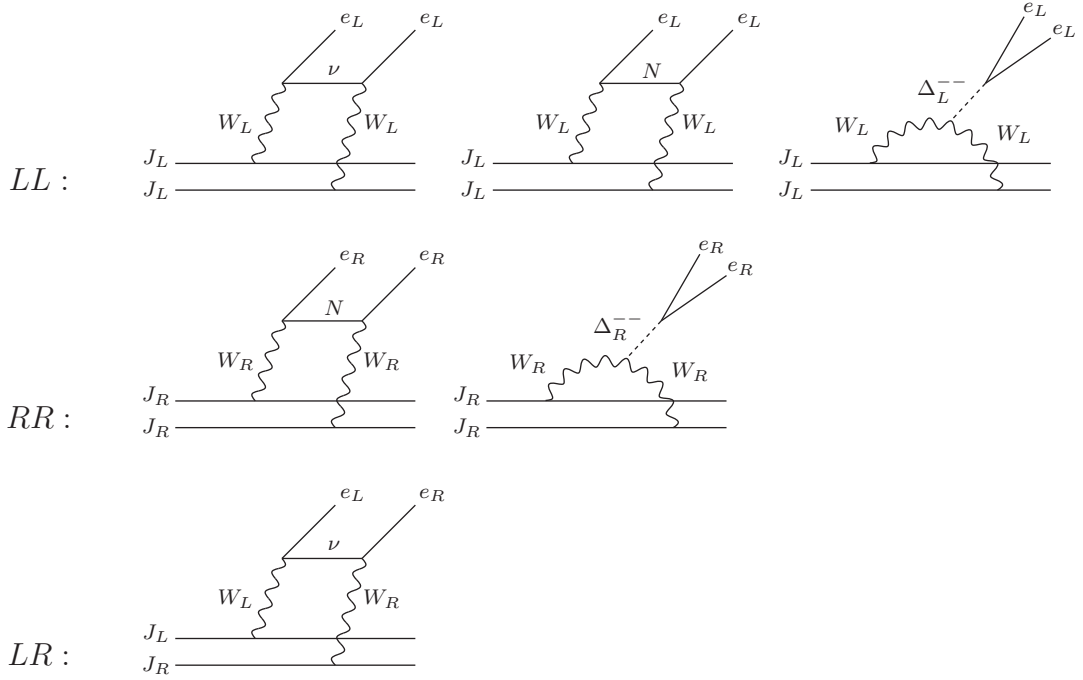


Figure 2.7: Feynman diagrams contributing to the amplitude of  $0\nu\beta\beta$ . As shown in the text the  $m_{LL}$  amplitude is dominated by light neutrinos. The  $RR$  and the  $LR$  amplitude can separately dominate the decay rate.

A straightforward calculation gives the following  $m_{LL}, m_{RR}$  and  $m_{LR}$  matrices:

$$m_{LL} = M_\nu^* - \Theta^\dagger \frac{k^2}{M_N} \left( 1 + 2 \frac{M_N M_N^*}{m_{\Delta_L^{++}}^2} \right) \Theta^* \quad (2.34a)$$

$$m_{RR} = \frac{M_W^4}{M_{W_R}^4} \frac{k^2}{M_N^*} \left( 1 + 2 \frac{M_N^* M_N}{m_{\Delta_R^{++}}^2} \right) \quad (2.34b)$$

$$m_{LR} = 2 \frac{M_W^2}{M_{W_R}^2} k \Theta^\dagger \quad (2.34c)$$

We remind here that  $M_\nu, M_N$  and  $\Theta$  are the light neutrino mass matrix, the heavy neutrino mass matrix and the light-heavy neutrino mixing matrix respectively. The effective Hamiltonian (2.33) contains only the  $ee$  element of the matrices (2.34). The  $m_{LL}$  amplitude is composed of three contributions. The first one is the canonical contribution due to the Majorana masses of light neutrinos. The second happens when each light neutrino is replaced by a heavy neutrino. The third contribution is due to the doubly charged left

triplet. The  $m_{RR}$  amplitude has heavy neutrinos coupled to the right-handed gauge boson in complete analogy to the canonical contribution. It also contains the contribution of the doubly charged right triplet. The  $m_{LR}$  amplitude occurs when a heavy neutrino is converted into a light neutrino.

In deriving (2.34) we did not take into account that the light neutrino masses are generated by the seesaw mechanism. The seesaw formula (1.42) with  $M_D = M_N \Theta$  reads:

$$M_\nu = \epsilon M_N - \Theta^T M_N \Theta \quad (2.35)$$

Inserting this equation in the amplitude  $m_{LL}$  we get:

$$m_{LL} = \epsilon^* M_N^* - \Theta^\dagger \left( 1 + \frac{k^2}{M_N^* M_N} + 2 \frac{k^2}{m_{\Delta_L^{++}}^2} \right) M_N^* \Theta^* \simeq M_\nu^* \quad (2.36a)$$

The last equality is valid when the masses  $m_{N_i}$  are not exactly of order  $k$ .<sup>3</sup>

In order to proceed with the decay rate, we need to have the value of the following nuclear matrix elements between the initial and the final nucleus:

$$\left\langle \frac{J_L^2}{p^2} \right\rangle, \left\langle J_R^2 \right\rangle, \left\langle J_L^\mu J_R^\nu \frac{p^\alpha}{p^2} \right\rangle \quad (2.37)$$

where the momentum  $p$  is implicitly being integrated. We define formally the value of  $k$  as the following ratio of matrix elements [11]:

$$k^2 = \langle J_R^2 \rangle \div \left\langle \frac{J_L^2}{p^2} \right\rangle \quad (2.38)$$

This  $k$  is the same in (2.33), physically it is expected to be of the order of the Fermi momentum of the nucleus. In Table 2.9 we took the matrix elements available in the literature and checked that the ratio (2.38) is indeed giving the right momentum. It turns

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<sup>3</sup>The case in which the heavy sterile neutrinos contribute substantially to the  $m_{LL}$  amplitude through the light-sterile neutrino mixing [73] requires independent choices of the mixing matrix  $\Theta$  (or equivalently of  $M_D$ ) and the heavy neutrino mass matrix  $M_N$  in order to make  $\Theta^T M_N \Theta$  smaller than  $k^2 \Theta^T M_N^{*-1} \Theta$ . In our case, the left-right symmetry takes over such arbitrariness with  $\Theta$  given by (1.46).

	ref.	<sup>76</sup> Ge	<sup>82</sup> Se	<sup>100</sup> Mo	<sup>130</sup> Te	<sup>136</sup> Xe	<sup>150</sup> Nd
$k$ in MeV	[75]	99	94	207	102	132	97
	[76]	190	186	189	180	280	210
	[77]	184	-	193	198	-	-
$G \mathcal{M}_\nu ^2$ in $\frac{1}{10^{25} \text{ yr} \times \text{eV}^2}$	[75]	4.5	17.3	8.2	21.4	4.7	310
	[76]	4.2	16.5	7.7	20.4	4.6	290
	[77]	10.8	-	60.8	48.8	-	-
$Q$ in MeV	[16]	2.039	2.995	3.034	2.529	2.476	3.367

Table 2.9: Nuclear factors relevant for  $0\nu\beta\beta$ .

out that the nuclear matrix element involving left and right-handed currents is suppressed, see for example [78]. Meaning that the naive estimation is off by one order of magnitude.

