Black holes: probes of the cosmos and fundamental physics

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Outline

• Overview general relativity and black holes

• Black holes as probes of the universe and fundamental physics
  – black hole mergers and gravitational wave astronomy
  – black holes and hydrodynamics – the instability of higher dimensional black strings
  – time permitting: black hole formation in super-Planck scale particle collisions

• Conclusions
General Relativity

- General relativity (GR) is Einstein’s theory about the nature of space and time, or spacetime

- GR is an extension of special relativity (SR):
  - SR describes spacetime ignoring the effects of “gravity”
    - The speed of light $c$ is a universal constant of nature in any reference frame
    - “Nothing” can travel faster than light
    - Energy and mass are intimately related concepts, quantified in the famous formula $E=mc^2$
  - One of Einstein’s motivations for developing GR was that Newton’s theory of gravity was inconsistent with SR, as it allowed instantaneous propagation of gravitational fields
Spacetime in general relativity

- In SR the geometric structure of spacetime is passive, i.e. it is the arena in which events happen, but none of the events can influence it.

- In GR spacetime becomes a dynamical structure, not only influencing the objects that exist in it, but matter/energy can now shape the geometric structure of spacetime.
  - Matter/energy causes the geometry of spacetime to become curved; exactly how is described by the Einstein field equations.
  - A small test mass that has no forces acting on it is hypothesized to follow a geodesic of the spacetime geometry, i.e. a curve extremizing the distance between two points.
  - Non-trivial geometry with the geodesic hypothesis is enough to completely describe what we think of as the “force of gravity”.

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Black Holes

• Consider a simple question in gravitational physics: what is the gravitational field produced by a point particle of mass $M$?

• In Newton’s theory, the answer is as simple: the field is an attractive force, experienced by all other matter in the universe, mathematically described by Newton’s universal law of gravitation $F = - \frac{G M m}{r^2}$

  – $F$ is the force experienced by a test particle of mass $m$, $G$ is Newton’s constant, and $r$ is the separation between $m$ & $M$

• In Einstein’s theory, the answer is almost as simple: a black hole!
Milestones in our understanding of black holes

- In 1916, just one year after Einstein published the theory of general relativity, Karl Schwarzschild published the first exact solution to the field equations.
  - though not fully appreciated until several decades later, Schwarzschild’s solution describes the spacetime of a non-rotating black hole.

- It is characterized by a single free parameter, the Schwarzschild Radius $r_s$, which can be related to the mass $m$ of the black hole via $r_s = \frac{2GM}{c^2}$.
  - $r_s$ is the location of the event horizon of the black hole. Nothing inside the event horizon, not even light, can escape.
  - $r_s$ also tells us much you have to compress an object before it will collapse into a black hole.
    - For the sun, $r_s \sim 3\text{km}$; for the earth, $r_s \sim 1\text{cm}$.
Golden Age of Black Hole Physics

• The 1960’s and 1970’s witnessed an explosion in our theoretical understanding of black holes, and the realization that black holes could be important denizens of our universe

• In 1963, Roy Kerr discovered a black hole solution describing rotating black holes

• In 1967, Werner Israel proved the “no-hair” theorem for non-rotating black holes in vacuum. Later extended to rotating black holes by Carter and Robinson; it said that only two numbers, mass $M$ and angular momentum $J$, uniquely describes any black hole geometry

  – The astonishing consequence of this is that the Kerr solution essentially describes the exact geometry of all isolated, macroscopic black holes in the universe
Golden Age of Black Hole Physics

- In 1974, Stephen Hawking showed that when quantum effects are included, black holes cease to be black! Instead, they radiate a thermal spectrum with temperature inversely proportional to their mass. This, together with the laws of black hole mechanics formulated by Bardeen, Carter, Hawking, and Bekenstein in the early 70’s, show that black holes are fundamentally thermodynamic objects.

- In 1964 Roger Penrose proved that singularities in the spacetime geometry generically arise inside black holes; this says that there is a time inside the black hole when general relativity fails to provide a correct description of nature.

- As an aside, all this work was done before the nomenclature “black hole” existed! John Wheeler is attributed to coining the name in 1968.
Golden Age of Black Hole Physics

- At roughly the same time that the theoretical nature of black holes was being uncovered, significant strides were being made in understanding the role of black holes in the universe.

- In 1971, Louise Webster and Paul Murdin, and Thomas Bolton independently discovered a massive hidden companion to a star associated with the x-ray source Cygnus X-1; the companion, with a mass about 15 solar masses can only be a black hole within conventional theory.

