Open superstring field theory and the supermoduli space of super-Riemann surfaces

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July 28, 2014
Talk at SFT 2014 in Trieste

work in progress with Kantaro Ohmori

1. Introduction

How should we deal with the superconformal ghost sector in open superstring field theory?

the small Hilbert space

- the Witten formulation

- the modified cubic theory

$\Rightarrow$ complications associated with local picture-changing operators

the large Hilbert space

- the Berkovits formulation
  $\rightarrow$ correct four-point amplitudes

  - the Ramond sector
  - the Batalin-Vilkovisky quantization

  $\Rightarrow$ comlicated

- open bosonic string field theory: A_∞
- closed bosonic string field theory: L_∞

closely related to the covering of the moduli space of Riemann surfaces

Many of the issues in superstring field theory we are confronted with seem to be coming from the lack of our understanding on the relation of superstring field theory to the supermoduli space of super-Riemann surfaces.

the supermoduli space of super-Riemann surfaces
$\rightarrow$ formulation based on the small Hilbert space

arXiv:1312.1677 with Iimori, Noumi and Torii

1. partial gauge fixing of the Berkovits formulation

$\Rightarrow$ regular formulation based on the small Hilbert space

ingredients:

- the BPZ inner product
- the BRST operator Q
- η_0
- a general line integral of ξ $\leftarrow$ surprise!

Further important developments by Erler, Konopka and Sachs

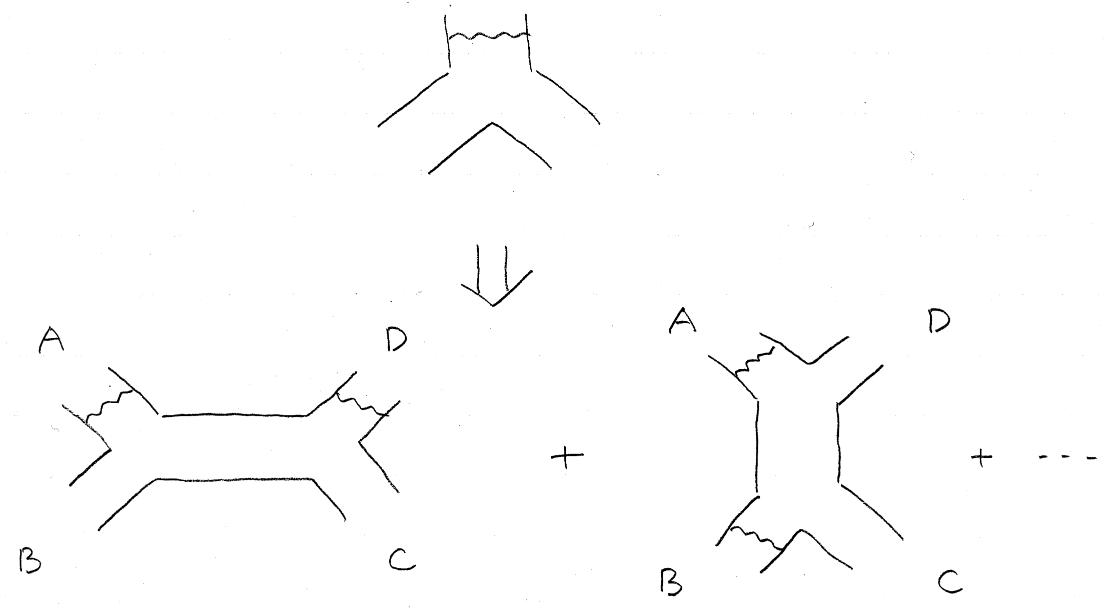
2. the roll of the quartic interaction in four-point amplitudes

$$\int_0^{\frac{1}{2}} dt \, \langle \, c V_A^{(0)}(0) \; V_B^{(-1)}(t) \; c V_C^{(-1)}(1) \; c V_D^{(0)}(\infty) \, \rangle$$

$$+ \int_{\frac{1}{2}}^{1} dt \, \langle \, c V_A^{(0)}(0) \; V_B^{(0)}(t) \; c V_C^{(-1)}(1) \; c V_D^{(-1)}(\infty) \, \rangle$$

The quartic interaction plays a roll of adjusting different behaviors of the picture-changing operators in the s channel and in the t channel.