The decay rate can then be written as follows:

$$\Gamma = G \cdot |\mathcal{M}_\nu| (|m_\nu^{ee}|^2 + |m_N^{ee} + m_\Delta^{ee}|^2 + |m_\Theta^{ee}|^2) + \text{interference terms} \quad (2.39)$$

where  $G$  is a phase factor,  $|\mathcal{M}_\nu|$  is the nuclear matrix element relevant for the light neutrino exchange, and with the amplitudes given by:

$$m_\nu^{ee} = (V_L^2)_{ie} m_{\nu_i} \quad (2.40a)$$

$$m_N^{ee} = \frac{M_W^4}{M_{W_R}^4} (V_R^2)_{ie} \frac{k^2 m_{N_i}}{k^2 + m_{N_i}^2} \quad (2.40b)$$

$$m_\Delta^{ee} = 2 \frac{M_W^4}{M_{W_R}^4} (V_R^2)_{ie} \frac{k^2 m_{N_i}}{m_{\Delta_R^{++}}^2} \quad (2.40c)$$

$$m_\Theta^{ee} = 2 \frac{M_W^2}{M_{W_R}^2} k \Theta_{ee}^* \quad (\text{suppressed by } \sim 10^{-1}) \quad (2.40d)$$

The values  $G|\mathcal{M}_\nu|$  for different nuclei are also shown in Table (2.38) together with the available kinetic energy  $Q$ . The interference terms [79] are small due to the difference in

chirality of the outgoing electrons. They are suppressed by a factor between  $m_e/Q$  and  $m_e^2/Q^2$ . The generic bounds arising from the limits on the total decay rate can then be translated from (2.31):

$$|(M_\nu)_{ee}| \lesssim 0.3 \text{ eV} \quad (2.41)$$

$$|(M_N)_{ee}| \gtrsim 10 \text{ GeV} \times \left( \frac{3.5 \text{ TeV}}{M_{W_R}} \right)^4 \quad (2.42)$$

$$|\Theta_{ee}| \lesssim 4 \times 10^{-5} \times \left( \frac{M_{W_R}}{3.5 \text{ TeV}} \right)^2 \quad (2.43)$$

From this last limit on the element  $\Theta_{ee}$  one can get a rough bound on the elements of the Dirac mass matrix:

$$m_D < \text{MeV} \times \frac{m_N}{100 \text{ GeV}} \times \left( \frac{M_{W_R}}{3.5 \text{ TeV}} \right)^2 \quad (2.44)$$

In the case of having a nearly diagonal  $V_R \sim 1$ , we could then allow  $m_N/m_{\Delta_R^{++}}$  to be large and at the same time keep the rates of LFV processes under control. The contribution of  $\Delta_R^{++}$  becomes important, specially when the heavy neutrino masses are close to  $M_{W_R}$ . So for  $V_R \sim 1$  we have additionally:

$$m_{\Delta_R^{++}} > 500 \text{ GeV} \times \left( \frac{3.5 \text{ TeV}}{M_{W_R}} \right)^2 \times \sqrt{\frac{m_{N_i}}{3 \text{ TeV}}} \quad (2.45)$$

## 2.6.2 Implication for the LR scale

The mass of  $W_R$  can not be arbitrarily large if the new physics is to generate substantial rates of  $0\nu\beta\beta$ . Here we aim to find maximum value of  $M_{W_R}$  that could do that [80]. We start by analysing the contribution of the doubly charged scalar  $\Delta_R^{++}$ :

$$m_\Delta^{ee} = 2 \frac{M_W^4}{M_{W_R}^4} (V_R^2)_{ie} \frac{k^2 m_{N_i}}{m_{\Delta_R^{++}}^2} \quad (2.46)$$

It can only be dominant when the heavy neutrino masses  $m_{N_i}$  are large. But the LFV processes  $\mu \rightarrow e^+ee$  and  $\mu \rightarrow e\gamma$  are proportional to  $m_N/m_{\Delta_R^{++}}$  and, as we have seen, when  $m_N$  is large the only way to satisfy the experimental constraints is to have a nearly diagonal

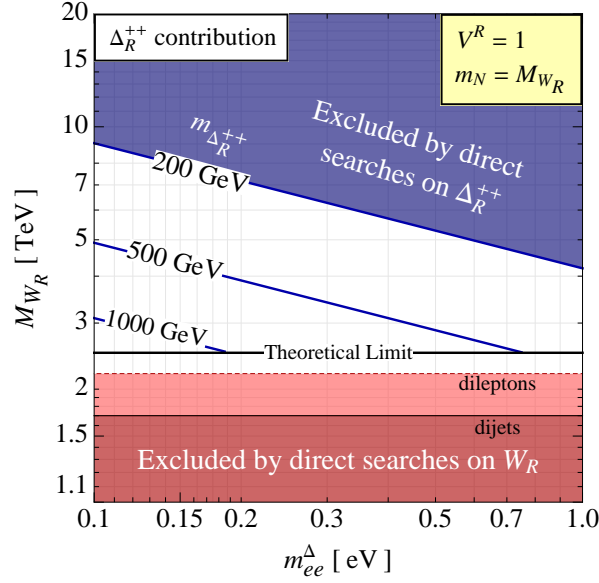


Figure 2.8: Upper bound from  $0\nu\beta\beta$  due to the doubly charged scalar  $\Delta_R^{++}$

right-handed mixing matrix. Figure 2.8 shows the contribution to the  $m_{\Delta}^{ee}$  amplitude for different values of  $m_{\Delta_R^{++}}$  and for a fixed value of  $m_N = M_{W_R}$  and with  $V_R$  diagonal. The limit  $m_{\Delta_R^{++}}$  follows from direct collider searches and is expected to increase in the near future, see Table 2.1. If this is the case, and if the direct searches in the collider continue pushing up the limits on  $M_{W_R}$ , we can see that the contribution of  $\Delta_R^{++}$  will lose space and may get ruled out before any evidence of  $0\nu\beta\beta$ . In any case this contribution is complementary to the contribution of the right-handed heavy neutrinos. The scale of the right-handed boson in any case can not exceed the 5-9 TeV if the rate is to be larger than 0.1eV.

The contribution of the heavy neutrinos becomes important when their masses are close to  $k \simeq 100$  MeV:

$$m_N^{ee} = \frac{M_W^4}{M_{W_R}^4} (V_R^2)_{ie} \frac{k^2 m_{N_i}}{k^2 + m_{N_i}^2} \quad (2.47)$$

For masses lower and larger than  $k$  the amplitude decreases and two values of  $m_N$  can give the same amplitude. Figure 2.9 shows the contribution of one  $N$  to the  $m_N^{ee}$  amplitude for the  $(V_R)_{eN} = 1$  which gives the most conservative bound on  $M_{W_R}$ . The lower regime in which  $m_N$  is lower than  $\sim 140$  MeV is disfavored by cosmology as it is explained below.