- In the early 60’s strong evidence was found that quasars, discovered a decade earlier, were at cosmological distances. This implied they had tremendous luminosities, and in 1969, before the term “black hole” had fully taken hold, Donald Lynden-Bell proposed that the gravitational potential energy of a “Schwarzschild throat” could explain quasars.
What is left to discover?

• In a sense, almost **everything**!

• Einstein’s theory is non-linear, and the field equations are impossible to solve analytically in generic scenarios
  
  – the general results using powerful global analysis methods (such as the singularity theorems) can’t give any details on what exactly happens
  
  – the strong-field properties of black holes were gleaned from stationary solutions or quasi-stationary scenarios
  
  – knowledge of dynamics, such as gravitational waves, came from perturbation theory

• Though strong circumstantial evidence that black holes exist in our universe, we have not yet “seen” them: can do so by observing the gravitational waves emitted when black holes collide
In a nutshell, gravitational waves are ripples, or distortions in the geometric structure of spacetime that travel at the speed of light.

In GR, matter/energy is responsible for producing curved-spacetime, so it is the motion of matter/energy that produces gravitational waves. In practice, it requires ultra-dense concentrations of energy moving at speeds close to the speed of light to produce “measurable” distortions of spacetime. Therefore, we need to look to astrophysical sources, as we can’t reproduce the above conditions in labs on earth. Collisions of black holes, neutron stars and white dwarfs, supernova explosions, events that may have occurred around the time of the Big Bang.
Weak field nature of gravitational waves

- Far from the source, the effect of a gravitational wave is to cause distortions in the geometry transverse to the direction of propagation.

- Two linearly independent polarizations (+ and x)
  - Schematic effect of a wave, traveling into the slide, on the distances between an initially circular ring of particles:
## The network of gravitational wave detectors

<table>
<thead>
<tr>
<th>LIGO/VIRGO/GEO/KAGRA</th>
<th>“LISA”</th>
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<tbody>
<tr>
<td>ground based laser interferometers (2015-18 for “advanced” LIGO)</td>
<td>space-based laser interferometer (future not certain at the moment … a few decades away?)</td>
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**LIGO Hanford**

**LIGO Livingston**

**“LISA”**

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<table>
<thead>
<tr>
<th>ALLEGRO/NAUTILUS/AURIGA/…</th>
<th>Pulsar timing network, CMB anisotropy</th>
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<tr>
<td>resonant bar detectors</td>
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**ALLEGRO**

**AURIGA**

**Segment of the CMB from WMAP**

**The Crab nebula ... a supernovae remnant harboring a pulsar**
Overview of expected gravitational wave sources

source frequency (Hz)

- CMB anisotropy
  - relic from the big bang, inflation

source "strength"

- Pulsar timing
  - >10^6 M_☉ BH/BH mergers
  - 10^2-10^6 M_☉ BH/BH mergers
  - EMR inspiral
  - NS binaries
  - WD binaries

- LISA
  - 1-10 M_☉ BH/BH mergers
  - NS/BH mergers
  - NS/NS mergers
  - pulsars, supernovae

- LIGO/...
  - Bar detectors

exotic physics in the early universe: phase transitions, cosmic strings, domain walls, …

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Anatomy of a Merger

• One can break down the evolution into 3 stages: inspiral, plunge/merger and ringdown

• **Inspiral $\Rightarrow$ quasi-circular inspiral (QSI)**
  
  – a binary system in general relativity is unstable: energy loss through gravitational wave emission causes the orbit to decay
  
  – if the initial pericenter of the orbit is sufficiently large, the orbit will lose its eccentricity long before merger and become quasi-circular
  
  – for equal mass, circular binaries the Keplerian orbital frequency offers a good approximation to the typical timescale until very close to merger

\[
\frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{M}{R^3}} \approx 11 \text{ kHz}\left(\frac{M_0}{M}\right)\left(\frac{R_0}{R}\right)^{3/2}
\]

• the dominant gravitational wave frequency is twice this
Anatomy of a Merger

• **Plunge/merger**

  – this is the time in the merger when the two event horizons coalesce into one

    • we know the two black holes must merge into one if cosmic censorship holds

  – full numerical solution of the field equations are required to solve for the geometry of spacetime in this stage

  – in all cases studied to date, this stage is exceedingly short, leaving its imprint in on the order of 1-2 gravitational wave cycles, at roughly twice the final orbital frequency
Anatomy of a Merger

• **Ringdown**

  – in the final phase of the merger, the remnant black hole “looses all its hair”, settling down to a Kerr black hole