Next step:
a new formulation based on the relation
to the supermoduli space of super-Riemann surfaces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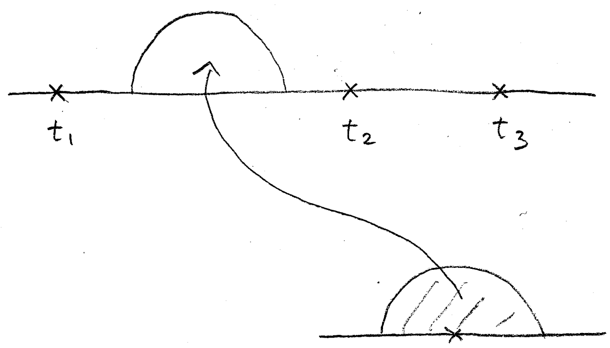
2. New cubic interaction

four-point amplitudes in the bosonic string

$$\int_{t_1}^{t_2} dt \, \langle \, cV_A(t_1) \; V_B(t) \; cV_C(t_2) \; cV_D(t_3) \, \rangle$$

t : modulus



$$V_B(t) = b_{-1} \cdot cV_B(t)$$
$$= b_{-1} \, e^{tL_{-1}} \cdot cV_B(0)$$

$$\{ Q, \; b_{-1} e^{tL_{-1}} \} = L_{-1} e^{tL_{-1}} = \partial_t \, e^{tL_{-1}}$$

$$\int dt \; b_{-1} e^{tL_{-1}} = \int dt \int d\tilde{t} \; e^{\tilde{t} b_{-1} + t L_{-1}}$$

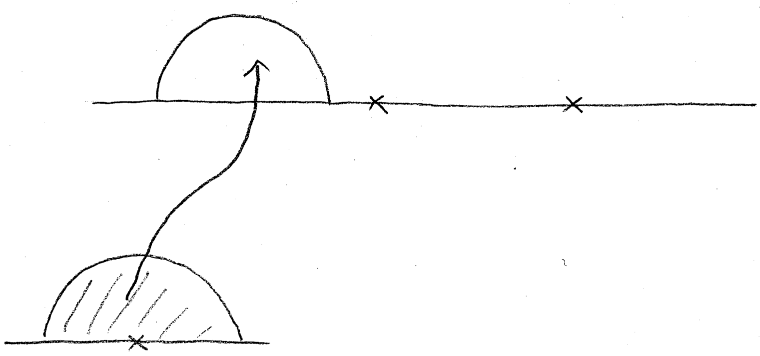
$\tilde{t}$: Grassmann odd

NS three-point amplitudes in the superstring

$V_A^{(0)}(t_1) \quad V_B^{(-1)}(t_2) \quad V_C^{(-1)}(t_3)$

one fermionic modulus ζ

-1 picture $\xrightarrow{\text{integration over } \zeta}$ 0 picture



$$\int da \int d\zeta \, \langle e^{-a\beta_{-1/2} + \zeta G_{-1/2}} \cdot V_A^{(-1)}(t_1) \, V_B^{(-1)}(t_2) \, V_C^{(-1)}(t_3) \rangle$$

a : Grassmann even (formal)

$$e^{-a\beta_{-1/2} + \zeta G_{-1/2}}$$

$$= e^{-a\beta_{-1/2}} + \int_0^1 dx \, e^{-(1-x)a\beta_{-1/2}} \zeta G_{-1/2} \, e^{-xa\beta_{-1/2}}$$