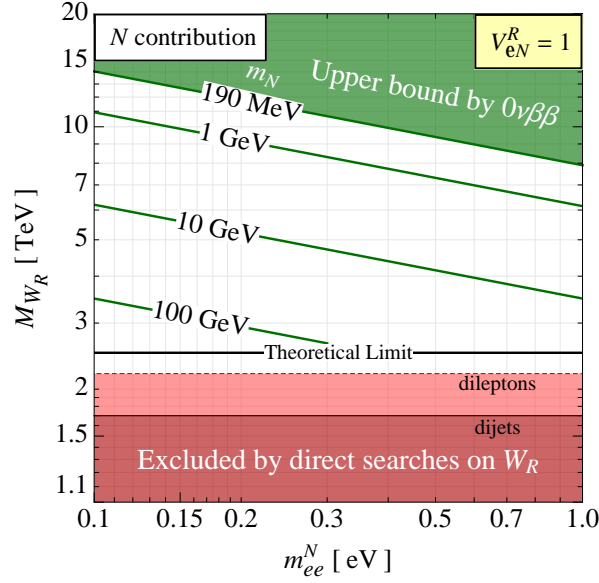


Figure 2.9: Upper bound from  $0\nu\beta\beta$  due to the heavy right handed neutrino  $N$

From the figure, we can see that an  $M_{W_R}$  of no more than  $\sim 10$  TeV can still produce large contribution of  $0\nu\beta\beta$ . However, there is a strong dependence in the mass hierarchy and the mixing matrix  $V_R$  analogous to the flavor dependence of the light neutrino contribution. In principle this rate could vanish itself even with low scale of  $m_N$  and  $M_{W_R}$ .

The contribution due to the light and heavy mixing, is less eloquent, and goes through the mixing matrix element:  $\Theta_{ee}$ :

$$m_{\Theta}^{ee} = 2 \frac{M_W^2}{M_{W_R}^2} k \Theta_{ee}^* \quad (\text{suppressed by } \sim 10^{-1}) \quad (2.48)$$

Nuclear physics calculations are giving a suppression to the naive estimation by a factor of 10% (see [78]). The light-heavy neutrino mixing matrix is given roughly by  $\sqrt{\epsilon - m_\nu/m_N}$  with  $\epsilon = v_L/v_R$ . Thus it becomes important when the vev  $v_L$  is large and when the masses of  $m_N$  are small. In the left frame of Figure 2.10 we show different values of  $v_L$  with the  $m_N$  fixed and large in order not to interfere with  $v_L$ , in the right frame we fix  $v_L$  to zero and see the dependence on light  $m_N$ . When  $v_L$  is large compared with  $M_{W_R} m_\nu/m_N$  there will be cancellations in the seesaw formula (between type I and type II seesaw). This  $\Theta$  contribution points in a direction which is hard to probe experimentally. The LHC can

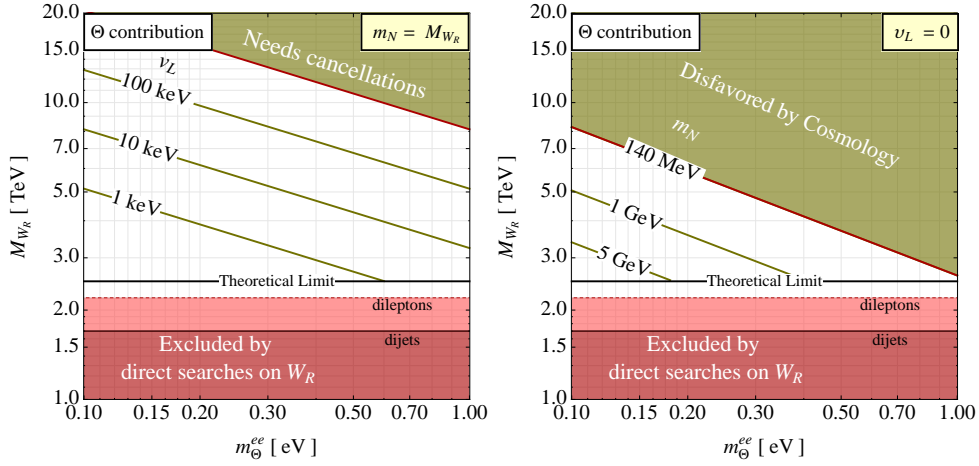


Figure 2.10: Upper bound from  $0\nu\beta\beta$  due to the light and heavy neutrino mixing  $\Theta$ .

indeed test right-handed heavy neutrinos above 10 – 100GeV (lower values will escape the detector because  $N$  will simply not have time to decay). Moreover large  $m_N$  give a subdominant contribution because they are already present in the amplitude of  $m_N^{ee}$ . The only interesting case is when  $v_L$  is large. This can in principle be seen at colliders because the coupling of  $\Delta_L^{++}$  to the  $W_L$  boson is through  $v_L$ . Nevertheless its value is bounded from above from electroweak precision tests  $v_L \lesssim 10\text{GeV}$ .

### 2.6.3 Cosmological constraints on $m_N$

The cosmological constraint on the masses of the heavy neutrinos requires the life time of  $N$  to be smaller than a second  $\tau_N \lesssim \text{sec}$ . This is needed in order not to ruin the abundance of light elements and the predictions of the BBN. When  $m_N$  is larger than the mass of the pion plus a lepton the decay rate of  $N$  into  $\pi\ell$  will be given by:

$$\Gamma(N \rightarrow \ell\pi) = \frac{G_F^2 f_\pi m_N^3}{8\pi} \frac{M_W^4}{M_{W_R}^4} |(V_R^q)_{ud}|^2 |(V_R)_{\ell N}|^2 \times [(1 - x_\ell^2)^2 - x_\pi^2(1 + x_\ell^2)] [(1 - (x_\pi + x_\ell)^2) (1 - (x_\pi - x_\ell)^2)]^{\frac{1}{2}} \quad (2.49)$$

where  $f_\pi = 130\text{MeV}$  is the pion decay constant, and  $x_{\pi,\ell} = m_{\pi,\ell}/m_N$ . In order to satisfy the cosmological constraint we will need generically the lightest heavy neutrinos heavier

than the sum of the decay products:

$$m_N > m_\pi + m_\ell \quad (2.50)$$

The precise limit depends on the leptonic right-handed mixing matrix  $V_R$ . In the case of having one more heavy neutrinos lower than (2.50) the only way to avoid the over-closure of the universe is to have them lighter than about few eV [81]. Moreover, if a light  $N$  is coupling substantially to electrons a low scale of  $W_R$  is disfavoured by supernova constraints [82]. The problem comes from the amount of energy released by the supernova. If  $N$  is light it will carry a considerable amount of energy unless  $W_R$  is heavy enough. The constraint reads [83]:

$$(V_R)_{eN} \left( \frac{3.5 \text{ TeV}}{M_{W_R}} \right) < 0.02 \quad (2.51)$$

If there is one light  $N$  at the eV scale, the other two will have an additional decay channel open. The decay rate of  $N$  into a lighter one is given by:

$$\Gamma(N_i \rightarrow N_j \ell \ell') = 2\Gamma_\mu \left( \frac{M_W}{M_{W_R}} \right)^4 \left( \frac{m_N}{m_\mu} \right)^5 |(V_R)_{\ell i} (V_R^*)_{\ell' j} + (V_R)_{\ell j} (V_R^*)_{\ell' i}|^2 \quad (2.52)$$

which numerically translates into a bound on the heavier neutrinos:

$$m_N \gtrsim 140 \text{ MeV} \left( \frac{M_{W_R}}{3.5 \text{ TeV}} \right)^{4/5} \quad (2.53)$$

meaning that again the neutrinos in the heavy regime need to have masses above  $\sim 100\text{MeV}$ .