  – one definition for when plunge/merger ends and ringdown begins, is when the spacetime can adequately be described as a Kerr black hole perturbed by a set of quasi-normal modes (QNM)

  – the ringdown portion of the waveform will be dominated by the fundamental harmonic of the quadrupole QNM, with characteristic frequency and decay time

\[
\frac{\omega_{QNM}}{2\pi} \approx 32 \text{ kHz} \left(\frac{M_{\alpha}}{M}\right) \quad \tau_{QNM} \approx 20 \mu s \left(\frac{M}{M_{\alpha}}\right)^2
\]

for Schwarzschild black holes
Example merger from numerical solution of the field equations: quasi-circular inspiral

This animation shows the lapse function in the orbital plane.

The lapse function represents the relative time dilation between a hypothetical observer at the given location on the grid, and an observer situated very far from the system --- the redder the color, the slower local clocks are running relative to clocks at infinity.

If this were in “real-time” it would correspond to the merger of two ~5000 solar mass black holes.

Initial black holes are close to non-spinning Schwarzschild black holes; final one is a Kerr black hole with spin parameter ~0.7, and ~4% of the total initial rest-mass of the system is emitted in gravitational waves.

Gravitational waves from the simulation

A depiction of the gravitational waves emitted in the orbital plane of the binary. Shown is the real component of the Newman Penrose scalar $\psi_4$, which in the wave zone is proportional to the second time derivative of the usual plus-polarization.

The plus-component of the wave from the same simulation, measured on the axis normal to the orbital plane.
What does the merger wave represent?

- Scale the system to two 10 solar mass (~ $2 \times 10^{31}$ kg) BHs
  - radius of each black hole in the binary is ~ 30km
  - radius of final black hole is ~ 60km
  - distance from the final black hole where the wave was measured ~ 1500km
  - frequency of the wave ~ 200Hz (early inspiral) - 800Hz (ring-down)
What does the merger wave represent?

- Fractional oscillatory “distortion” in space induced by the wave transverse to the direction of propagation has a maximum amplitude \( \Delta L/L \sim 3 \times 10^{-3} \)
  - a 2m tall person will get stretched/squeezed by \( \sim 6 \text{ mm} \) as the wave passes
  - LIGO’s arm length would change by \( \sim 12 \text{ m} \). Wave amplitude decays like \( 1/\text{distance} \) from source; e.g. at 10Mpc the change in arms \( \sim 5 \times 10^{-17} \text{ m} \) (1/20 the radius of a proton, which is well within the ballpark of what LIGO is trying to measure!)

- Despite the seemingly small amplitude for the wave, the energy it carries is enormous — around \( 10^{30} \text{ kg} \ c^2 \sim 10^{47} \text{ J} \sim 10^{54} \text{ ergs} \)
  - peak luminosity is about \( 1/100^{th} \) the Planck luminosity of \( 10^{59} \text{ ergs/s} \)
  - luminosity of the sun \( \sim 10^{33} \text{ ergs/s} \), a bright supernova or milky-way type galaxy \( \sim 10^{42} \text{ ergs/s} \)
  - if all the energy reaching LIGO from the 10Mpc event could directly be converted to sound waves, it would have an intensity level of \( \sim 80 \text{ dB} \)
Motivation: why study higher dimensional gravity?

- If string theory is providing the correct path to a consistent theory of nature valid at Planck scales, the universe is fundamentally higher dimensional.

- Even if string theory is not correct, there has recently been a lot of work using the holographic dual correspondences of string theory (AdS/CFT in particular) to describe many aspects of conventional non-gravitational 4D physical processes in terms of higher dimensional gravity:
  - superconductors, superfluidity, quark-gluon plasmas, etc.
  - interestingly, the gravitational dual to all the processes studied to date involves black holes.

- Much interesting geometry in higher dimensional Ricci-flat Lorentzian manifolds, in particular the zoo of “black objects” – black spheres, rings, strings, saturns, drops, ...
Higher dimensional black holes

- Higher dimensional black holes have many properties in common with their 4D counterparts, e.g.
  - have event horizons
  - contain geometric singularities
  - quasi-stationary processes are governed by the usual laws of black hole mechanics
  - can be formed by gravitational collapse of dense distributions of matter
  - Hawking radiate when quantum processes included

- However, a few properties are in general drastically different, including
  - no strict uniqueness of stationary solutions
  - many black objects are unstable to perturbations
Black Strings

- Black strings are a particularly simple class of higher dimensional black hole solutions
  - In 5 spacetime dimensions, the geometry is just the 4D dimensional Schwarzschild solution, with an extra, flat Euclidean direction $w$

