

## April 2010 – Entrance Examination: Physics and Chemistry of Biological Systems

Solve **one** of the following problems (no extra credit is given for attempts to solve more than one problem). Write out solutions clearly and concisely. State each approximation used. Diagrams welcome. Number page, problem, and question clearly. All essays/solutions should be written in English. Do not write your name on the problem sheet, but use extra envelope.

### Problem n. 1 – Interconverting molecules

Consider an ensemble of identical, non-interacting molecules, each of them with three available internal states  $A$ ,  $B$  and  $C$ . The total number of molecules ( $N = N_A + N_B + N_C$ ) is kept constant and satisfies  $N \gg 1$ . The transition rate for the process  $A \rightarrow B$  is  $k_{BA}$ , whereas the transition rate for the process  $B \rightarrow C$  is  $k_{CB}$ . All the other rates are negligible.

At time  $t = 0$  all the  $N$  molecules in the ensemble are in state  $A$ .

1. Calculate  $N_A$  as a function of time.
2. Calculate  $N_A$ ,  $N_B$  and  $N_C$  in the long time limit.
3. Sketch the time dependence of  $N_A$ ,  $N_B$  and  $N_C$ .
4. Calculate the explicit expressions for  $N_B$  and  $N_C$  as a function of time.
5. Now assume that  $k_{BA} = k_{CB} \equiv k$  and that, due to the action of an unspecified external force, state  $C$  can convert to state  $A$  with rate  $k_{AC} = k$ . Calculate  $N_A$ ,  $N_B$  and  $N_C$  in the long time limit, and estimate the typical time required to reach the steady state from an arbitrary initial condition.

## Problem 11. 2 – Hückel MO Theory

Using Hückel MO theory a molecule can be thought of as a set of 6 sites on each of which it is placed a C atom. Each C atom shares one electron in a delocalized  $p_z$  orbital. Considering the benzene and hexatriene molecules:

1. calculate the  $\pi$ -electron energies
2. draw an energy diagram for the electronic states.
3. indicate which orbitals are occupied
4. draw a picture of the molecular orbitals.
5. calculate the delocalization energy per  $\pi$  electron.
6. determine which 6  $\pi$  electron system is more stable