## 2.7 Interplay between the LHC and Neutrinoless double beta decay.

There is a region in the  $(M_{W_R}, m_N)$  plane in which the same parameter space is being explored simultaneously by direct searches at the LHC and by indirect searches on  $0\nu\beta\beta$



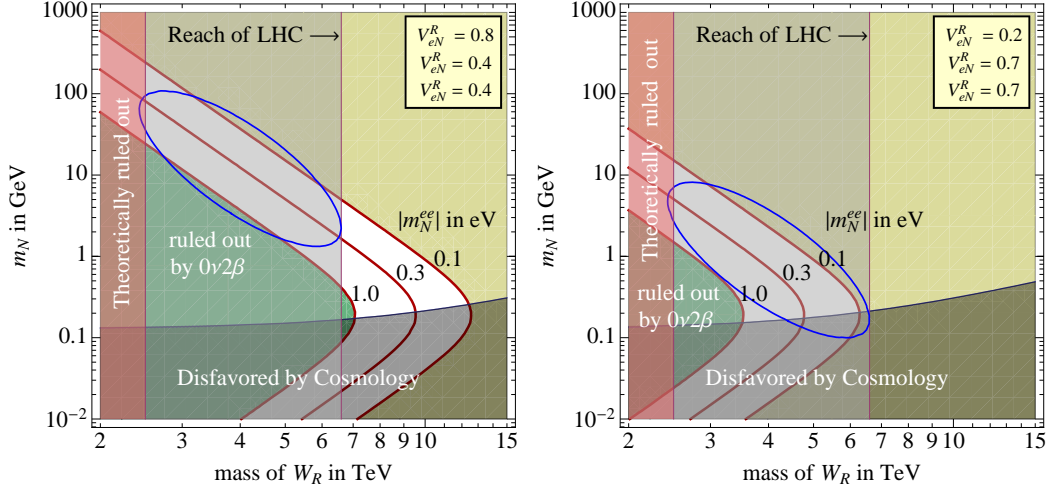


Figure 2.11: Contours of  $m_N^{ee}$  in the  $(M_{W_R}, m_N)$  plane. Illustrated for large (left frame) and small (right frame) couplings to the electron  $(V_R)_{eN}$ .

[80]. The common region is highlighted with a blue ellipse in Figure 2.11. There we show two different choices of the coupling of the lightest  $N$  to the electron. The reach of LHC is thus covering most of the space in which  $m_N^{ee}$  is dominating the rate of  $0\nu\beta\beta$ . There is only a small window with  $7 \text{ TeV} < M_{W_R} < 15 \text{ TeV}$  and  $300 \text{ MeV} < m_N < 10 \text{ GeV}$  in which we could encounter a large rate of  $0\nu\beta\beta$  due to  $m_N^{ee}$ , but no sign of  $W_R$  at the LHC. In Figure 2.11 we have also shown the theoretical constraint on the mass of  $M_{W_R} > 2.6 \text{ TeV}$  coming from the right-handed contribution to  $K_L-K_S$  mass difference [29]. From the two values that  $m_N$  can have to produce the same amplitude  $m_N^{ee}$  one of them is disfavored by cosmology and only those with masses larger than  $\sim 100 \text{ MeV}$  can contribute to the decay rate. This increases the chance for searches at the LHC as the same-sign lepton channel prefers heavy neutrino decaying inside the detector.

## 2.8 Illustrative example: Type II seesaw dominance

As we have seen, we know nothing about the V+A leptonic mixing angles. There is hope that LHC can probe the mixing angles together with the heavy neutrino masses. This will allow a complete reconstruction of the heavy neutrino mass matrix  $M_N$ . Therefore no harm will be done by going ahead with a definite mixing matrix. We could, for example,

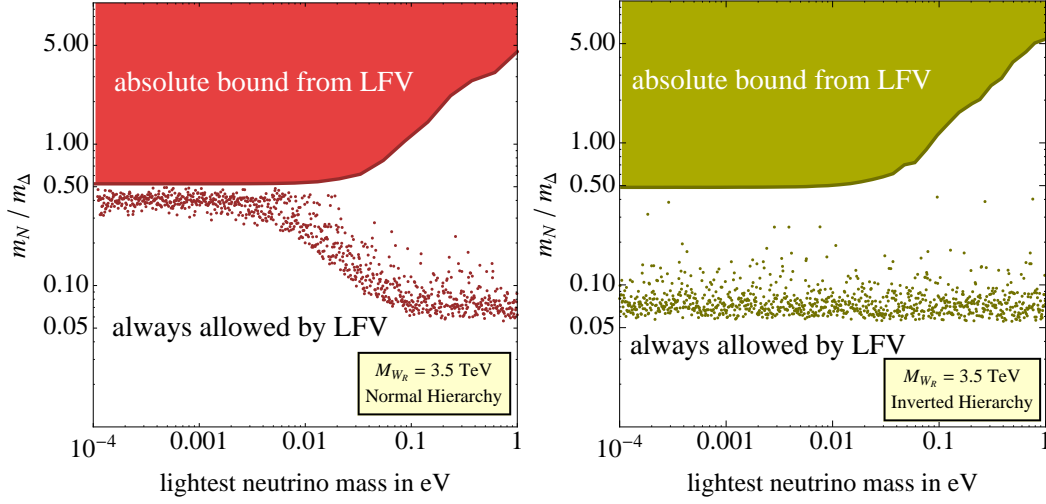


Figure 2.12: Combined bounds on  $m_N^{heaviest}/m_{\Delta_{++}}$  from LFV. The dots show the most probable upper bounds resulting for different mixing angles and phases. They were varied in the interval  $\{\theta_{12}, \theta_{23}, \theta_{13}\} = \{31^\circ\text{-}38^\circ, 36^\circ\text{-}55^\circ, 7^\circ\text{-}10^\circ\}$ ,  $[0, 2\pi]$  respectively. The plot scales as  $M_{W_R}/3.5\text{TeV}$

imagine to have  $V_R = V_L^*$  analogous to what happens in the quark sector where the Left-Right symmetry does imply, up to phases, that  $V_R^q = V_L^{q*}$ . Then one would get after some simplifications the following total decay rate of  $0\nu\beta\beta$ :

$$\Gamma_{0\nu\beta\beta} \propto |(V_L)_{ei}^2 m_{\nu_i}|^2 + \frac{M_W^8}{M_{W_R}^8} \left| (V_L)_{ei}^2 \frac{k^2}{m_{N_i}} \right|^2 + 4|\epsilon| \frac{M_W^4}{M_{W_R}^4} (|V_L|_{ei}^2 s_i)^2 k^2 \cdot 10^{-2} \quad (2.54)$$