- A $w =$-constant slice is exactly Schwarzschild, though the mass parameter $m$ is now interpreted as mass per unit length, so a segment of length $\Delta w = L$ of the spacetime has total mass $M = mL$

- In the 4D subspace, taking a planar slice through the center of the black hole (e.g. $\theta =$-constant) and then adding the dimension, the horizon looks like an infinite cylinder
Gregory-Laflamme instability

- Gregory and Laflamme [PRL 70 (1993)] first showed that black strings are linearly unstable to long-wavelength perturbations

\[ g = g_0 + \delta g e^{\Omega t/m + i\mu w/m} \]


- the D=4 curve corresponds to the 5D black string, and the critical wavelength above which modes are unstable is

\[ \lambda_c \approx 14.3 m \]
Based on an entropic argument, GL argued the black string would “pinch-off” into a sequence of spherical black holes. This cannot happen without the appearance of a naked singularity → a generic example of cosmic censorship violation.

However, Horowitz and Maeda [PRL 87, 131301 (2001)] proved that black string horizons cannot shrink to zero cross-sectional radius in finite affine time of horizon generators. “out” : could happen in infinite affine time, which may or may not correspond to infinite asymptotic time.

Conjectured the end-state would be a new, static, non-uniform solution with the same topology as the black string.

A few were subsequently found, though they had too high an entropy to be the end-state of the GL instability [S. S. Gubser, CQG. 19, 4825 (2002), T. Wiseman, CQG. 20, 1137 (2003), E. Sorkin, PRD74:104027].
Further (anecdotal) evidence in favor of the pinch-off scenario has gathered in the form of various correspondences between equations governing viscous hydrodynamics and black hole horizon dynamics:

- The membrane paradigm [Damour; Thorne, Price, Macdonald, Eds. (1986)] shows that the dynamics of a “stretched horizon” is governed by the Navier-Stokes equations for a relativistic fluid with shear-viscosity to entropy-density ratio $\eta/s = 1/4\pi$.

- Cardoso and Dias [PRL 96 (2006)] showed that the spectrum of unstable modes of a cylindrical flow of fluid with surface tension, subject to the Rayleigh-Plateau instability, was qualitatively similar to that of black strings.
The reason why this could be considered evidence for pinch-off is that unstable fluid streams generically break up.

The caveat with the fluid analogues is they’re either just that — analogies — or at a level of approximation of the field equations that are not applicable to the late-time, non-linear development of the instability.

- bottom line: both end-state possibilities remain, and one needs to solve the full field equations to discover the answer ... will show results from recent simulations.

[work with L. Lehner, PRL 105 (2010); building on earlier work of Choptuik et al. PRD 68, 044001 (2003)]
Embedding Diagram of Apparent Horizon Unstable 5D Black String

- map the geometric 1D shape of each \( t=x=y=\text{constant} \) slice of the apparent horizon to a flat \((R,Z)\) Euclidean space; i.e. in parametric form

\[
(R, Z) = (R(\xi), Z(\xi))
\]

- \( R(\xi) \) is the areal radius of that point on the horizon, and \( Z(\xi) \) is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry

- the movie shows this curve spun around \( R=0 \) to form a surface for visual aid

- color is mapped to \( R \)

- note that time is “slowing down”
Embedding Diagram of Apparent Horizon Unstable 5D Black String, close-up and in “real time”

- map the geometric 1D shape of each \( t=x=y=constant \) slice of the apparent horizon to a flat \((R,Z)\) Euclidean space; i.e. in parametric form

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- color is mapped to \( R \)
In the Rayleigh-Plateau hydrodynamic analogue, a self-similar cascade can also occur

- the lower the viscosity of the fluid, the more generations of self-similar behavior are observed before break-up; it is unknown at present whether below some critical viscosity the behavior continues indefinitely

- the membrane paradigm suggests black holes have much lower viscosity that any “real-world” fluid

An exact scaling solution of the Navier Stokes equation [Eggers, PRL 71 (1993); Miyamoto, JHEP 1010 (2010)] near pinch-off is known, giving

\[ r \propto (t_0 - t) \]

and this is consistent with the numerical results of Gregory-Laflamme

Image from Review article by Eggers [Rev.Mod.Phys 59 (1997)], from work of Tjahjadi, Stone, and Ottino, [Fluid Mech. 243 (1992)]
Black hole formation at the LHC and in the atmosphere?

