Open Many-body Non-Equilibrium Systems

OMNES

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Proposal duration: 60 months

Proposal summary:

We shall study non-equilibrium many-body quantum systems, considering local interactions in one or two spatial dimensions in situations in which the generator of time evolution in the bulk of the system is unitary whereas the incoherent processes are limited to the system’s boundaries. We foresee a mathematical theory of dynamical quantum phases of matter with applications in the theory of quantum transport and nanoscale devices that manipulate heat, information, charge or magnetization.

Our steady-state setup represents a fundamental paradigm of mathematical statistical physics, which has been pioneered by the PI, who gave the first explicit solution [Phys. Rev. Lett. 106, 217206 (2011); ibid. 107, 137201 (2011)] for boundary-driven/dissipative strongly interacting many-body problem (XXZ spin 1/2 chain), which answered a long debated question on the strict positivity of the spin Drude weight at high temperature.

The focus of OMNES will be centered on exploring the following three interconnected pathways: Most importantly, we shall develop a general framework for exact solutions of non-equilibrium integrable quantum many-body models, in particular the steady states and relaxation modes, and develop quantum integrability methods for non-equilibrium many-body density operators. Fundamentally new concepts, which are expected to emerge from these studies, relevant beyond the context of boundary-driven/dissipative systems, are novel quasilocal conservation laws of the bulk Hamiltonian dynamics. Secondly, we shall investigate relevance of exact solutions in physics of generic systems which are small perturbations of integrable models and explore the problem of stability of local and quasilocal conserved quantities under generic integrability-breaking perturbations. Thirdly, we shall formulate and study the problem of quantum chaos in clean lattice systems, in particular to establish a link between random matrix theory of level statistics and kinematic and dynamical features of lattice models with sufficiently strong integrability breaking.

Tomaž Prosen
Open Many-body Nonequilibrium Systems
The main result on which OMNES is being built:
Strict fractal lower bound on ballistic transport at high temperatures

TP, Phys. Rev. Lett. 106 (2011);
Ilievski and TP, Commun. Math. Phys. 318 (2013);
Fundamental hypothesis and the guideline of OMNES

In equilibrium quantum physics the observables are characterized by Hermitian operators. In integrable situations of exactly solvable systems, this implies functional relations among the spectra of large sets of mutually commuting operators. However, in far-from-equilibrium physics, such as in the boundary-driven steady state paradigm, one naturally encounters operators that are non-Hermitian, non-normal, and most importantly, non-diagonalizable. Important examples of this kind, discovered by PI $^{7,8}$, are the quasi-local almost-conserved operators of anisotropic Heisenberg spin chains. This is a fundamental reason why integrable non-equilibrium quantum systems are much more challenging: Namely, non-diagonalizable commuting transfer operators contain important physical information beyond specifying the dependence of their eigenvalues on the so-called spectral parameter. Moreover, this distinction goes beyond the context of integrability. We shall develop new methods for precise the characterisation of commuting families of non-diagonalizable many-body operators and focus on fundamental non-equilibrium physical contexts in which they can be applied.
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Three interconnected pillars:

- **Pillar I**: Exact solutions out of equilibrium – integrability of open quantum many-body systems
- **Pillar II**: Stability/robustness of quantum integrability – towards quantum analogue of Kolmogorov-Arnold-Moser theorem
- **Pillar III**: Quantum chaos in clean many-body systems