However, without the right-handed neutrino masses we are only able to draw generic conclusion. To go a step further, we consider the case in which the light neutrino mass matrix is generated mostly by  $\epsilon$ . This is the so called type-II seesaw scenario. In this case we have a proportionality between the light and heavy neutrino mass matrices:

$$M_\nu \simeq \epsilon M_N \quad (2.55)$$

This automatically implies that the left and right-handed mixing matrices are equal (up to a complex phase):

$$V_R = \pm e^{-\frac{i\theta_L}{2}} V_L^* \quad (2.56)$$

with  $\theta_L$  the spontaneous phase of  $\langle \Delta_L^0 \rangle$ . The light and heavy neutrino masses are proportional:

$$\frac{m_{\nu_1}}{m_{N_1}} = \frac{m_{\nu_2}}{m_{N_2}} = \frac{m_{\nu_3}}{m_{N_3}} = |\epsilon| \quad (2.57)$$

To check this scenario, one needs to measure  $V_R, m_{N_i}$  and  $\epsilon$ . The last quantity  $\epsilon$  will be tiny and can be translated into a  $v_L$  of the order of few keV. This  $v_L$  can allow for the production of  $\Delta_L^{++}$  through gauge interaction in colliders. But in any case, if the masses are proportional and the mixing angles are the same it will be a strong indication in favor of the type II scenario and not in the general case where these relations are a merely coincidence.

With the masses and mixing defined as in (2.56) and (2.57), we can then use all the information available from neutrino oscillations presented in Table 2.2. The only free variables are then the lightest neutrino mass, the Dirac and the Majorana phases. Also two kinds of hierarchies are allowed, depending on the sign of  $\Delta m_{13}^2$ . In Figure 2.12 we show a numerical analysis for the bound arising from  $\mu \rightarrow e^+ee$ ,  $\mu \rightarrow e\gamma$ , muon conversion in Au, and also the rare leptonic tau decay. The Majorana phases are chosen in such a way to minimize the rate of all these LFV processes simultaneously. The particular values of the left mixing angles, specially the large  $\theta_{13}$  make the limit arising from  $\mu \rightarrow e^+ee$  alone already very robust. In some points where the Majorana phases can make the branching ratio small there will be another LFV process with different flavor dependence which put a limit on  $m_N/m_{\Delta^{++}}$ . However, for degenerate neutrino masses a proper choice of the Majorana phases can make the LFV rates small enough for any  $m_N/m_{\Delta^{++}}$ .

The decay rate of  $0\nu\beta\beta$  is then given by:

$$\Gamma_{0\nu\beta\beta} \propto |m_{\nu+N}^{ee}|^2 = (V_L)_{ei}^2 (V_L^*)_{ej}^2 \left( m_{\nu_i} m_{\nu_j} + \frac{M_W^8}{M_{W_R}^8} \frac{|\epsilon|^2 k^4}{m_{\nu_i} m_{\nu_j}} \right) \quad (2.58)$$

where we have introduced the effective mass parameter  $|m_{\nu+N}^{ee}|$  [11]. This quantity supersedes the standard matrix element  $m_\nu^{ee}$  in the parameter space accessible to LHC. In Figure 2.13 we show  $|m_{\nu+N}^{ee}|$  for a fixed value of  $\epsilon$  as a function of the lightest neutrino

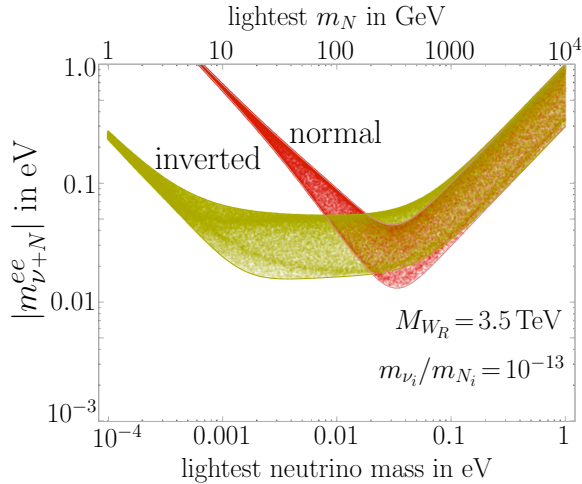


Figure 2.13: Effective  $0\nu\beta\beta$  mass parameter  $|m_{\nu+N}^{ee}|$ , a measure of the total  $0\nu\beta\beta$  rate including contribution from both left and right-handed currents.

mass. The effect of the heavy neutrinos is that of inverting the role of the hierarchies. Contrary to the canonical light neutrino contribution the new physics has more chance to occur for highly hierarchical neutrinos. The normal hierarchy being ahead of the inverted hierarchy in a wide region of the parameter space. The combination of light and heavy neutrinos implies that the total rate of  $0\nu\beta\beta$  can not vanish for any choice of the Majorana phases. The upper axis of Figure 2.13 shows the range of the lightest  $m_N$ . When it is below 100 GeV it would lead to interesting displaced vertices at the LHC [29]. This examples shows, that  $0\nu\beta\beta$  may be naturally governed by the new physics, and this may eventually be confirmed by the LHC. Thus it is possible that the light neutrino masses give only a subleading contribution to the decay rate. This demonstrates that the theorem of [84] in which a non-zero  $0\nu\beta\beta$  rate can be translated into a non-zero neutrino Majorana mass has no practical purpose. Another example was provided by the minimal supersymmetric standard model [85].

## 2.9 Electron electric dipole moment

A non zero value of EDM will signal the violation of P and T (or CP) symmetry. For a review see [86]. The experimental constraints for the electric dipole moment of charged

leptons are shown in Table 2.10. For the electron, the contribution of the Standard Model appears at 4 loop order with an approximate value of [87]:

$$d_e^{\text{SM}} \simeq 10^{-27} \frac{e}{2m_e} \quad (2.59)$$

And thus it is far from being relevant. In the Left-Right model, on the contrary, we encounter a non-zero value already at the 1 loop level. To perform the calculation we need the effective interaction  $l_1 \rightarrow l_2 \gamma$ . For this, we define the vertex  $\Gamma^\mu$  as:

$$\langle f_2(p_2) | J_\mu^{em} | f_1(p_1) \rangle = \bar{u}_2(p_2) \Gamma^\mu u_1(p_1) \quad (2.60)$$

The conservation of electromagnetic current implies that  $\Gamma^\mu$  depends only in four form factors:

$$\Gamma^\mu = (q^2 \gamma^\mu - q^\mu \not{q}) (F_1 + F_2 \gamma_5) + i \sigma^{\mu\nu} q_\nu (F_3 + F_4 \gamma_5) \quad (2.61)$$

with  $q = p_1 - p_2$ . When the initial and final particles are the same, the coefficient  $F_3$  and  $F_4$  are real and correspond to the electric and magnetic dipole moments, respectively.