- In certain extra dimension scenarios [Arkani-Hamed, S. Dimopoulos & G.R. Dvali; Randall & Sundrum] the true Planck scale can be very different from what is then only an effective 4-dimensional Planck scale of $10^{19}$ GeV.

- A TeV Planck scale is a “natural” choice to solve the hierarchy problem.

- The LHC will probe energies to ~14 TeV; much higher energies can occur in collisions of cosmic rays with the Earth.

- Collisions sufficiently above the Planck scale are expected to be dominated by the gravitational interaction, and arguments suggest the generic outcome would be black hole formation.

- These black holes will be small and decay rapidly via Hawking radiation, which is the most promising route to detection.
But do super-Planck scale particle collisions

- The argument that the ultra-relativistic collision of two particles should form a black hole is purely classical, and is essentially based on Thorne’s hoop conjecture
  
  - (4D) if an amount of matter/energy $E$ is compacted to within a sphere of radius $R=2GE/c^4$ corresponding to the Schwarzschild radius of a black hole of mass $M=E/c^2$, a black hole will form

  - applied to the head-on collision of two “particles” each with rest mass $m$, characteristic size $W$, and center-of-mass frame Lorentz gamma factors $\gamma$, this says a black hole will form if the Schwarzschild radius corresponding to the total energy $E=2mc^2\gamma$ is greater than $W$

  - the quantum physics comes in when we say that the particle’s size is given by its de Broglie wavelength $W = hc/E$, from which one gets the Planck energy $E_p=(hc^5/G)^{1/2}$
Hoop Conjecture and Particle Collisions

\[ W = \frac{2G\gamma m/c^2}{c} \]

\[ \gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 1 \]
Hoop Conjecture and Particle Collisions

\[ \gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 2 \]
Hoop Conjecture and Particle Collisions

\[ \gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 5 \]
Hoop Conjecture and Particle Collisions

\[ \gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 10 \]

Black hole forms!
Evidence to support this

- From the classical perspective, evidence to support this would be solutions to the field equations demonstrating that weakly self-gravitating objects, when boosted toward each other with large velocities, generically form a black hole when the interaction occurs within a region smaller than the Schwarzschild radius.

  - **generic**: the outcome would have to be independent of the particular details of the structure and non-gravitational interactions between the particles, if the classical picture is to have any bearing on the problem.

  - **interaction**: the non-linear interaction of the gravitational kinetic energy of the boosted particles will be key in determining what happens.

  - consider the trivial counter-examples to the hoop conjecture applied to a single particle boosted to ultra-relativistic velocities, or a white hole “explosion”.

  - until recently the only evidence to support this has been in the infinite boost limit of colliding plane-fronted gravitations waves (Penrose ‘74).
High speed soliton collision simulations

• Test this hypothesis by colliding self-gravitating solitons, boson stars in this case (M.W. Choptuik & FP, PRL 104, (2010))

• Very computationally expensive to run high-$\gamma$ simulations, so need to start with a relatively compact boson star that will reach hoop-conjecture limits with reasonable $\gamma$’s.

• choose parameters to give a boson star with $R/2M \sim 22$
  
  – thus, hoop-conjecture suggests a collision of two of these with $\gamma=11$ in the center of mass frame will be the marginal case
Case 1: free-fall collision from rest

- Here, gravity dominates the interaction, causing the boson stars to coalesce into a single, highly perturbed boson star (this case eventually collapses to form a black hole).

Symmetry axis

Both the color and height of the surface represent the magnitude of the scalar field. Scale M is the total rest-mass of the boson stars.
Case 2: $\gamma = 2$

- Here, though gravity strongly perturbs the boson stars, kinetic energy "wins" and causes them to pass through each other
  - soliton-like interference pattern can be seen as the boson star matter interacts
Case 3: $\gamma = 4$

- Here, the early matter interaction looks similar, but now the gravitational interaction of the kinetic energy of the solitons causes gravitational collapse and black hole formation.
  - Note: different coordinate system than previous case: the apparent spreading of the solitons before collision, and shrinking of the horizon afterwards, are just coordinate effects.
Conclusions

- Black holes are one of the more startling and remarkable predictions of general relativity.

- Anticipated gravitational wave observations are expected to be a driving force providing new understanding of the universe over the next couple of decades; compact object mergers involving black holes are among the more promising sources.

- Aside from their obvious role in gravitational physics, ongoing research over the past few decades have revealed they “know” much more about many phenomena in physics than one could have naively anticipated.

  - whether this is telling us something profound about nature, or “merely” that the phenomenology of null surfaces in Einstein geometries is so rich one can map many physical processes to horizon dynamics, remains to be discovered.