$$[\beta_{-1/2}, G_{-1/2}] = 2b_{-1}$$

$$[\beta_{-1/2}, [\beta_{-1/2}, G_{-1/2}]] = 0$$

$$= e^{-a\beta_{-1/2}} + \zeta e^{-a\beta_{-1/2}} (G_{-1/2} + ab_{-1})$$

integration over ξ

$$\int da \; e^{-a\beta_{-1/2}} \left(G_{-1/2} + a\, b_{-1} \right)$$

① $V^{(-1)} = -c\,\delta(\gamma)\,\hat{V}_{1/2}$

$$\int da \; e^{-a\beta_{-1/2}} \left(G_{-1/2} + a\, b_{-1} \right) \cdot V^{(-1)} = V^{(0)}$$

$$V^{(0)} = c\, G^{m}_{-1/2} \cdot \hat{V}_{1/2} + \gamma\, \hat{V}_{1/2}$$

② $\left[Q,\; e^{-a\beta_{-1/2}} \left(G_{-1/2} + a\, b_{-1} \right) \right]$

$$[\, Q,\; \beta_{-1/2} \,] = G_{-1/2}$$

$$= -\, e^{-a\beta_{-1/2}}\, a \left(G_{-1/2} + a\, b_{-1} \right)^2 + e^{-a\beta_{-1/2}}\, a\, L_{-1}$$

$$= 0$$

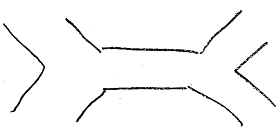
BRST-invariant !

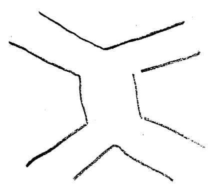
- correct on-shell three-point amplitudes
- We can construct a gauge-invariant action up to cubic order.

3. Strategy for the quartic interaction

on-shell four-point amplitudes

$$\int d\xi$$ 

$$\Rightarrow \int d\xi_1 \int d\xi_2 \int dt$$ 

$$+ \int d\xi_1 \int d\xi_2 \int dt$$ 

Schematically,

$$\int d\xi_1 \int d\xi_2 \int_0^{\frac{1}{2}} dt \quad A(\xi_1, \xi_2, t)$$

$$+ \int d\xi_1 \int d\xi_2 \int_{\frac{1}{2}}^{1} dt \quad A'(\xi_1, \xi_2, t)$$

four points

$$(z_1 | \theta_1) \quad (z_2 | \theta_2) \quad (z_3 | \theta_3) \quad (z_4 | \theta_4)$$

We can fix three bosonic and two fermionic coordinates.

$$(t_1 | \xi_1) \quad (t | 0) \quad (t_2 | \xi_2) \quad (t_3 | 0)$$

$$\Big\downarrow \text{ superconformal transformation}$$

$$(\underset{\sim}{t_1'} | \xi_1') \quad (t | 0) \quad (t_2 | 0) \quad (t_3 | \xi_2')$$

$\uparrow$ Schematically, $t_1' = t_1 + \xi_1 \xi_2$

$$\int d\xi_1 \int d\xi_2 \int_{\frac{1}{2}}^{1} dt \; A'(\xi_1, \xi_2, t)$$

$$= \int d\xi_1 \int d\xi_2 \int_{\frac{1}{2}+\xi_1\xi_2}^{1} A(\xi_1, \xi_2, t)$$

Atick, Rabin & Sen '88
[NPB 299 (1988) 279]

$$\int_0^1 dt = \underbrace{\int_0^{\frac{1}{2}} dt}_{\uparrow\ \text{s channel}} + \underbrace{\boxed{\int_{\frac{1}{2}}^{\frac{1}{2}+\xi_1\xi_2} dt}}_{\uparrow\ \text{quartic interaction}} + \underbrace{\int_{\frac{1}{2}+\xi_1\xi_2}^{1} dt}_{\uparrow\ \text{t channel}}$$

4. Future programs

Extension to the Ramond sector

Extension to a formulation without imposing the GSO projection