Using complex quantities, we could write the effective dipole interactions as:

$$H = \bar{\ell} \sigma_{\mu\nu} [\sigma \gamma_L + \sigma^* \gamma_R] \ell F^{\mu\nu} \quad (2.62)$$

In that case we have:

$$d = \text{Re}(\sigma) \quad \text{electric dipole moment} \quad (2.63)$$

$$\mu = \text{Im}(\sigma) \quad \text{magnetic dipole moment} \quad (2.64)$$

We can use the form factors of  $\mu \rightarrow e \gamma$  already calculated in (2.15) and simply take the imaginary part. There we found that the only complex form factor was given by the mixed amplitude  $\sigma_{LR}$ . This corresponds to the diagram in which the  $W_L$  boson and the

charged lepton	$ d $
$e$	$< 1 \times 10^{-16} \frac{e}{2m_e}$
$\mu$	$< 1 \times 10^{-6} \frac{e}{2m_\mu}$
$\tau$	$< 2 \times 10^{-2} \frac{e}{2m_\tau}$

Table 2.10: Limits on charged leptons electric dipole moments

heavy neutrinos are propagating in the loop. The rest of the diagrams are real and do not contribute to the EDM. The amplitude  $\sigma$  was found to be:

$$\sigma = \sigma_{LR} = -\frac{eG_F}{4\sqrt{2}\pi^2} M_N V_R^* F_2(t) V_R^T \xi \Theta \quad (2.65)$$

The EDM of the electron  $d_e$  is simply the imaginary part of  $\sigma_{ee}$ , and can be written after some simplifications in the following way:

$$d_e = \frac{eG_F}{4\sqrt{2}\pi^2} \frac{M_W^2}{M_{W_R}^2} \sin(2\beta) \text{Im} \left[ e^{ia} (V_R)_{ei} (V_L^*)_{ej} F_2(t_i) (M_D^{\text{phys}})_{ij} \right] \quad (2.66)$$

Similar expression can be derived for the muon and tau leptons by simply replacing the index  $e$  by  $\mu$  and  $\tau$  in the formula above. The flavor dependence is controlled by the left and right-handed mixing angles and also by the neutrino Dirac mass matrix which we reproduce here for convenience:

$$M_D^{\text{phys}} = V_R^\dagger M_N \sqrt{\epsilon - M_N^{-1} M_\nu V_L} \quad (2.67)$$

Therefore apart from the overall factor  $\sin(2\beta)$ , which fixes the overall scale of the EDM, with the data from LHC we could be able to predict the ratio  $d_e : d_\mu : d_\tau$  between different EDM [21]:

$$\frac{d_\alpha}{d_\beta} = \frac{\sum_{ij} \text{Im} \left[ e^{ia} (V_R)_{\alpha i} (V_L^*)_{\alpha j} F_2(t_i) (M_D^{\text{phys}})_{ij} \right]}{\sum_{ij} \text{Im} \left[ e^{ia} (V_R)_{\beta i} (V_L^*)_{\beta j} F_2(t_i) (M_D^{\text{phys}})_{ij} \right]} \quad (2.68)$$

From (2.66) we can write also:

$$d_e \simeq 10^{-16} \frac{e}{2m_e} \times \left( \frac{3.5 \text{ TeV}}{M_{W_R}} \right)^2 \sin(2\beta) \text{Im} \left[ e^{ia} (V_R)_{ei} (V_L^*)_{ej} F_2(t_i) \frac{(M_D^{\text{phys}})_{ij}}{\text{MeV}} \right] \quad (2.69)$$

Using the experimental constraint on the electron EDM from Table 2.10 we can see that, for a  $W_R \simeq 3.5 \text{ TeV}$ , the experiment on the electron EDM are probing Dirac masses of order:

$$m_D \sin(2\beta) \sim \text{MeV} \quad (2.70)$$

This complements the limit from  $0\nu\beta\beta$  in (2.44). The additional dependence on  $\beta$  can be seen from a positive point of view. In principle if  $d_e$  is measured and  $m_N$  and  $V_R$  are determined from the collider, we could predict  $\beta$  as well. It can then be used to determine  $d_\mu, d_\tau$  and also the  $W_L$ - $W_R$  mixing angle  $\xi \simeq \sin 2\beta (M_W^2/M_{W_R}^2)$ . A direct test of this mixing would then have to wait for further experiments.

### 2.9.1 Neutrino dipole moments

For Majorana particles only the transition dipole moments are different from zero. What happens is that the dipole and magnetic moment of the neutrino are cancelled by equally opposite moments coming from the antineutrino.

The calculation of the transition dipole moments goes in the same way as before. We can again make use of the general formulas in [50]. The only difference is in the number of diagrams. We need the diagrams with neutrinos as well as with antineutrinos. Transition amplitudes  $\mu_{ij}$  and  $d_{ij}$  are given by [88, 21]:

$$\mu_{ij} = \frac{3eG_F i}{16\sqrt{2}\pi^2} \left\{ (m_{\nu_i} + m_{\nu_j}) \text{Im}[(V_L)_{ki} (V_L^*)_{kj}] \frac{m_{\ell_k}^2}{M_W^2} + \frac{16}{3} m_{\ell_k} \text{Im}[\xi (\Theta^* V_L^*)_{ki} (V_L^*)_{kj} + \xi^* (\Theta V_L)_{kj} (V_L)_{ki}] \right\} \quad (2.71a)$$

$$d_{ij} = \frac{3eG_F}{16\sqrt{2}\pi^2} \left\{ (m_{\nu_j} - m_{\nu_i}) \text{Re}[(V_L)_{ki} (V_L^*)_{kj}] \frac{m_{\ell_k}^2}{M_W^2} + \frac{16}{3} m_{\ell_k} \text{Re}[\xi (\Theta^* V_L^*)_{ki} (V_L^*)_{kj} - \xi^* (\Theta V_L)_{kj} (V_L)_{ki}] \right\} \quad (2.71b)$$

One can check that when  $i = j$  these quantities vanish as they should. The nice feature is that the new contribution is not suppressed by the light neutrino masses. The new physics through the left-right mixing is expected to dominate over the light neutrino masses by a factor of roughly:

$$\sim 10^{10} \sin(2\beta) \Theta \left( \frac{3.5 \text{TeV}}{M_{W_R}} \right)^2 < 10^5 \sin(2\beta) \quad (2.72)$$

Here on the last step, we have used the generic constraint (2.41) from  $0\nu\beta\beta$  on the mixing matrix  $\Theta$ . So unless the angle  $\beta$  is very close to zero the dominant contribution will come from the left-right mixing. However, the experimental limit on the magnetic moment of neutrinos is rather weak [89]:

$$\mu_{12} < 10^{-10} \frac{e}{2m_e} \quad (2.73)$$

and at the present does not provide new constraints. More important, once more we will be able to predict its value up to the overall factor  $\sin\beta$  with the outcome from LHC on  $V_R$  and  $m_N$ . As we have said in the previous section, if we combine the searches from LHC with the measurement of the electron EDM we could shed light also on the  $\beta$  angle and fully predict the neutrino transition dipole moments.



# Conclusion

The left-right model offers a natural answer to one of the most puzzling facts about neutrino masses. The smallness of their mass scale is connected with the scale of right-handed gauge currents. With the lepton number violated in both places we are left with two complementary manifestations of the Majorana nature of neutrinos, namely, neutrinoless double beta decay ( $0\nu\beta\beta$ ) and the KS production of the same-sign charged lepton pairs at colliders. Both processes will signal the violation of lepton number by two units. But the low energy manifestation of this large scale in which the parity is restored, can also show up in  $0\nu\beta\beta$ . We have shown quantitatively in this thesis that it can even dominate the total decay rate. Moreover, it can do it in the parameter space in which the KS signal is in turn within reach of the LHC, thus doubling the physical impact of the scale of the right-handed gauge currents.

Moreover we have also shown how the low scale of lepton flavor violating processes such as  $\mu \rightarrow 3e$ ,  $\mu \rightarrow 3\gamma$  and the  $\mu$ - $e$  conversion in the nuclei, can be used to assist the searches of the scalar particles and right-handed neutrinos in the collider. For example, one should expect in general masses of the right-handed neutrinos much below the masses of the doubly charged scalar particles. This was shown quantitatively for a particular scenario in which the neutrino masses are generated mostly by the vacuum expectation value of the left scalar triplet. For the same case, we also illustrated graphically the interconnection between  $0\nu\beta\beta$  and the KS signal outlined above. We found that the neutrino mass hierarchy in  $0\nu\beta\beta$  can not be extracted with a simple measurement of the total decay rate. Obviously this is not the best place to look for oscillation parameters.

On the contrary, we argued and repeatedly emphasized throughout this thesis, that the best place to look for the leptonic right-handed mixing parameters is the LHC through the KS lepton number (and also lepton flavor) violating signal.

In the scenario in which the right-handed neutrino mass matrix is determined from the LHC we showed that one can in turn predict the coupling of light and heavy neutrinos to the Higgs boson. We found a definite expression for the Dirac mass matrix depending on oscillation parameters and high energy collider observables. It was precisely the left-right symmetry which eliminated the unphysical parameters present in generic type I seesaw, that allowed us to invert the seesaw formula in favor of the Dirac mass matrix in a definite way. This is of fundamental importance now that we have entered in the era of the Higgs boson. The light-heavy neutrino mixing matrix is proportional to the Dirac mass matrix and therefore, with the outcome of the LHC, one will be able to quantify at the same time all the low energy manifestation depending on this small mixing.

## Appendix A

# Scalar Potential

The most general scalar potential invariant under  $\mathcal{G}_{LR}$  and the  $\mathcal{C}$  symmetry in which  $\Phi \leftrightarrow \Phi^T$  and  $\Delta_L \leftrightarrow \Delta_R^*$  can be written as follows:

$$\begin{aligned}
V = & -\mu_1^2 \text{Tr} |\Phi|^2 - \mu_2^2 \det \Phi - \mu_2^{*2} \det \Phi^* - \mu_3^2 (\text{Tr} |\Delta_L|^2 + \text{Tr} |\Delta_R|^2) \\
& + \lambda_1 (\text{Tr} |\Phi|^2)^2 + \lambda_2 (\det \Phi)^2 + \lambda_2^* (\det \Phi^*)^2 + \lambda_3 |\det \Phi|^2 \\
& + (\lambda_4 \det \Phi + \lambda_4^* \det \Phi^*) \text{Tr} |\Phi|^2 \\
& + \rho_1 [(\text{Tr} |\Delta_L|^2)^2 + (\text{Tr} |\Delta_R|^2)^2] + \rho_2 (|\det \Delta_L|^2 + |\det \Delta_R|^2) \\
& + \rho_3 \text{Tr} |\Delta_L|^2 \text{Tr} |\Delta_R|^2 + \rho_4 \det \Delta_L \det \Delta_R^* + \rho_4^* \det \Delta_L^* \det \Delta_R \\
& + (\alpha_1 \text{Tr} |\Phi|^2 + \alpha_2 \det \Phi + \alpha_2^* \det \Phi^*) (\text{Tr} |\Delta_L|^2 + \text{Tr} |\Delta_R|^2) \\
& + \alpha_3 (\text{Tr} \Phi \Phi^\dagger \Delta_L \Delta_L^\dagger + \text{Tr} \Phi^\dagger \Phi \Delta_R \Delta_R^\dagger) \\
& + \beta_1 \text{Tr} \Phi \Delta_R \Phi^\dagger \Delta_L^\dagger + \beta_1^* \text{Tr} \Phi^\dagger \Delta_L \Phi \Delta_R^\dagger \\
& + \beta_2 \text{Tr} \Phi^c \Delta_R \Phi^\dagger \Delta_L^\dagger + \beta_2^* \text{Tr} \Phi^{c\dagger} \Delta_L \Phi \Delta_R^\dagger \\
& + \beta_3 \text{Tr} \Phi \Delta_R \Phi^{c\dagger} \Delta_L^\dagger + \beta_3^* \text{Tr} \Phi^\dagger \Delta_L \Phi^c \Delta_R^\dagger
\end{aligned}$$

The minimum of the potential is attained when:

$$\langle \Phi \rangle = \begin{pmatrix} v \cos b & 0 \\ 0 & -ve^{-ia} \sin b \end{pmatrix}, \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\delta} & \end{pmatrix}, \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \quad (\text{A.1})$$

under the conditions that  $V'_v = V'_b = V'_a = V'_{v_R} = V'_{v_L} = V'_\delta = 0$ . The first four will give us  $\mu_1, \mu_2, \mu_2^*, \mu_3$  in term of  $v, v_R$ . Using these values in fifth equation we will find  $v_L \propto v^2/v_R$ . The sixth equation gives a relation between phases. To get the mass spectrum we can use the Higgs mechanism, and rotate the charge fields in the following way:

$$\begin{pmatrix} \phi_1^- \\ \phi_2^- \\ \Delta_R^- \end{pmatrix} \simeq \begin{pmatrix} \cos b & -\sin b & -\frac{M_W}{M_{W_R}} e^{-ia} \sin b \\ e^{ia} \sin b & e^{ia} \cos b & -\frac{M_W}{M_{W_R}} \cos b \\ \frac{M_W}{M_{W_R}} e^{ia} \sin 2b & \frac{M_W}{M_{W_R}} e^{ia} \cos 2b & 1 \end{pmatrix} \begin{pmatrix} G_L^- \\ H^- \\ G_R^- \end{pmatrix} \quad (\text{A.2})$$

where  $G_L^-$  and  $G_R^-$  are the unphysical scalars which are absorbed by  $W_L$  and  $W_R$  respectively. In the same way, we can rotate the neutral scalars as follows:

$$\begin{pmatrix} \text{Im}(\phi_1^0) \\ \text{Im}(e^{-ia} \phi_2^0) \\ \text{Im}(\Delta_R^0) \end{pmatrix} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \cos b & -\sin b & \frac{M_Z}{M_{Z'}} \frac{\sqrt{c_{2W}}}{c_W^2} \cos b \\ \sin b & \cos b & \frac{M_Z}{M_{Z'}} \frac{\sqrt{c_{2W}}}{c_W^2} \sin b \\ -\frac{M_Z}{M_{Z'}} \frac{\sqrt{c_{2W}}}{c_W^2} \sin 2b & 0 & 1 \end{pmatrix} \begin{pmatrix} G_L^0 \\ H_\phi^0 \\ G_R^0 \end{pmatrix} \quad (\text{A.3})$$

where the  $G_L^0$  and  $G_R^0$ , are the unphysical scalars absorbed by  $Z$  and  $Z'$  respectively. In Table A.1 are show the masses of all the physical scalars. The heavy doublet  $H_\phi$  is mostly  $\varphi_2$ , which is the notation used in the text.

Original field	Physical field	mass squared
Light doublet	$h_\phi^0$	$v^2 [4\lambda_1 + (\lambda_3 + \text{Re}(\lambda_2 e^{-ia})) \sin^2 2b - 2\text{Re}(\lambda_4 e^{-ia}) \sin 2b]$
Heavy doublet	$H_\phi^0$	$\alpha_3 v_R^2 \sec 2b + v^2 (\lambda_3 + \text{Re}(\lambda_2 e^{-2ia})) \cos^2 2b$
	$A_\phi^0$	$\alpha_3 v_R^2 \sec 2b + v^2 (\lambda_3 - \text{Re}(\lambda_2 e^{-2ia}))$
	$H_\phi^+$	$\alpha_3 v_R^2 \sec 2b + \alpha_3 v^2 \cos 2b$
Left triplet	$H_L^0$	$(\rho_3 - 2\rho_1) v_R^2$
	$A_L^0$	$(\rho_3 - 2\rho_1) v_R^2$
	$\Delta_L^+$	$(\rho_3 - 2\rho_1) v_R^2 + \frac{1}{2} \alpha_3 v^2 \cos 2b$
	$\Delta_L^{++}$	$(\rho_3 - 2\rho_1) v_R^2 + \alpha_3 v^2 \cos 2b$
Right triplet	$H_R^0$	$4\rho_1 v_R^2$
	$\Delta_R^{++}$	$\rho_2 v_R^2 + \alpha_3 v^2 \cos 2b$

Table A.1: Physical scalar particles

## Appendix B

# The square root of a matrix

Here we follow the method of [91]. To find the square root of a matrix  $A$  one first needs to find the matrix  $X$  and  $J$  such that:

$$A = XJX^{-1} \tag{B.1}$$

where  $J$  is called the normal form of  $A$ . The matrix  $J$  is a block diagonal matrix made out of Jordan blocks:

$$J = \text{diag}(J_1, \dots, J_i), \quad J_i = \lambda_i I_i + E_i \tag{B.2}$$

Here  $I_i$  is a diagonal matrix and  $E_i$  a matrix which has ones just above the diagonal and zeros elsewhere, both matrices have the dimension of the block. For non-singular matrices, the square root of  $J$  is given by <sup>1</sup>:

$$\sqrt{J_i} = \lambda_i^{\frac{1}{2}} I_i + \frac{1}{2} \lambda_i^{\frac{1}{2}-1} E_i + \frac{1}{2!} \frac{1}{2} \left( \frac{1}{2} - 1 \right) \lambda_i^{\frac{1}{2}-1} E_i^2 + \dots \tag{B.3}$$

One needs the first  $m$  terms of the series for each  $m \times m$  block.

---

<sup>1</sup>This formula fails when  $\lambda_i = 0$  but  $E_i \neq 0$ .

The square root of  $A$  is then:

$$\sqrt{A} = XY\sqrt{J}Y^{-1}X^{-1} \quad (\text{B.4})$$

where  $Y$  is a matrix which commutes with  $J$  and contains arbitrary continuous parameters. It can be shown that when all  $\lambda_i$  are different from each other, the matrix  $Y$  has a block diagonal form and then if it commutes with  $J$  it commutes also with  $\sqrt{J}$ . Therefore *when all  $\lambda_i$  are different the square root of a  $n \times n$  matrix contains only discrete solutions.*<sup>2</sup>

We now show focus in some special cases.

- Consider the case the matrix  $A$  is diagonalizable. This means all  $E_i$  are zero. If all  $\lambda_i$  are different from each other (zeros are also possible). The square root is:

$$\sqrt{A} = X \text{diag}(\pm\sqrt{\lambda_1}, \dots, \pm\sqrt{\lambda_n})X^{-1} \quad (\text{B.5})$$

- Roots of the unity matrix. In this case  $X = 1$ ,  $J = 1$  and  $Y$  is an arbitrary matrix. Then  $\sqrt{I} = Y \text{diag}(\pm 1, \dots, \pm 1)Y$ . For example in 2 dimensions we have:

$$\sqrt{I}_{2 \times 2} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}^{-1} = \begin{pmatrix} a & b \\ \frac{1-a^2}{b} & -a \end{pmatrix}$$

where  $a = (y_{11}y_{22} + y_{12}y_{21})/\det Y$  and  $b = -2y_{11}y_{22}/\det Y$  are the independent complex parameters.

In the same way, in 3 dimensions one has for example  $\sqrt{I} = Y \text{diag}(1, 1, -1)Y$ , and after some redefinitions one arrives at:

$$\sqrt{I}_{3 \times 3} = \frac{1}{a+b+c} \begin{pmatrix} a & -(b+c)x & -(b+c)y \\ -(a+c)x^{-1} & b & -(a+c)x^{-1}y \\ -(a+b)y^{-1} & -(a+b)xy^{-1} & c \end{pmatrix}$$

---

<sup>2</sup>The singular case  $\lambda_i = 0$  but  $E_i \neq 0$  does not always have a root but when it does it contains continuous parameters because  $Y$  will not commute with  $\sqrt{J}$ .

with  $a, b, c, x, y$  arbitrary complex parameters.

In our physical case, we have:

$$\Theta = \sqrt{\epsilon - M_N^{-1} M_\nu} \tag{B.6}$$

The arbitrary nature of the square root comes from the matrix  $Y$  in (B.4) which commutes with  $J$ . If this matrix does not commute with  $\sqrt{J}$  there will be arbitrary parameters. We have seen that for non-singular matrices, this would require at least two identical eigenvalues. But this requirement is far from being physical because even the slightest perturbation will destroy the degeneracy. The singular case is also disfavoured because  $\epsilon$  is already in the main diagonal and only a perfect cancellations would make the matrix singular. Therefore, we conclude that the continuous solutions of the square root should not be taken seriously.

Obviously  $M_D$  does not have to be perfectly symmetric. It is enough if it is mostly symmetric. A small asymmetry can always be generated radiatively but provide it is small our formula is still valid.



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